Scaling Laws at Large Transverse Momentum*

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The application of simple dimensional counting to bound states of pointlike particles enables us to derive scaling laws for the asymptotic energy dependence of electromagnetic and hadronic scattering at fixed c.m. angle which only depend on the number of constituent fields of the hadrons. Assuming quark constituents, some of the $s\to\infty$, fixed-t/s predictions are $(d\sigma/dt)_{\pi p\to\pi p}\sim s^{-8}$, $(d\sigma/dt)_{pp\to pp}\sim s^{-10}$, $(d\sigma/dt)_{\gamma p\to\pi p}\sim s^{-7}$, $(d\sigma/dt)_{\gamma p\to\gamma p}\sim s^{-6}$, $F_\pi(q^2)\sim(q^2)^{-1}$, and $F_{1p}(q^2)\sim(q^2)^{-2}$. We show that such scaling laws are characteristic of renormalizable field theories satisfying certain conditions.

Nature has presented us with a tantalizing glimpse of the hadrons. On one hand, they are evidently composite since the meson and nucleon form factors fall with increasing momentum transfer. On the other hand, the carriers of the currents within the hadron seem to be structureless, as indicated by the apparent scaling behavior of the deep-inelastic structure functions. In this note we shall show that asymptotic properties of electromagnetic form factors and large-angle exclusive and inclusive scattering amplitudes can also be understood in terms of renormalizable field theories with certain conditions, namely, asymptotically scale-invariant interactions among the constituents, and hadronic wave functions which are finite at the origin, as discussed below. Accordingly, the application of dimensional counting to the minimum quark-field components of a hadron will be shown to account for many of the experimental consequences of its compositeness.

Our central result for exclusive scattering[1] is

$$(d\sigma/dt)_{AB\to CD}\sim s^{2-n}f(t/s) \tag{1}$$

($s\to\infty$, t/s fixed). Here n is the total number of leptons, photons, and quark components (i.e., elementary fields) of the initial and final states. This result follows heuristically if the only physical dimensional quantities are particle masses and momenta. We begin by considering a world in which a hadron would become a collection of free quarks with equal momenta if the strong interactions were turned off. Note that the dimension[2] of the connected invariant amplitude $M_{n_i\to n_f}$ with n_i+n_f external lines is $(\text{length})^{n_i+n_f-4}$. If all the invariants are large relative to the masses and are proportional to s, then $M_{n_i\to n_f}\sim s^{-(n_i+n_f)/2+2}$. If we now assume that the introduction of binding between the quarks of the hadrons does not modify this basic scaling result (i.e., no fundamental scale is introduced), then Eq. (1) follows, using $d\sigma/dt\sim s^{-2}|M|^2$. Multiparticle production is described by the obvious generalization of Eq. (1), when all invariants are large: We obtain

$$\Delta\sigma\sim s^{-1-N_M-2N_B} \text{ at } s\to\infty,$$

for the exclusive cross section integrated over a (c.m. angular) region where the $p_i\cdot p_j/s$ are held fixed and N_M and N_B are the total number of external mesons and baryons in the process.

The validity of this heuristic argument for Eq. (1) can be examined in renormalizable field theories. First construct the minimum connected (Born) diagrams for the amplitude $M_{n_i\to n_f}$ with the quarks of each hadron carrying a finite fraction of the total momentum and having the appropriate total spin. Typical graphs are shown in Figs. 1(a)–1(c). In these simple graphs, the mass terms play no role in the asymptotic fixed-angle region and the total Born amplitude scales with s as argued above, since the coupling constant is dimensionless.[3] The essential assumption required to justify Eq. (1) is that the scaling behavior of the physical scattering amplitude (when $s\to\infty$, t/s fixed) is the same as the scaling behavior for the free-quark amplitude in Born approximation. There are two classes of higher-order diagrams which give the full hadronic amplitude[4] in terms of quark scattering and which could invalidate our result.

(i) The irreducible diagrams [see Fig. 1(d)]: These are the loop corrections to the Born amplitude $M_{n_i\to n_f}$ which involve interactions between

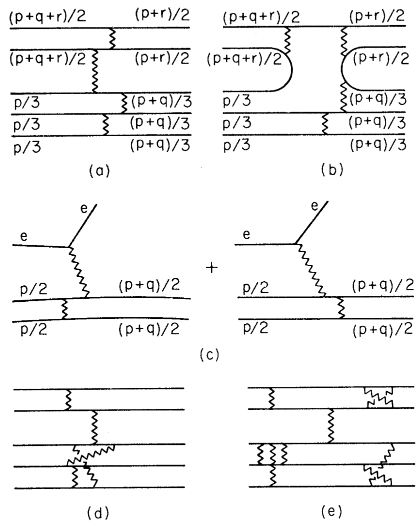

FIG. 1. Typical Born diagrams for large-momentum-transfer elastic scattering in the quark picture. (a) $\pi p \to \pi p$ (quark scattering), (b) $\pi p \to \pi p$ (quark interchange), (c) $e\pi \to e\pi$, (d) an irreducible loop diagram, (e) a reducible loop diagram.

the quarks of different hadrons and are not two- (or three-) particle reducible in the meson (or baryon) legs. We will assume that the *sum* of all irreducible corrections to the n-particle scattering amplitude has the same scaling behavior, up to a finite number of logarithms, as the Born amplitude. The scaling of deep-inelastic scattering depends on the behavior of diagrams of this type, so if anomalous dimensions or an infinite number of logarithms turn out to be necessary to understand deep-inelastic scattering, they will probably be necessary here.

(ii) The reducible diagrams [see Fig. 1(e)]: These are higher-order diagrams involving interaction between the quarks of the same hadron, and they are automatically summed if the total irreducible amplitude is convoluted with the full meson and baryon Bethe-Salpeter wave functions. If the wave functions are finite for zero quark separation, i.e.,

$$\int d^4k\, \psi_M(k) = \psi_M(x=0) < \infty \tag{2}$$

and similarly for baryons (integrating over both relative momenta), then the full hadronic amplitude has the same scaling behavior in s as the irreducible n-point amplitude. Although Eq. (2) has not been proved in renormalized field theory, it is very reasonable physically. It requires that the effective Bethe-Salpeter kernel is slightly less singular at short distances than indicated by perturbation theory.[5] Breaking of (2) by a finite number of logarithms modifies Eq. (1) by a finite number of logarithms. The values of the $\psi(x_i = 0)$ determine the absolute normalization of the asymptotic cross sections.

Some experimental consequences of Eq. (1) for specific high-energy processes at fixed t/s, using minimal quark representations for mesons and baryons, are (a) $d\sigma/dt \sim s^{-8}$ for meson-baryon scattering ($\pi p \to \pi p$, $Kp \to \pi\Sigma$, etc.), (b) $d\sigma/dt \sim s^{-10}$ for baryon-baryon scattering ($pp \to pp$, $pp \to pN^*$, etc.), (c) $d\sigma/dt \sim s^{-7}$ for meson photoproduction ($\gamma p \to \pi p$, $\gamma p \to \rho p$, etc.), (d) $d\sigma/dt \sim s^{-6}$ for Compton scattering ($\gamma p \to \gamma p$). All these predictions should hold when s and t are much larger than the masses of the particles involved. The results (a) and (c) are in excellent agreement with recent experiments.[6] For pp elastic scattering the fitted power-law exponent is between -10 and -12.[7] The result (d) is in agreement with dominance of a $J=0$ fixed pole in the Compton amplitude with form-factor residue.[8]

In the case of electron scattering and e^+e^- annihilation, we shall define the effective electromagnetic elastic (or transition) form factors by (at fixed t/s)

$$d\sigma/dt \propto t^{-2} |F_{HH'}(t)|^2.$$

Then, from (1), we predict for large t $|F_{HH'}(t)| \sim t^{1-n_H}$, where n_H is the number of quark fields in hadron H. This holds for multiple as well as single photon exchange. In renormalizable spin-$\frac{1}{2}$ theories, $F_{2p} \sim t^{-3}$ and thus we obtain G_E/G_M asymptotic scaling for the nucleon, consistent with experiment.[9] The meson results are in agreement with recent Frascati data[10] for $e^+e^- \to \pi^+\pi^-, K^+K^-$. The results for the transition form factors agree with Bloom-Gilman scaling.[11] We predict the asymptotic power t^{-5} for the spin-averaged deuteron form factor.

A specific model which gives many of the above predictions for hadron scattering is the interchange model.[12] Although the electromagnetic form factors of the hadrons involved must be input from experiment, the interchange model predicts the detailed angular dependence of the differential cross section, $f(t/s)$, as well as its falloff in s. The results are consistent with (1) for processes involving photons and mesons. However, because the calculations of Ref. 12 treat all hadrons as a two-component "parton-core" system, those results differ from the approach here for processes involving only baryons. Namely, we have $(d\sigma/dt)_{pp\to pp} \sim s^{-10}$, where-as the interchange prediction is $(d\sigma/dt)_{pp\to pp} \sim s^{-12}$. Thus it is particularly useful to do this experiment carefully to see if the proton is better described at short distances as a parton plus core or as three quarks.

Turning now to scaling laws for inclusive processes at large transverse momentum and missing mass, we see that we cannot proceed without additional assumptions. In contrast to exclusive scattering, the specification of the observed particle momenta in an inclusive process (away from the exclusive edge of phase space) does not uniquely determine the number of elementary fields which interact at short distances since not all the invariants (of unobserved particles) need be large. However, in the absence of the accumulation of logarithms, i.e., the validity of the dimensional scaling arguments, the most important mechanism for producing high-$p_\perp$ inclusive events involves the large-s and -t scattering of a subset n of the constituents (giving $\mathfrak{M} \sim s^{2-n/2}$), plus many scale-invariant low-momentum-transfer scatterings of spectators. Models based on renormalizable field theory, however, do fall into two distinct categories with respect to the minimum numbers of interacting fields and hence their asymptotic behavior (see Fig. 2).

(I) *Quark-quark scattering.*—If a direct, hard scattering of quarks which are constituents of different hadrons is allowed, then all inclusive reactions are asymptotically scale invariant:

$$E\, d\sigma/d^3p \sim s^{-N} f(t/s, M^2/s), \tag{3}$$

with $N = n-2 = 2$ for $s\to\infty$, when t/s and M^2/s are fixed. In such models[13] a quark from A and one from B scatter to large angles "for free" and then, by (scale invariant) fragmentation, produce the observed large transverse momentum particle C. Experimental evidence already excludes Eq. (3) with $N=2$ for $pp\to\pi^0X$,[14] so that we should take seriously the other category of scaleless models.

(II) *Quark-hadron scattering.*—If elementary pointlike or gluon interactions between quarks of *different* hadrons are suppressed,[12] either by a selection rule[15] or dynamically, then the number of elementary fields which are required to receive large momentum transfers will depend on the nature of the particles involved. For example, in pp (or πp) $\to \pi X$, the minimal large-s and -t exclusive scattering subprocess, assuming quark-partons, would be $q\overline{q}\to\pi\pi$, $q\pi\to q\pi$, or possibly $qq\to\pi qq$, giving $n=6$ and $N=4$ in Eq. (3) as predicted by the interchange model.[16] This result for $pp\to\pi^0X$ is in agreement with recent data.[14] Similarly, for pp (or πp) $\to pX$, the minimal exclusive scattering process could be $qq\to p\overline{q}$,[17] again giving $N=4$. However, if, as in the interchange model of Ref. 16, the subprocess $qq\to p\overline{q}$ were suppressed, then one would consider the subprocesses, $qp\to qp$, qp, $q\pi\to p\overline{q}\overline{q}$, or $q\overline{q}\to p\overline{p}$, giving $n=8$, $N=6$. For $\gamma p\to\pi X$ we find $N=3$, based on the minimal process $\gamma q\to\pi q$. Deep inelastic processes $ep\to eX$, $\gamma p\to\gamma X$, and $pp\to\mu X$ are scale invariant since $n=4$.

Near the exclusive edge of phase space the results for inclusive (M^2 large) scattering must match smoothly onto the corresponding exclusive (M^2 small) formula.[11] Comparing Eqs. (1) and (3), this implies that for small M^2, $f(t/s, M^2/s) = (M^2/s)^p f(t/s)$, with $p = n-N-3$.

If class-II theories are correct, the absence of hard quark-quark collisions between hadrons has striking implications for the t/s dependence of exclusive processes. Let us consider the region $s \gg |t|$ with $|t|$ large and parametrize $|M(AB\to A'B')| \sim s^{\alpha(t)}\beta(t)$, or for exotic channels, $\sim u^{\alpha(t)}\times\beta(t)$. As shown in Ref. 12, in the absence of scalar or vector gluon exchange between hadrons, $\beta(t)\sim F_{BB'}(t)$ [or $F_{AA'}(t)$, whichever falls faster]. Then from Eq. (1) we find that the leading effective trajectory for very large $|t|$ is $\alpha(t) = 1-\frac{1}{2}\times(n_A+n_{A'})$. These results, $\alpha_{\pi p\to\pi p}\to -1$, $\alpha_{pp\to pp}\to -2$, and $\alpha_{\gamma p\to\gamma p}\to 0$, are consistent with experiment.[18,19]

The approach presented here[20] is quite remarkable for so simply explaining such a large number of observations. We have shown that experiment justifies the application of parton scaling ideas in a larger regime than previously thought. Even when the parton which receives a large momentum transfer remains bound within a hadron, the system behaves like a collection of free quarks. It is an interesting possibility that the results given here can be obtained by applying the technique of operator-product expansions on the light cone and at short distances to bound-state prob-

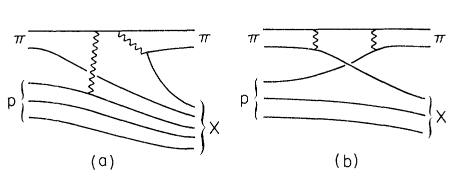

FIG. 2. Typical diagrams for $\pi p\to\pi X$ inclusive scattering at large momentum transfer and large missing mass. (a) Quark scattering, (b) quark interchange. Interactions among the spectator quarks are not shown.

lems.

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[1]This result for elastic scattering has been obtained independently by V. Matveev, R. Muradyan, and A. Tavkhelidze, Joint Institute for Nuclear Research Report No. D2-7110, 1973 (to be published). We thank J. Kiskis for bringing this work to our attention.

[2]Spinors are normalized to $u^\dagger u = 2E$.

[3]It is in general crucial to include *all* Born diagrams since (e.g., in renormalizable Yang-Mills theories) canonical scaling is true only for the total Born amplitude and not necessarily for each diagram separately.

[4]A complete discussion of scattering amplitudes in the Bethe-Salpeter formalism is given by S. Mandelstam, Proc. Roy. Soc., Ser A 233, 248 (1953).

[5]An interesting possibility is that such a condition may be satisfied in non-Abelian gauge theories. See also S. D. Drell and T. D. Lee, Phys. Rev. D 5, 1738 (1972) and references therein for discussion of the validity of Eq. (2).

[6]If the data are parametrized at fixed c.m. angle as $d\sigma/dt = A/s^n$, then R. Anderson *et al.*, SLAC Report No. SLAC-PUB-1178 (unpublished), fit $n(\gamma N \to \pi^+ N) = 7.3 \pm 0.4$; G. Brandenberg *et al.*, SLAC Report No. SLAC-PUB-1203 (unpublished), fit $n(K_L{}^0 p \to K_S{}^0 p) = 8.5 \pm 1.4$, $n(\bar{K}^0 p \to \pi^+ \Lambda^0) = 7.4 \pm 1.4$ and $n(\bar{K}^0 p \to \pi^+ \Sigma^0) = 8.1 \pm 1.4$.

[7]For references, see R. Blankenbecler, S. Brodsky, and J. Gunion, Phys. Lett. 39B, 649 (1972), and Phys. Rev. D 8, 187 (1973).

[8]S. Brodsky, F. Close, and J. Gunion, Phys. Rev. D 6, 177 (1972).

[9]P. Kirk *et al.*, Phys. Rev. D 8, 63 (1973). Although the data are not consistent with a dipole fit over the entire q^2 range, they are consistent with our prediction of a pure power, $(q^2)^{-2}$, when $|q^2| \gtrsim 3$ GeV^2.

[10]See N. Silvestrini, in *Proceedings of the Sixteenth International Conference on High Energy Physics, The University of Chicago and National Accelerator Laboratory, 1972,* edited by J. D. Jackson and A. Roberts (National Accelerator Laboratory, Batavia, Ill., 1973), Vol. 4. A fit to the combined π, K points gives $|F(t)| = A/t^n$ with $n = 1.08 \pm 0.15$.

[11]E. Bloom and F. Gilman, Phys. Rev. Lett. 25, 1140 (1970); J. Bjorken and J. Kogut, SLAC Report No. SLAC-PUB-1213 (unpublished).

[12]Blankenbecler, Brodsky, and Gunion, Ref. 7. See also P. V. Landshoff and J. C. Polkinghorne, Phys. Rev. D 8, 927 (1973).

[13]Examples are S. Berman, J. Bjorken, and J. Kogut, Phys. Rev. D 4, 3388 (1971); S. Berman and M. Jacob, Phys. Rev. Lett. 25, 1683 (1970); D. Horn and F. Moshe, Nucl. Phys. 48B, 557 (1972). This mechanism will always occur on the electromagnetic level.

[14]B. Blumenfeld *et al.*, to be published.

[15]This would happen in a model with fictitious colored quarks and colored gluons as discussed by H. Fritzsch and M. Gell-Mann, in *Proceedings of the Sixteenth International Conference on High Energy Physics, The University of Chicago and National Accelerator Laboratory, 1972,* edited by J. D. Jackson and A. Roberts (National Accelerator Laboratory, Batavia, Ill., 1973), Vol. 2, and could happen in a model with real colored quarks and gluons as discussed by Y. Nambu, in *Preludes in Theoretical Physics,* edited by A. de-Shalit, H. Feshbach, and L. Van Hove (North-Holland, Amsterdam, 1966), and recently advocated by H. Lipkin, Weizmann Institute of Science Report No. WIS 73/13, 1973 (to be published).

[16]These are the inclusive process predictions of the interchange model. See R. Blankenbecler, S. Brodsky, and J. Gunion, Phys. Rev. D 6, 2652 (1972), and Phys. Lett. 42B, 461 (1972).

[17]We thank J. D. Bjorken for suggesting this possibility.

[18]Except for the value for α_{pp}, which depends on the details of the proton wave function, these results are consistent with those of Ref. 12.

[19]D. Coon, J. Tran Thanh Van, J. Gunion, and R. Blankenbecler, to be published.

[20]A detailed report of this work is in preparation.