Macroscopic Quantum Mechanics in a Classical Spacetime

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We apply the many-particle Schrödinger-Newton equation, which describes the co-evolution of an many-particle *quantum* wave function and a *classical* space-time geometry, to macroscopic mechanical objects. By averaging over motions of the objects' internal degrees of freedom, we obtain an effective Schrödinger-Newton equation for their centers of mass, which are degrees of freedom that can be monitored and manipulated at the quantum mechanical levels by state-of-the-art optoemchanics experiments. For a single macroscopic object moving quantum mechanically within a harmonic potential well, we found that its quantum uncertainty evolves in a different frequency from its classical eigenfrequency — with a difference that depends on the internal structure of the object, and can be observable using current technology. For several objects, the Schrödinger-Newton equation predicts semiclassical motions just like Newtonian physics, yet they do not allow quantum uncertainty to be transferred from one object to another through gravity.

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Introduction.— Prescribing and testing modifications to non-relativistic macroscopic quantum mechanics due to self gravity has been one, although minor, approach towards the exploration of nontrivial effects of quantum gravity. Apart from the standard formulation of linearized quantum gravity [1], which seems rather implausible to test in the lab, several types of schemes have been proposed [2–12]. The first type is *gravity decoherence* [2–6, 13–15], where the existence of gravity is conjectured to introduce decoherence to macroscopic quantum superpositions. The second type modifies canonical quantization motivated by the existence of a minimum length scale [7–9]. A third type, which will be the subject of this paper, is often referred to as *semiclassical gravity* [10–12]. As originally suggested by Moller [10] and Rosenfeld [11], spacetime structure might *still remain classical* even if it is sourced by matters which have quantized motion. The idea is to impose the following condition ($G = c = 1$):

$$G_{\mu\nu} = 8\pi\langle\psi|\hat{T}_{\mu\nu}|\psi\rangle\,, \tag{1}$$

where $G_{\mu\nu}$ is the Einstein tensor of the (3+1)-D classical spacetime, $\hat{T}_{\mu\nu}$ is an operator for the energy-stress tensor, and $|\psi(t)\rangle$ is the wave function of all matter in the universe, which evolves within this classical spacetime.

Many arguments exist *against* semiclassical gravity. Some rely on the conviction that a classical system (gravity) cannot properly interact with a quantum system (electroweak plus strong interactions) without creating contradictions. Others are based on "instrinsic" mathematical inconsistencies, the most famous one between Eq. (1), state collapse, and the Bianchi Identity [16]. Towards the former type of argument, it is exactly the aim of this paper to work out the effects of gravity being classical on the quantum mechanics of macroscopic objects. Although we will find them counter intuitive, they do not seem to us as completely dismissible right away. In fact, we shall find these effects "right on the horizon of testability" by current experimental technology. Towards the latter type of arguments, we shall remain open minded regarding the possibility of getting rid of state reduction while at the same time avoiding the many-world interpretation [17].

We will consider the non-relativistic version of Eq. (1), the so-called Schrödinger-Newton (SN) equation, for macroscopic objects each consisting of many particles, and show that within certain experimental parameter regimes, the center-of-mass (CM) wavefunction approximately satisfies the following nonlinear Schrödinger-Newton equation (with $\hbar = 1$):

$$i\frac{\partial\Psi}{\partial t} = \left[-\frac{\nabla^2}{2M} + \frac{1}{2}M\omega_c^2 x^2 + \frac{1}{2}C(x - \langle x\rangle)^2\right]\Psi\,. \tag{2}$$

Here $\langle x\rangle \equiv \langle\Psi|\hat{x}|\Psi\rangle$ is the expectation value of CM position; ω_c is the eigenfrequency of CM motion in absence of self gravity; we shall refer to C as the SN coupling constant, and $\omega_{SN} \equiv \sqrt{C/M}$ the SN frequency.

For a single macroscopic object prepared in a squeezed Gaussian state, Eq. (2) leads to a different evolutions of expectation values and quantum uncertainties, as illustrated in Fig. 1. Such a distinctive deviation could be tested by optomechanical devices in the quantum regime [18–21]. For two macroscopic objects interacting through gravity, we show further, using the two-body counterpart of Eq. (2), that classical gravity cannot be used to transfer quantum information — although experimental demonstration of this effect will be much more difficult than the modification to single-object dynamics.

Here we emphasize that it is *not* our aim to *explain* the collapse of quantum states using the SN equation, as has been attempted in the literature [22–24]. In fact, we will avoid altogether the regime in which wavefunctions can be highly distorted [25–27] by the nonlinearity of the SN equation, or other proposed quantum-gravity effects, and constrain ourselves to Gaussian states whose evolutions only deviate very little from predictions of standard quantum mechanics. The experimental tests we propose will require less isolation from the environment, for much shorter periods of time, although the price we pay is higher level of dependence on the models we are testing, and less dramatic symptoms for those models.

Many-particle SN equation.— For n particles, we denote their joint wave function as $\psi(t, \mathbf{X})$ with $3n$-D vector $\mathbf{X} \equiv$

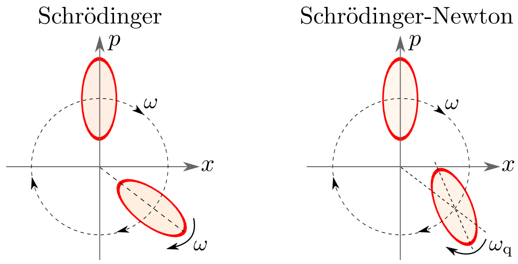

FIG. 1. (Color online). By contrast with the standard Schrödinger evolution (left panel), the covariance ellipse of a Gaussian state, under the Schrödinger-Newton Equation (right panel), rotates at an angular frequency $\omega_{\rm q} \equiv (\omega_{\rm c}^2 + \omega_{\rm SN}^2)^{1/2}$ higher than $\omega_{\rm c}$, at which the vector $(\langle x\rangle, \langle p\rangle)$ rotates.

$(\mathbf{x}_1, \cdots, \mathbf{x}_n)$ and $\mathbf{x}_k$ the 3-D spatial coordinate of k-th particle. From Refs. [10, 11], the co-evolution of the (3+1)-D spacetime background with coordinate $(t, \mathbf{x})$ and the n-particle wave function is given by [cf. Eq. (1)]

$$G_{\mu\nu}(t, \mathbf{x}) = 8\pi \int d^{3n}\mathbf{X}\, T_{\mu\nu}(\mathbf{x}, \mathbf{X}) |\psi(t, \mathbf{X})|^2 , \tag{3}$$

$$0 = \sum_k (\Box_k + m_k^2)\psi(t,\, \mathbf{X}) , \tag{4}$$

where $\Box_k$ is the (curved-spacetime) D'Alembertian with respect to the k-th spacetime coordinate and m_k is the mass of the k-th particle. In the non-relativistic limit, Diosi and Penrose [5, 22] factored out a fast-varying phase, $\psi(t,\, \mathbf{X}) \equiv e^{-i\sum_k m_k t}\varphi(t,\, \mathbf{X})$, and obtained

$$i\partial_t\varphi = \sum_k \left[-\nabla_k^2/(2m_k) + m_k\, U(t, \mathbf{x}_k)/2\right]\varphi + V(\mathbf{X})\varphi , \tag{5}$$

where $V(\mathbf{X})$ is the energy for non-gravitational interactions, and the Newtonian potential $U(t, \mathbf{x}_k)$ is given by

$$\nabla^2 U(t,\, \mathbf{x}) = 4\pi \sum_j \int d^{3n}\mathbf{X}\, |\varphi(t, \mathbf{X})|^2 m_j\, \delta(\mathbf{x} - \mathbf{x}_j) . \tag{6}$$

Equations (5) and (6) are still not concrete enough for experimental studies, because we cannot separately probe or drive the motion of each particle of a macroscopic object. In optomechanical devices, a light beam often probes (and hence acts back onto) the average displacements of atoms within the first few layers of the reflective coating of a mirror-endowed mechanical resonator. Motion of this effective surface location can often be well-approximated by the CM motion; the error of this approximation is referred to as the "internal thermal noise", and has been shown to be suppressible below the quantum level of CM motion [28–30].

Separation of scales.— Before actually deriving the CM equation of motion, let us survey the time and length scales present in our system, which are critical for separating the CM motion from the internal motions. To be specific, let us consider a macroscopic object (crystal) with equal-mass particles (atoms, mass m) positioned on a uniform 3-D lattice.

For *time scales*, the CM motion is determined externally by the measurements we chose to perform. In this paper, we consider motions from Hz to kHz scale. By contrast, the internal motion — the oscillation of the nuclei around their equilibrium positions on the lattice — takes place at the Debye frequency ω_D of the material, which is around tens of THz — much faster than the CM motion [31]. The coherence time of CM motion can be made very high, up to $\tau_{\rm CM} \sim Q/\omega_{\rm CM} \sim 10^{14}/\omega_{\rm CM}^2$, where Q is the mechanical quality factor and we have used the fact that $Q\,\omega_{\rm CM} \approx 10^{12} \sim 10^{14}$ for low-loss materials [20]; $\tau_{\rm CM}$ can be minutes or even hours, and this is orders of magnitude longer than the coherence time $\tau_{\rm int}$ of the internal motion, which is usually less than μs, even under cryogenic temperature [32].

For *length scales*, the typical CM motion, during a quantum-limited measurement process, is characterized by its zero-point motion, and $\Delta x_{\rm CM}(10^{-3}$–$10\,{\rm kg}, 1$–$1000\,{\rm Hz}) \sim 10^{-17}$–$10^{-19}\,$m. In comparison, the zero-point motion for an internal degree of freedom (DOF), i.e., a nucleus, is given by $\Delta x_{\rm int} \sim \sqrt{\hbar/(2m\,\omega_{\rm int})}$ where m is the mass of the nucleus, usually between 10 and 100 times the mass of the proton ($\sim$ 28 for a Silicon crystal). Since the typical frequency of the internal motion $\omega_{\rm int}$ is close to the Debye frequency $\omega_{\rm D} \sim 10^{14}\,{\rm s}^{-1}$, we have $\Delta x_{\rm int} \sim 10^{-12}\,$m, which is much larger than $\Delta x_{\rm CM}$. Therefore, the quantum mechanical CM motion we impose externally is tiny compared with internal motions.

SN equation for the CM.— Keeping the above separations of scales in mind, we derive the SN equation for the CM. Suppose we have a crystal with n atoms, then the CM is at $\mathbf{x}_{\rm CM} = (1/n)\sum_k \mathbf{x}_k$ and the internal motion of the k-th particle with respect to the CM is defined as $\mathbf{y}_k \equiv \mathbf{x}_k - \mathbf{x}_{\rm CM}$. In standard quantum mechanics, given a Hamiltonian that only depends on the separation of particles, the CM and the internal DOFs are separable: $\varphi(t, \mathbf{X}) = \Psi_{\rm CM}(t, \mathbf{x}_{\rm CM})\Psi_{\rm int}(t,\, \mathbf{Y})$, with $3(n-1)$-D vector $\mathbf{Y} \equiv (\mathbf{y}_1, \cdots, \mathbf{y}_{n-1})$. The two wavefunctions evolve independently when the external force acts directly on the center of mass [through $V(\mathbf{x})$]:

$$i\partial_t\Psi_{\rm CM}(t, \mathbf{x}) = \left[-\nabla^2/(2M) + V(\mathbf{x})\right]\Psi_{\rm CM}(t, \mathbf{x}) , \tag{7}$$

$$i\partial_t\Psi_{\rm int}(t, \mathbf{Y}) = H_{\rm int}\Psi_{\rm int}(t, \mathbf{Y}) . \tag{8}$$

Here $H_{\rm int}$ acts only on the the internal DOFs.

In the case of the SN equation, however, the Newtonian potential U in Eq. (5) does not allow a strictly separable evolution of $\Psi_{\rm CM}$ and $\Psi_{\rm int}$. Nevertheless, as we will show, the CM and the internal DOFs can be *approximately separable* at time scales long compared with the coherence time of the internal DOFs, but much shorter than the CM dynamical time scale. Suppose at time t, we have a product of pure states of CM and the internal DOFs, and let us consider an evolution from t to $t + \Delta t$ with increment Δt satisfying

$$\tau_{\rm CM} \gg \omega_{\rm CM}^{-1} \gtrsim \Delta t \gg \tau_{\rm int} \gg \omega_{\rm int}^{-1} , \tag{9}$$

therefore focusing on the evolution time scale of the CM. Within Δt, the internal DOFs suffer from decoherence. To accommodate this fact, let us evaluate the state evolution by using density matrices. At moment t, the joint density matrix is given by $\hat{\rho}(t) = |\varphi(t)\rangle\langle\varphi(t)| = \hat{\rho}_{\rm CM}(t) \otimes \hat{\rho}_{\rm int}(t)$, where we have introduced $\hat{\rho}_{\rm CM}(t) = |\Psi_{\rm CM}(t)\rangle\langle\Psi_{\rm CM}(t)|$ and $\hat{\rho}_{\rm int}(t) = |\Psi_{\rm int}(t)\rangle\langle\Psi_{\rm int}(t)|$. Since we are mainly interested in the effect of the Newtonian potential U, we move into an interaction picture in which U is the only interaction Hamiltonian. Up to the first order of Δt, we have

$$\hat{\rho}_I(t+\Delta t) - \hat{\rho}_I(t) = \frac{i}{2}\int_t^{t+\Delta t} {\rm d}t' [\hat{U}_I(t'), \hat{\rho}_I(t)] \tag{10}$$

with $\hat{\rho}_I(t) = \hat{\rho}(t) = \hat{\rho}_{\rm CM}(t) \otimes \hat{\rho}_{\rm int}(t)$ initially. Here the Newtonian potential operator $\hat{U}_I(t)$ is a simplified notation for $\hat{U}_I(t, \hat{\mathbf{Y}}(t), \hat{\mathbf{x}}_{\rm CM}(t))$, defined as [cf. Eqs. (5) and (6)]

$$\hat{U}_I(t) = m^2 \sum_{k,j} \int {\rm d}^3\mathbf{x} \frac{{\rm Tr}\left\{\hat{\rho}_I(t)\,\delta[\hat{\mathbf{y}}_j(t) + \hat{\mathbf{x}}_{\rm CM}(t) - \mathbf{x}]\right\}}{|\mathbf{x} - \hat{\mathbf{y}}_k(t) - \hat{\mathbf{x}}_{\rm CM}(t)|}. \tag{11}$$

Note that $\hat{\mathbf{y}}_k(t)$ and $\hat{\mathbf{x}}_{\rm CM}(t)$ are operators in the interaction picture with time evolution under the free Hamiltonian and the influence of thermal decoherence.

Because $\Delta t \gg \tau_{\rm int}$, the internal DOFs undergo strong decoherence and we can therefore assume the internal motion $\hat{\mathbf{Y}}(t)$ in the interaction picture is *ergodic*, and the time average can be approximately by the ensemble average:

$$\frac{1}{\Delta t}\int_t^{t+\Delta t} {\rm d}t' \hat{U}_I(t') = {\rm Tr}_{\rm int}[\hat{\rho}^{\rm en}_{\rm int}\,\hat{U}_I] + \epsilon_{\rm err}. \tag{12}$$

The ensemble density matrix of the internal motion $\hat{\rho}^{\rm en}_{\rm int}$, differing from $\hat{\rho}_I(t)$, is equal to the one averaged over thermal equilibrium. The error term $\epsilon_{\rm err}$ in this approximation is suppressed by two factors: (i) the small ratio between the coherence time $\tau_{\rm int}$ and Δt, which breaks $\epsilon_{\rm err}$ into many incoherent contributions in time, and (ii) the large number of independent atoms, which further breaks $\epsilon_{\rm err}$ into many independent contributions from different atoms. Appendix B provides more details for estimating $\epsilon_{\rm err}$.

Under assumption (12), at the time scale we choose, the CM and the internal DOFs will remain separable; the CM state will remain pure up to the decoherence time $\tau_{\rm CM}$, while the internal DOF will undergo decoherence at a much faster time scale. After singling out the CM, we obtain the SN equation for the CM in the interaction picture,

$$i\partial_t \Psi_{\rm CM} = {\rm Tr}_{\rm int}[\hat{\rho}^{\rm en}_{\rm int}\,\hat{U}_I/2]\Psi_{\rm CM}. \tag{13}$$

In Appendix A, we show that when $\Delta x_{\rm CM} \ll \Delta x_{\rm int}$, keeping the leading order in $x_{\rm CM}$, we can write

$${\rm Tr}_{\rm int}[\hat{\rho}^{\rm en}_{\rm int}\,\hat{U}_I] \approx \mathcal{E}''(0)/2\left[\langle\hat{x}^2_{\rm CM}\rangle - 2\langle\hat{x}_{\rm CM}\rangle\hat{x}_{\rm CM} + \hat{x}^2_{\rm CM}\right] \tag{14}$$

after having restricted center-of-mass motion along one particular direction. Here $\langle\hat{x}_{\rm CM}\rangle \equiv \langle\Psi_{\rm CM}|\hat{x}_{\rm CM}|\Psi_{\rm CM}\rangle$; $\mathcal{E}''$ is the double (directional) derivative of the Newtonian potential energy $\mathcal{E}(\mathbf{x})$ between the mass distribution according to the internal motion and the same distribution but translated by $\mathbf{x}$,

$$\mathcal{E}(\mathbf{x}) = \iint d^3\mathbf{x}' d^3\mathbf{x}'' \frac{\varrho_{\rm int}(\mathbf{x}')\varrho_{\rm int}(\mathbf{x}'')}{|\mathbf{x}' - \mathbf{x}'' + \mathbf{x}|}, \tag{15}$$

with $\varrho_{\rm int}(\mathbf{x})$ the matter density at position $\mathbf{x}$, given by:

$$\varrho_{\rm int}(\mathbf{x}) = \sum_k m_k \int {\rm d}^{3n-3}\mathbf{Y}\,{\rm Tr}\left[\langle\mathbf{Y}|\hat{\rho}^{\rm en}_{\rm int}|\mathbf{Y}\rangle\delta(\mathbf{x} - \mathbf{y}_k)\right]. \tag{16}$$

Neglecting the c-number term $\langle x^2_{\rm CM}\rangle - \langle x_{\rm CM}\rangle^2$ as it only adds an overall phase to the wave function and putting back the free part of the Hamiltonian for the CM, we obtain Eq. (2) shown at the very beginning with SN coupling constant defined by:

$$C \equiv \mathcal{E}''(0)/2. \tag{17}$$

Estimates for $\omega_{\rm SN}$.— Let us now estimate the magnitude of $\omega_{\rm SN}$. Suppose we have an object with mass M and size L; assuming an absolutely homogeneous mass distribution, we obtain

$$C^{\rm hom} \approx 3GM^2/(2L^3) \approx 3GM\rho_0/2,\ \omega^{\rm hom}_{\rm SN} \approx \sqrt{3G\rho_0}, \tag{18}$$

where we put back the Newton's constant G and ρ_0 is the density of the homogenous material. This is a typical order of magnitude obtained in the literature. However, according to results obtained so far in the paper, we must account for the concentration of material near the equilibrium positions of the nuclei, with $\Delta x_{\rm int} \ll a_{\rm lattice}$, we can roughly model the object's matter distribution according to $\hat{\rho}_{\rm int}$ as a square lattice of solid balls with radius $\Delta x_{\rm int}$ and constant density — with lattice spacing equal to $a_{\rm lattice}$. It is easy to argue that the self gravitational energy $\mathcal{E}$ of such a lattice is dominated by the sum of the self energies of each ball, which gives

$$C^{\rm crystal} \approx C^{\rm hom}\Lambda, \quad \Lambda \equiv (a_{\rm lattice}/\Delta x_{\rm int})^3. \tag{19}$$

In other words, we have an amplification factor $\Lambda \gg 1$ for the SN constant. For Silicon at $T = 100\,$K, we already have $\hbar\omega_D \sim 7k_BT$ and the variance of nucleus thermalized motion is about the same as $\Delta x^2_{\rm int}$. Noting further that $a_{\rm lattice} \approx 5 \times 10^{-10}$m, $\rho_0 \approx 2.7$g/cm^3, and $\Delta x_{\rm int} \sim 5 \times 10^{-12}$m, we have

$$\Lambda \sim 10^6, \quad \omega^{\rm crystal}_{\rm SN} \sim 0.4\,{\rm s}^{-1}. \tag{20}$$

We shall use this $\omega^{\rm crystal}_{\rm SN}$ as our baseline estimate for $\omega_{\rm SN}$ throughout the rest of the paper.

Evolutions of Gaussian States.— As one can easily prove, an initial Gaussian state evolves under Eq. (2) will remain Gaussian. Here we write down the self-contained evolution equations for first- and second-moments of x and p, which completely determine the evolving Gaussian state:

$$Md\langle x\rangle/dt = d\langle p\rangle/dt, \quad d\langle p\rangle/dt = -M\omega_{\rm c}^2\langle x\rangle, \tag{21}$$

$$\dot{V}_{xx} = 2V_{xp}/M, \quad \dot{V}_{pp} = -2M(\omega_{\rm c}^2 + \omega_{\rm SN}^2)V_{xp}, \tag{22}$$

$$\dot{V}_{xp} = V_{pp}/M - M(\omega_{\rm c}^2 + \omega_{\rm SN}^2)V_{xx}. \tag{23}$$

For covariance we have defined $V_{AB} \equiv \langle\hat{A}\hat{B} + \hat{B}\hat{A}\rangle/2 - \langle\hat{A}\rangle\langle\hat{B}\rangle$. Equation (21) indicates that expectation values of $\hat{x}$ and $\hat{p}$ evolve the same way as a standard simple harmonic oscillator with angular frequency $\omega_{\rm c}$; this means any *semiclassical* measurement of the oscillator on expectation values of x and p will confirm classical physics. Evolutions of second moments, i.e., the covariance matrix, which represent *quantum uncertainties*, however, are modified; they evolve the same way as a harmonic oscillator with a modified frequency:

$$\omega_q \equiv \sqrt{\omega_c^2 + \omega_{SN}^2}\,, \tag{24}$$

as illustrated in Fig. 1. In Appendix C, we show that the difference between first- and second-moment evolution may show up in the output spectrum of an optomechanical system in the quantum regime. In order to measure the effect of the SN term, one must be able to distinguish between ω_c and ω_q, by requiring $\omega_q - \omega_c > \omega_c/Q$, where Q is the quality factor of the mechanical oscillator. This requires

$$Q \gtrsim (\omega_{SN}/\omega_c)^2\,, \tag{25}$$

which is experimentally achievable if $\omega_{SN} \approx 0.4\,\mathrm{s}^{-1}$, $\omega_{CM} \approx 2\pi \times 10^3$Hz, and $Q \gtrsim 2\times 10^8$ [33].

SN equation for two macroscopic objects.— If gravity is introduced as an interaction potential between two objects which are confined within potential wells with common eigenfrequency ω_0, and moving along the same direction as the separation vector $\mathbf{L}$ between their equilibrium positions, we have

$$\hat{V} = C_{12}(\hat{x}_{CM_1} - \hat{x}_{CM_2})^2 \tag{26}$$

with

$$C_{12} \equiv \frac{1}{2}\frac{\partial^2 \mathcal{E}_{12}}{\partial L^2},\ \mathcal{E}_{12} \equiv \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 \frac{G\tilde{\rho}^{(1)}_{\rm int}(\mathbf{x}_1)\tilde{\rho}^{(2)}_{\rm int}(\mathbf{x}_2)}{|\mathbf{x}_1 - \mathbf{x}_2 - \mathbf{L}|}, \tag{27}$$

where $\tilde{\rho}^{(1)}_{\rm tot}$ and $\tilde{\rho}^{(2)}_{\rm tot}$ are ensemble-averaged mass distribution densities for objects 1 and 2, respectively. This weak coupling makes differential mode of the two CM motions oscillate at a slightly higher frequency — thereby allowing quantum state to slosh between the two objects, at a frequency of $\Delta = |\omega_+ - \omega_-| = C_{12}/(2M\omega_0)$. In order for information to successfully transfer, one requires both objects to be in equilibrium with a heat bath with $k_B T \lesssim \hbar\Omega_c$, and $\Delta \gtrsim \omega_0/Q$, which requires

$$Q \gtrsim M\omega_0^2/C_{12}\,. \tag{28}$$

Suppose we instead use the SN equation for two macroscopic objects in this scenario. Following the same procedure as above, we obtain the SN equation for the CMs of two objects:

$$i\partial_t \Psi_{12} = [H_{11} + H_{22} + \mathcal{E}_{12} - H_{12}]\Psi_{12}\,. \tag{29}$$

Here $H_{11,22}$ are the sum of the free Hamiltonian and the SN interaction term for each individual CM [as in Eq. (2)] , $\mathcal{E}_{12}$ is the zero point of the mutual Newtonian interaction energy and H_{12} is given by,

$$H_{12} = C_{12}\left[(x_{CM_1} - \langle x_{CM_2}\rangle)^2 + (x_{CM_2} - \langle x_{CM_1}\rangle)^2\right]/2. \tag{30}$$

Equation (30) makes sure that only expectation value of position gets transferred between the two objects (same way as in classical physics), while *quantum uncertainty does not transfer from one object to the other.* In retrospect, the expectation value gets transferred because the semiclassical limit of the SN equation is simply Newtonian physics; quantum uncertainty does not get transferred because classical space-time is incapable of storing, let along passing on, quantum uncertainty.

However, because $C_{12} \sim GM^2/L^3 \lesssim GM\rho_0 \ll C$, the lack of the amplification factor Λ [Eq. (19)] in C_{12}, makes the Q required [Eq. (28)] for testing the inability of classical gravity to transfer quantum information is much larger than the the threshold for testing the self-gravity effect. We will be forced to require $\omega_c \approx 2\pi \times 1\,$Hz, if $Q \sim 2\times 10^8$, which seems very challenging for ground-based experiments, yet might not be impossible [34].

Discussions— In this paper, we have taken the liberty to assume that spacetime might be classical [12]. In our opinion, due to the lack of precision experimental tests on the quantum coherence of dynamical gravity, semiclassical gravity is still worth testing. In fact, our calculations show that detecting the consequences of classical gravity using quantum mechanical phenomena requires carefully designed experiments using techniques that have not been available until recently.

A concrete mathematical formulation of non-relativistic semiclassical gravity, namely the SN equation, combined with basic knowledge of condensed-matter physics, has revealed self-gravity effects in macroscopic objects that can be tested much more easily than performing the more obvious (but less model dependent) experiment of demonstrating impossibility of information transfer. Thanks to the amplification factor Λ, this "much easier test" seems only moderately challenging with current technology.

We speculate that the rate gravity decoherence should also be expedited by the same $\Lambda^{1/2} \sim 1000$, because gravitational self energy is also used as an indicator of decoherence time [5]. However, due to the lack of a widely-accepted microscopic model of gravity decoherence, this only makes it more hopeful to make experimental attempts, but would not enforce a very powerful bound if decoherence were not to be found.

Since our classical gravity model literally requires the existence of a global wave function of the universe that does not collapse, in the arguably unlikely case the proposed experimental test shows any positive result, we will open up the opportunity of investigating the fundamental nature of quantum measurement using gravity.

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APPENDICES

A. Details in calculating $\mathrm{Tr}_{\rm int}[\hat\rho^{\rm en}_{\rm int}\hat U_I]$

In this section, we will fill in details that would lead to Eq. (2). Let us first define

$$\mathbf{x}_k = \mathbf{y}_k + \mathbf{x}_{\rm CM}\,,\quad k=1,\ldots,n\,. \tag{A.1}$$

Here $\mathbf{x}_k$ are positions of the k-th particle, $\mathbf{x}_{\rm CM}$ is the center-of-mass position, while $\mathbf{y}_k$ are *internal motions*, with the constraint that $\sum_{k=1}^n \mathbf{y}_k = 0$. For a density matrix $\hat\rho$ that is factorable into $\hat\rho = \hat\rho^{\rm en}_{\rm int}\otimes\hat\rho_{\rm CM}$, the CM and the internal motions are independent from each other. For this $\hat\rho$, we can easily obtain the matter-density distribution

$$\begin{aligned}\varrho_{\rm tot}(\mathbf{x})/m &= \sum_{k=1}^n \mathrm{tr}\left[\delta^{(3)}(\mathbf{x}-\mathbf{x}_k)\hat\rho^{\rm en}_{\rm int}\otimes\hat\rho_{\rm CM}\right]\\ &= \sum_{k=1}^n\int d^{3n-3}\mathbf{Y}d^3\mathbf{x}_{\rm CM}\tilde\rho_{\rm int}(\mathbf{Y})\tilde\rho_{\rm CM}(\mathbf{x}_{\rm CM})\delta^{(3)}[\mathbf{x}-(\mathbf{y}_k+\mathbf{x}_{\rm CM})]\,,\end{aligned} \tag{A.2}$$

where for convenience we have defined

$$\tilde\rho_{\rm int}(\mathbf{Y})\equiv\langle\mathbf{Y}|\hat\rho^{\rm en}_{\rm int}|\mathbf{Y}\rangle\,,\ \tilde\rho_{\rm CM}(\mathbf{x}_{\rm CM})\equiv\langle\mathbf{x}_{\rm CM}|\hat\rho_{\rm CM}|\mathbf{x}_{\rm CM}\rangle\,. \tag{A.3}$$

In a similar way, we can also define matter-density distributions given by the internal motion alone (assuming $\mathbf{x}_{\rm CM}=0$), as well as the center-of-mass matter-density distribution:

$$\varrho_{\rm int}(\mathbf{x}) = \sum_{k=1}^N m\int d^{3n-3}\mathbf{Y}\tilde\rho_{\rm int}(\mathbf{Y})\delta^{(3)}[\mathbf{x}-\mathbf{y}_k]\,, \tag{A.4}$$

$$\varrho_{\rm CM}(\mathbf{x}) = m\,\tilde\rho_{\rm CM}(\mathbf{x})\,. \tag{A.5}$$

Together, Eqs. (A.2) and (A.4) gives

$$\varrho_{\rm tot}(\mathbf{x}) = \int\varrho_{\rm int}(\mathbf{x}-\mathbf{y})\varrho_{\rm CM}(\mathbf{y})d^3\mathbf{y}\,, \tag{A.6}$$

which is rather intuitive. We can then write

$$\begin{aligned}\sum_{k=1}^N U(\hat\rho,\hat{\mathbf{x}}_k) &= m\sum_{k=1}^N\int\frac{\varrho_{\rm tot}(\mathbf{x})}{|\mathbf{x}-\hat{\mathbf{y}}_k-\hat{\mathbf{x}}_{\rm CM}|}d^3\mathbf{x}\\ &= m\sum_{k=1}^N\int\frac{\varrho_{\rm tot}(\mathbf{x})\delta^{(3)}(\mathbf{z}-\hat{\mathbf{y}}_k)}{|\mathbf{x}-\mathbf{z}-\hat{\mathbf{x}}_{\rm CM}|}d^3\mathbf{x}d^3\mathbf{z}\end{aligned} \tag{A.7}$$

This leads to

$$\begin{aligned}&\mathrm{tr}_{\rm int}\left[\hat\rho^{\rm en}_{\rm int}\sum_{k=1}^N U(\hat\rho,\hat{\mathbf{x}}_k)\right]\\ &= \int\frac{\varrho_{\rm int}(\mathbf{x}-\mathbf{y})\varrho_{\rm cm}(\mathbf{y})\varrho_{\rm int}(\mathbf{z})}{|\mathbf{x}-\mathbf{z}-\hat{\mathbf{x}}_{\rm CM}|}d^3\mathbf{x}d^3\mathbf{y}d^3\mathbf{z}\\ &= \int d^3\mathbf{y}\,\varrho_{\rm cm}(\mathbf{y})\int d^3\mathbf{x}d^3\mathbf{z}\frac{\varrho_{\rm int}(\mathbf{x})\varrho_{\rm int}(\mathbf{z})}{|\mathbf{x}-\mathbf{z}+\mathbf{y}-\hat{\mathbf{x}}_{\rm CM}|}\,.\end{aligned} \tag{A.8}$$

At this stage, if we define

$$\mathcal{E}(\mathbf{x})\equiv\int d^3\mathbf{y}d^3\mathbf{z}\frac{\varrho_{\rm int}(\mathbf{y})\varrho_{\rm int}(\mathbf{z})}{|\mathbf{y}-\mathbf{z}+\mathbf{x}|}, \tag{A.9}$$

this is the mutual gravitational potential energy between a matter distribution $\varrho_{\rm int}$ and an identical one translated by $\mathbf{x}$. If translation is along one direction, with a magnitude much less than the scale in which $\mathcal{E}$ changes, then

$$\mathcal{E}(\mathbf{x})\approx\mathcal{E}''(0)x^2/2+\mathcal{E}(0) \tag{A.10}$$

The constant energy piece does not affect wave function evolution except adding an overall phase — which can also be absorbed into the definition of the internal wavefunction. This means

$$\begin{aligned}\mathrm{tr}_{\rm int}[\hat\rho^{\rm en}_{\rm int}\hat U_I] &= \frac12\mathcal{E}''(0)\int(\mathbf{y}-\hat{\mathbf{x}}_{\rm CM})^2\varrho_{\rm cm}(\mathbf{y})d^3\mathbf{y}\\ &= \frac12\mathcal{E}''(0)\left[\langle\hat x^2_{\rm CM}\rangle-2\langle\hat x_{\rm CM}\rangle\hat x_{\rm CM}+\hat x^2_{\rm CM}\right].\end{aligned} \tag{A.11}$$

B. Estimation of the Fluctuation Term

In this section, we estimate the error of the fluctuation which appears in Eq. (12):

$$\frac{1}{\Delta t}\int_t^{t+\Delta t}\mathrm{d}t'\hat U_I(t') = u+\epsilon_{\rm err}\,, \tag{B.1}$$

with

$$u = \mathrm{tr}_{\rm int}\left[\hat\rho^{\rm en}_{\rm int}\sum_{k=1}^n m\,U(\hat\rho,\hat{\mathbf{x}}_k)\right]\approx\frac{Gnm^2}{\Delta x^3_{\rm int}}\Delta x^2_{\rm CM} \tag{B.2}$$

$$\epsilon_{\rm err} = \frac{m}{\Delta t}\int_t^{t+\Delta t}dt'\sum_{k=1}^n\left[\hat U_k(t')-\langle\hat U_k(t')\rangle\right]. \tag{B.3}$$

Here the $\langle\ldots\rangle$ in $\epsilon_{\rm err}$ represents ensemble average. It would not be difficult to estimate, by squaring Eq. (B.3) and taking expectation value, that

$$\sqrt{\langle\epsilon^2_{\rm err}\rangle}\sim m\sqrt{\frac{n\tau_*}{\Delta t}}(\Delta U)\sim\frac{Gnm^2}{\Delta x_{\rm int}}\sqrt{\frac{\tau_*}{n\Delta t}} \tag{B.4}$$

Here τ_* is the coherence time of $\hat U_k(t)$, ΔU is the likely range in which U can fluctuate, while the number of particles n enters this way because motions of atoms are largely independent from each other. This means we have

$$\frac{\epsilon_{\rm err}}{u}\sim\left(\frac{\Delta x_{\rm int}}{\Delta x_{\rm CM}}\right)^2\sqrt{\frac{\tau_*}{n\Delta t}} \tag{B.5}$$

Here the separation of time scales, plus the large number of n, can often overcome $\Delta x_{\rm int}/\Delta x_{\rm CM}$ factor and make $\epsilon_{\rm err}\ll u$. This means we can ignore the error introduced by using ensemble average.

C. Effective Heisenberg Equations of Motion and Coupling with Optical Field

The fact that Gaussian states leads to Gaussian states encourages us to look for effective Heisenberg equations of motion, which will at least be valid for Gaussian states. It is easy to find that

$$d\hat{x}/dt = \hat{p}/M \tag{C.1}$$
$$d\hat{p}/dt = -M\omega_c^2\hat{x} + M\omega_{NS}^2(\hat{x} - \langle\hat{x}\rangle) \tag{C.2}$$

will give the same set of first- and second-moment equations of motion. Note that in the Heisenberg picture, the initial state of the oscillator remains constant.

Let us now consider a more realistic scenario, in which the oscillator is damped with decay rate γ_m, driven with classical thermal noise and other classical driving; we also consider using light to sense the position of the mirror, in which we also suffer from sensing noise. The entire process can be described by the following set of equations:

$$d\hat{x}/dt = \hat{p}/M\,, \tag{C.3}$$
$$d\hat{p}/dt = -M[\omega_c^2\hat{x} + 2\gamma_m\hat{p} + \omega_{NS}^2(\hat{x} - \langle\hat{x}\rangle)] + \alpha\hat{a} + f\,, \tag{C.4}$$
$$\hat{b}_2 = \hat{a}_2 + n + \alpha\hat{x}\,, \quad \hat{b}_1 = \hat{a}_1\,. \tag{C.5}$$

Here γ_m is the rate of damping, α is the optomechanical coupling constant, f is classical driving (we ignore the tiny quantum contribution from the thermal force). We have used $a_{1,2}$ to represent the quadratures of the incoming optical field, and $b_{1,2}$ those of the out-going field. We have used n to denote classical or quantum sensing noise.

This set of equations can be solved first for $\langle\hat{x}\rangle$ by taking the expectation value of the first two equations, and then insert this back to obtain the entire solution. If we define

$$\chi_0 = -\frac{1}{M(\omega^2 + 2i\gamma_m\omega - \omega_c^2)} \tag{C.6}$$
$$\chi_g = -\frac{1}{M(\omega^2 + 2i\gamma_m\omega - \omega_q^2)}\,, \quad \omega_q = \sqrt{\omega_c^2 + \omega_{NS}^2}\,, \tag{C.7}$$

then we can write, in the frequency domain,

$$b_2 = \hat{a}_2 + \alpha\chi_g\hat{a}_1 + \chi_0 f + n\,. \tag{C.8}$$

Because both χ_g and χ_0 in the time domain are Green functions of stable systems, Eq. (C.8) represent the steady-state solution for the out-going field, which is only determined by the in-going optical field and the classical driving field — initial states of the mechanical oscillator does not matter [similar to the case of Ref. [35, 36]].

Equation (C.8) carries the separation between classical and quantum rotation frequencies in the previous section (Fig. 1) into the frequency spectrum of our measuring device: quantum radiation-pressure noise spectrum in the output port of the continuous measuring device is the same as an oscillator with frequency ω_q, and therefore peaks around ω_q — while classical noise follows that of an oscillator with frequency ω_c, and peaks at ω_c. In order to look for such a signature, we will need classical force noise to be comparable in level to quantum noise, and have the two peaks to be resolvable,

$$\omega_q - \omega_c \gtrsim \gamma_m \tag{C.9}$$

which means

$$Q \gtrsim (\omega_c/\omega_m)^2 \tag{C.10}$$

where Q is the quality factor of the mechanical oscillator.

D. SN Equation for Two Macroscopic Objects

Following the analysis in Appendix. A, here we deal with two macroscopic objects. For simplicity, let us assume the two objects as identical, and define

$$\mathbf{x}_k^{(I)} = \mathbf{y}_k^{(I)} + \mathbf{x}_{CM}^{(1)}\,, \quad k = 1, \ldots, n\,, \quad I = 1, 2. \tag{C.11}$$

As the distance between the two objects is macroscopic, their wavefunctions have no overlap. We also have

$$\varrho_{tot}^{(I)}(\mathbf{x}) = \int \varrho_{int}^{(I)}(\mathbf{x} - \mathbf{y})\,\varrho_{CM}^{(I)}(\mathbf{y})d^3\mathbf{y}\,, \quad I = 1, 2\,. \tag{C.12}$$

The total SN term in the joint SN equation will be

$$\sum_{I=1}^{2}\sum_{k=1}^{n} U(\hat{\rho}, \hat{x}_k^{(I)}) = \sum_{I,J=1}^{2}\sum_{k=1}^{n}\int \frac{\varrho_{tot}^{(I)}(\mathbf{x})}{|\mathbf{x} - \hat{\mathbf{x}}_k^{(J)}|}d^3\mathbf{x} \tag{C.13}$$

It is easy to identify contributions to the self-SN terms:

$$\sum_{k=1}^{n}\int \frac{\varrho_{tot}^{(1)}(\mathbf{x})}{|\mathbf{x} - \hat{\mathbf{x}}_k^{(1)}|}d^3\mathbf{x} \to H_{11}, \ \sum_{k=1}^{n}\int \frac{\varrho_{tot}^{(2)}(\mathbf{x})}{|\mathbf{x} - \hat{\mathbf{x}}_k^{(2)}|}d^3\mathbf{x} \to H_{22}. \tag{C.14}$$

The mutual SN terms can be evaluated via ensemble averaging over $\hat{\rho}_{int}^{en} = \hat{\rho}_{int}^{(1)\,en} \otimes \hat{\rho}_{int}^{(2)\,en}$:

$$\begin{aligned}&\mathrm{tr}_{int}\left[\hat{\rho}_{int}^{en}\sum_{k=1}^{n}\int \frac{\varrho_{tot}^{(2)}(\mathbf{x})}{|\mathbf{x} - \hat{\mathbf{x}}_k^{(1)}|}\right]\\ \approx&\mathcal{E}_{12}(0) + \mathcal{E}_{12}''/2\left[\hat{x}_{CM_1}^2 - 2\hat{x}_{CM_1}\langle\hat{x}_{CM_2}\rangle + \langle\hat{x}_{CM_2}^2\rangle\right]\end{aligned} \tag{C.15}$$

$$\begin{aligned}&\mathrm{tr}_{int}\left[\hat{\rho}_{int}^{en}\sum_{k=1}^{n}\int \frac{\varrho_{tot}^{(1)}(\mathbf{x})}{|\mathbf{x} - \hat{\mathbf{x}}_k^{(2)}|}\right]\\ \approx&\mathcal{E}_{12}(0) + \mathcal{E}_{12}''/2\left[\hat{x}_{CM_2}^2 - 2\hat{x}_{CM_2}\langle\hat{x}_{CM_1}\rangle + \langle\hat{x}_{CM_1}^2\rangle\right]\end{aligned} \tag{C.16}$$

where $\mathcal{E}_{12}$ is given by

$$\mathcal{E}_{12}(\mathbf{x}) \equiv \int d^3\mathbf{y}d^3\mathbf{z}\frac{\varrho_{int}^{(1)}(\mathbf{y})\,\varrho_{int}^{(2)}(\mathbf{z})}{|\mathbf{y} - \mathbf{z} + \mathbf{L} + \mathbf{x}|} \tag{C.17}$$