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PM = EM? Partially massless duality invariance

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We show that higher spin, maximal depth, partially massless systems defined in $d = 4$ de Sitter space enjoy Maxwellian electric-magnetic duality. These photonlike models can also couple to charged matter.

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I. INTRODUCTION

The notion of duality invariance,

$$\vec{E} \to \vec{B}, \qquad \vec{B} \to -\vec{E},$$

is almost coeval with Maxwell's equations themselves, although a proof of its validity awaited over a century [1,2]. In addition to countless generalizations of "duality" in field and string theory, it has led to an enormous variety of more precise analogs, in particular to spin 2 [2,3], then to all free massless (integer or half-integer) spin $s > 0$ systems in flat space [4,5].

In de Sitter (dS) space, electromagnetic (EM) interactions can be mediated by generalized Maxwell systems [9]. These are the maximal depth partially massless (PM) fields of Refs. [10,11] which enjoy many characteristics of EM such as lightlike propagation [12], gauge invariance [11], conformal invariance [13] and stability [14]. Unlike EM, these particular PM models describe higher spin s propagating helicities $\pm s, \ldots, \pm 1$ [11]. Here we show that they are invariant under Maxwellian duality rotations, whence our title.

II. DS MAXWELL DUALITY

For Maxwell systems, duality in dS (or even generally curved) backgrounds [15] is formally obvious in a covariant form notation, $F \to \star F$; it just interchanges the Maxwell equations with the Bianchi identity,

$$\delta F = 0 = dF,$$

where $\delta = \star d \star$. To establish the symmetry correctly, that is within the action principle in dS, we could simply appeal to the conformal invariance of the four dimensional Maxwell theory in order to employ the flat space proof of Refs. [1,2]. For our purposes, an explicit dS proof is needed for our generalization to PM: in the particular dS coordinate frame

$$ds^2 = -dt^2 + \exp(2\sqrt{\Lambda/3}\, t)\, d\vec{x}^2, \tag{1}$$

the first order Maxwell action just reduces to

$$S[E, A; \Lambda] = \int d^4x \Big[E\dot{A} - \frac{1}{2} e^{-\sqrt{\Lambda/3}\, t} \{E^2 + B(A)^2\} \Big]. \tag{2}$$

Here E and A are transverse (i.e., $A_i = A_i^T$, $\partial_i A_i^T = 0$) by virtue of the Gauß constraint; the labels "^T_i" are omitted throughout. Also $B(A)$ denotes $\vec{\nabla} \times A$; for transverse V we have $B(\vec{\nabla} \times V) = -\Delta V$.

Duality invariance is now exhibited as being a canonical transformation interchanging (with a helicity twist) the conjugate (E, A) pair, while leaving $S(E, A; \Lambda)$ unchanged, but rotating (E, B). Infinitesimally,

$$\delta E = B(A), \quad \delta A = \vec{\nabla} \times (\Delta^{-1} E) \Rightarrow \delta B(A) = -E. \tag{3}$$

(Spatial nonlocality is of course allowed.) Invariance of (2) under (3) is nearly manifest: The kinetic integrands' variations are easily seen to be total derivatives of the form $V \cdot \vec{\nabla} \times \dot{V}$ (basically $F \star F$, in covariant language) up to possible but harmless Coulomb $\sim |r - r'|$ factors, while the Hamiltonian's $\{E^2 + B^2\}$ is the very embodiment of rotation invariance.

III. PM SYSTEMS

PM systems originate from free mass m fields propagating in dS backgrounds ($\Lambda > 0$) for which special $m:\Lambda$ tunings yield additional gauge invariance(s) [10,11], thereby eliminating one or more lower helicity components from the unavoidable flat space $(2s + 1)$ total. For maximal depth PM systems, the helicity zero excitation is thereby removed, leaving only helicity $\pm(s, \ldots, 1)$ modes. We simply write the final form of their (gauge-invariant) actions when all constraints are solved. (This is the critical

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step that requires sourceless fields: in the original Maxwell example, $\vec{B}$ is identically transverse ($\vec{\nabla}\cdot\vec{B}=0$), so the duality rotation is only well defined [let alone an invariance] when the electric field is likewise transverse, with vanishing longitudinal-Coulomb component.) It is also why zero mass is required in flat space.) Reduced PM actions in terms of transverse-traceless (TT) tensors were first given in Eq. (29) of Ref. [14] for PM spin 2 or Eq. (24) of Ref. [13] for arbitrary s maximal depth PM fields,

$$S=\sum_{\varepsilon=1}^{s} S[\pi^{\mathrm{TT}}_{i_1\ldots i_\varepsilon},\varphi^{\mathrm{TT}}_{i_1\ldots i_\varepsilon};\Lambda],\qquad S[p,q;\Lambda]:=\int d^4x\Big[p\dot{q}-\frac{1}{2}\Big\{p^2+e^{-2\sqrt{\Lambda/3}\,t}B(q)^2-\frac{\Lambda}{12}q^2\Big\}\Big]. \tag{4}$$

Here the ε sum runs over the helicities $1,\ldots,s$ of a (maximal depth) partially massless field and thus avoids the dangerous helicity zero mode. All indices are suitably contracted in each helicity's action and $B(q):=\vec{\nabla}\times q$ denotes the "magnetic" field, namely the symmetrized curl [4,18]

$$\vec{\nabla}\times\varphi^{\mathrm{TT}}_{i_1\ldots i_\varepsilon}:=\epsilon_{(i_1|kl}\partial_k\varphi^{\mathrm{TT}}_{l|i_2\ldots i_\varepsilon)}. \tag{5}$$

Note that (only) in dimension $d=3+1$ do the tensor ranks of each $B(\varphi)$ still match those of their potentials, and so of their corresponding "electric" companions π. In what follows, the (easily verified) identity for transverse-traceless tensors

$$B(\vec{\nabla}\times q)=-\Delta q$$

will play an essential role.

In the dS coordinates (1) used in Refs. [13,14], the only metric dependence of the action (4) is through Λ. Note also that, as shown in Ref. [14], although the Hamiltonian in (4) is neither time independent nor manifestly positive, the generator of time translations constructed from the composition $\xi^\mu T_{\mu\nu}$ of the timelike dS Killing vector ξ^μ and the stress energy tensor $T_{\mu\nu}$ [19] is both conserved and positive within the intrinsic horizon.

IV. PM DUALITY

We now generalize the above scheme to PM. The essential point is that the duality rotations occur separately within each helicity sector. The key maneuver, therefore, is to bring the action $S[p,q;\Lambda]$ displayed in (4) to the manifestly duality invariant form $S[E,A;\Lambda]$ of (2). This is achieved via the field redefinition

$$E:=e^{\frac{1}{2}\sqrt{\Lambda/3}\,t}\Big\{p-\frac{\sqrt{\Lambda/3}}{2}q\Big\},\qquad A:=e^{-\frac{1}{2}\sqrt{\Lambda/3}\,t}q.$$

The proof that duality invariance is a canonical transformation is now identical to that of the dS Maxwell theory given above, save that the vector curl is replaced by its higher rank symmetrized counterpart (5). Thus PM duality rotation invariance is, like Maxwell's, traceable to conformal invariance. [For the nonconformal, nonmaximal depth models the above field redefinition produces the Maxwell form (4) with a Hamiltonian modified by a time dependent mass term $\sim e^{-2\sqrt{\Lambda/3}\,t}A^2$, which invalidates the duality symmetry under (3).]

V. SUMMARY

We have explicitly established the (extended) duality invariance of maximal depth PM systems. These models live in dS and share many EM features there. Hence, despite their unusual coupling to charges [9], one may speculate on their possible cosmological relevance [20]. If present, the PM field's radiative interactions could have observable consequences in the relevant era, possibly even requiring rethinking of current cosmological scenarios.

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