Supplementary material for Macroscopic Quantum Mechanics in a Classical Spacetime

A. Incompatibility between the many-world interpretation of quantum mechanics and classical gravity

At this moment, the only well-known (and widely accepted) interpretation of quantum mechanics that explains the phenomenology of quantum measurement without resorting to quantum-state reduction is the many-world interpretation of quantum mechanics [20]: the entire universe's wavefunction contains many branches, incorporating all possible measurement outcomes; each observer, however, can only perceive one of the branches, therefore experiencing the phenomenon of quantum-state reduction.

If we were to combine the many-world interpretation of quantum mechanics and classical gravity, the classical spacetime geometry will have to be determined by the expectation value of stress-energy tensor, which effectively averages over all possible measurement outcomes. Following Page and Geilker [21], let us consider the following gedankenexperiment. Suppose quantum measurement of $\hat{\sigma}_z$ of a spin-1/2 particle at a state of $(|+\rangle + |-\rangle)/\sqrt{2}$ determines whether we put a mass on the left or right side of a scale, then the combination of the many-world interpretation and classical gravity will predict a leveled scale, because the expectation value of matter densities on both sides are equal. This is in stark contrast with experimental facts.

Nevertheless, for many, including some of the authors, neither the concept of state reduction nor the many-world interpretation seems a satisfactory explanation of why we cannot predict the outcome of a quantum measurement. We therefore remain open minded towards the possibility of further interpretations/clarifications/modifications of quantum mechanics and the quantum measurement process that offer better explanations. For us, the Schrödinger-Newton equation is not ruled out right away, and is therefore still worth testing.

B. Separation between CM and internal degrees of freedom

In Eq. (11), we obtained the Schrödinger-Newton potential kept at quadratic order (in $\Delta x_{\rm CM}/\Delta x_{\rm zp} \ll 1$):

$$\begin{aligned} V_{\rm SN} &= \sum_k \varepsilon(\mathbf{y}_k) + (x_{\rm CM} - \langle x_{\rm CM}\rangle)\sum_k \varepsilon'(\mathbf{y}_k) \\ &+ \frac{1}{2}(x_{\rm CM}^2 - x_{\rm CM}\langle x_{\rm CM}\rangle + \langle x_{\rm CM}^2\rangle)\sum_k \varepsilon''(\mathbf{y}_k)\,. \end{aligned} \tag{B.1}$$

As we shall argue, truncation at this order will give us the leading correction to CM motion, and it is also separable from corrections to the internal motion, therefore justifying the assumption of separability between the CM motion and internal DOFs. Higher order terms, being suppressed by powers of $\Delta x/\Delta x_{\rm zp}$ are therefore negligible.

The first term

$$V_{\rm SN}^{(0)} = \sum_k \varepsilon(\mathbf{y}_k) \tag{B.2}$$

is readily absorbed into $H_{\rm int}$, which now becomes nonlinear. This is already the leading correction for the internal motion. Let us calculate the modulus of its contribution,

$$\begin{aligned} \|V_{\rm SN}^{(0)}\varphi\| &= \left[\int \sum_{k,j} \varepsilon(\mathbf{y}_k)\varepsilon(\mathbf{y}_j)|\Psi_{\rm int}(\mathbf{Y})|^2 d^{3n-3}\mathbf{Y}\right]^{1/2} \\ &\approx n\left(Gm^2/\Delta x_{\rm zp}\right) \end{aligned} \tag{B.3}$$

The second term

$$V_{\rm SN}^{(1)} = (x_{\rm CM} - \langle x_{\rm CM}\rangle)\sum_k \varepsilon'(\mathbf{y}_k) \tag{B.4}$$

describes the interaction between the CM and the internal motion of each of the atoms. In order to estimate its effect, let us first calculate the modulus of the change in φ it induces:

$$\begin{aligned}\|V_{\rm SN}^{(1)}\varphi\| =&\Delta x_{\rm CM}\left[\int\sum_{k,j}\varepsilon'(\mathbf{y}_k)\varepsilon'(\mathbf{y}_j)|\Psi_{\rm int}(\mathbf{Y})|^2 d^{3n-3}\mathbf{Y}\right]^{1/2}\\ \approx&\sqrt{n}\left(Gm^2/\Delta x_{\rm zp}\right)\left(\Delta x_{\rm CM}/\Delta x_{\rm zp}\right)\,. \end{aligned} \tag{B.5}$$

Here we only have $\sqrt{n}$ (instead of n) because the integral will not vanish only when distribution of $\mathbf{y}_k$ and $\mathbf{y}_j$ are correlated, which only happens for nearby atoms. The other factor $\Delta x_{\rm CM}/\Delta x_{\rm zp}$ is due to the fact that this is the next order in the Taylor expansion. From the point of view of internal motion, $V_{\rm SN}^{(1)}$ clearly gives a higher-order correction than $V_{\rm SN}^{(0)}$, hence negligible. We will show that, even for CM motion, the contribution of $V_{\rm SN}^{(1)}$ is also less than contribution from the next Taylor-expansion term $V_{\rm SN}^{(2)}$.

Now turning to $V_{\rm SN}^{(2)}$, let us split it into two terms

$$V_{\rm SN}^{(2)}=\bar{V}_{\rm SN}^{(2)}+\delta V_{\rm SN}^{(2)}, \tag{B.6}$$

with

$$\bar{V}_{\rm SN}^{(2)}=\frac{1}{2}(x_{\rm CM}^2-2x_{\rm CM}\langle x_{\rm CM}\rangle+\langle x_{\rm CM}^2\rangle)\Big\langle\sum_k\varepsilon''(\mathbf{y}_k)\Big\rangle\,, \tag{B.7}$$

defined as the ensemble average, where

$$\Big\langle\sum_k\varepsilon''(\mathbf{y}_k)\Big\rangle\equiv\int\sum_k\varepsilon''(\mathbf{y}_k)|\Psi_{\rm int}(\mathbf{Y})|^2d^{3n-3}\mathbf{Y}\equiv\mathcal{C}\,, \tag{B.8}$$

and

$$\mathcal{C}=-\frac{1}{2}\frac{\partial^2}{\partial z^2}\left[\int\frac{G\tilde{\rho}_{\rm int}(\mathbf{y})\tilde{\rho}_{\rm int}(\mathbf{y}')}{|\mathbf{z}+\mathbf{y}-\mathbf{y}'|}d\mathbf{y}d\mathbf{y}'\right]_{\mathbf{z}=0}\,. \tag{B.9}$$

Note that $\bar{V}_{\rm SN}^{(2)}$ does not depend explicitly on $\mathbf{Y}$, and hence is a correction to the Hamiltonian for the CM motion. It is straightforward to estimate that

$$\|\bar{V}_{\rm SN}^{(2)}\varphi\|=nGm^2/\Delta x_{\rm zp}\left(\Delta x_{\rm CM}/\Delta x_{\rm zp}\right)^2\,. \tag{B.10}$$

This means, at any given time,

$$\|V_{\rm SN}^{(1)}\varphi\|/\|\bar{V}_{\rm SN}^{(2)}\varphi\|\approx\frac{1}{\sqrt{n}}\frac{\Delta x_{\rm zp}}{\Delta x_{\rm CM}}\approx\sqrt{\frac{\omega_{\rm CM}}{\omega_D}}\ll 1\,. \tag{B.11}$$

In addition, as we evolve in time, the effect of $V_{\rm SN}^{(1)}$ oscillates around zero over a very fast time scale, while the effect of $\bar{V}_{\rm SN}^{(2)}$ does not oscillate around zero — this further suppresses the relative contribution of $V_{\rm SN}^{(1)}$. For this reason, we ignore $V_{\rm SN}^{(1)}$ completely.

As for $\delta V_{\rm SN}^{(2)}$, its effect is suppressed from $\bar{V}_{\rm SN}^{(2)}$ by $\sqrt{n}$, because, much similar to Eq. (B.5), effects of different atoms do not accumulate unless they are very close to each other.

C. Effective Heisenberg Equations of Motion and Coupling with Optical Field

The fact that Gaussian states leads to Gaussian states encourages us to look for *effective* Heisenberg equations of motion, which will at least be valid for Gaussian states. It is easy to find that

$$\dot{\hat{x}}=\hat{p}/M \tag{C.1}$$

$$\dot{\hat{p}}=-M\omega_{\rm CM}^2\hat{x}-\mathcal{C}(\hat{x}-\langle\hat{x}\rangle) \tag{C.2}$$

will give the same set of first- and second-moment equations of motion as the SN equation. Note that in the Heisenberg picture, the initial state of the oscillator remains constant.

Let us now consider a more realistic scenario, in which the oscillator is damped with decay rate γ_m, driven with classical thermal noise and other classical driving; we also consider using light to sense the position of the mirror, in which we also suffer from sensing noise. The entire process can be described by the following set of equations [cf. Eqs. (21), (22) and (23)]:

$$\dot{\hat{x}} = \hat{p}/M\,, \tag{C.3}$$

$$\dot{\hat{p}} = -M\omega_{\rm CM}^2\hat{x} - 2\gamma_m\hat{p} - \mathcal{C}(\hat{x} - \langle\hat{x}\rangle) + \hat{F}_{\rm BA} + F_{\rm th}\,, \tag{C.4}$$

$$\hat{b}_2 = \hat{a}_2 + n_x + (\alpha/\hbar)\hat{x}\,, \quad \hat{b}_1 = \hat{a}_1\,. \tag{C.5}$$

This set of equations can be solved first for $\langle\hat{x}\rangle$ by taking the expectation value of the first two equations, and then insert this back to obtain the entire solution. If we define

$$\chi_0 = -\frac{1}{M(\omega^2 + 2i\gamma_m\omega - \omega_{\rm CM}^2)} \tag{C.6}$$

$$\chi_g = -\frac{1}{M(\omega^2 + 2i\gamma_m\omega - \omega_{\rm q}^2)}\,, \tag{C.7}$$

then we can write, in the frequency domain,

$$\hat{b}_2 = \hat{a}_2 + (\alpha/\hbar)\chi_g\hat{F}_{\rm BA} + (\alpha/\hbar)\chi_0 F_{\rm th} + n_x\,. \tag{C.8}$$

Because both χ_g and χ_0 in the time domain are Green functions of stable systems, Eq. (C.8) represent the steady-state solution for the out-going field, which is only determined by the in-going optical field and the classical driving field — initial states of the mechanical oscillator does not matter (similar to the case of Ref. [49, 50]).

Equation (C.8) carries the separation between classical and quantum rotation frequencies in the previous section (Fig. 1) into the frequency spectrum of our measuring device: quantum back-action (radiation-pressure) noise $\hat{F}_{\rm BA}$ spectrum in the output port of the continuous measuring device is the same as an oscillator with frequency $\omega_{\rm q}$, and therefore peaks around $\omega_{\rm q}$ — while classical noise $F_{\rm th}$ follows that of an oscillator with frequency $\omega_{\rm CM}$, and peaks at $\omega_{\rm CM}$. In order to look for such a signature, we will need classical force noise to be comparable in level to quantum noise, and have the two peaks to be resolvable,

$$\omega_{\rm q} - \omega_{\rm CM} \gtrsim \gamma_m \tag{C.9}$$

which means

$$Q \gtrsim (\omega_{\rm CM}/\omega_{\rm SN})^2 \tag{C.10}$$

where Q is the quality factor of the mechanical oscillator.

D. SN Equation for Two Macroscopic Objects

Following the analysis in Appendix. A, here we deal with two macroscopic objects, and define

$$\mathbf{x}_k^{(I)} = \mathbf{X}^{(I)} + \mathbf{y}_k^{(I)} + \mathbf{x}_{\rm CM}^{(I)}\,, \quad k = 1, \ldots, n_I\,, \quad I = 1, 2. \tag{D.1}$$

Here $\mathbf{X}^{(I)}$ is the zero point we use for describing object I, and n_I is the number of atoms object I contains. Following the same argument for deriving the single-object SN equation, we can still write the joint wavefunction as a product between the joint CM wavefunction and the internal wavefunctions,

$$\varphi = \Psi_{\rm CM}\left[\mathbf{x}^{(1)}, \mathbf{x}^{(2)}\right]\Psi_{\rm int}^{(1)}\left[\mathbf{Y}^{(1)}\right]\Psi_{\rm int}^{(2)}\left[\mathbf{Y}^{(2)}\right]\,. \tag{D.2}$$

and show that this form will be preserved during evolution, even adding the SN term, which is now

$$V_{\rm SN} = \frac{Gm^2}{2}\sum_{I,J=1}^{2}\sum_{i=1}^{n_I}\sum_{j=1}^{n_J}\frac{\left|\Psi_{\rm CM}(\tilde{\mathbf{z}}^{(1)}, \tilde{\mathbf{z}}^{(2)})\Psi_{\rm int}^{(1)}(\tilde{\mathbf{Y}}^{(1)})\Psi_{\rm int}^{(2)}(\tilde{\mathbf{Y}}^{(2)})\right|^2}{\left|\mathbf{L}^{(JI)} + \mathbf{x}^{(I)} + \mathbf{y}_i^{(I)} - \tilde{\mathbf{z}}^{(J)} + \tilde{\mathbf{y}}_j^{(J)}\right|}d\tilde{\mathbf{z}}^{(1)}d\tilde{\mathbf{z}}^{(2)}d\tilde{\mathbf{Y}}^{(1)}d\tilde{\mathbf{Y}}^{(2)} \tag{D.3}$$

Here we have denoted

$$L^{(IJ)} \equiv X^{(J)} - X^{(I)} . \tag{D.4}$$

Terms with $I = J$ have already been dealt with, and gives rise to the SN correction within object I. We will only have to deal with cross terms. In doing so, we shall assume each object's CM moves very little from its zero point, and carry out Taylor expansion. Note that because these objects are already macroscopically separated, with $L^{(IJ)}$ comparable to or greater than the size of each object, the expansion here will be valid for the cross term as long as the CM motion of each object is much less than its size.

The zeroth order expansion in CM motion, $V_{\rm SN}^{(0)}$ gives rise to SN coupling between the objects' internal motions. Fortunately, that does not entangle their internal motions, and preserves the form of Eq. (D.2).

The first order in CM motion gives (after conversion of summation over atoms into ensemble average, the same as we did in Appendix B, and removing a constant):

$$\bar{V}_{\rm SN}^{(1)} = -x_{\rm CM}^{(1)}\mathcal{E}'_{21} - x_{\rm CM}^{(2)}\mathcal{E}'_{12} = \left[x_{\rm CM}^{(1)} - x_{\rm CM}^{(2)}\right]\mathcal{E}'_{12} \tag{D.5}$$

where $\mathcal{E}_{12}$ is the interaction energy between the objects, as a function of their separation

$$\mathcal{E}_{12}(\mathbf{x}) \equiv -\int d^3\mathbf{y}d^3\mathbf{z}\frac{G\tilde{\varrho}_{\rm int}^{(1)}(\mathbf{y})\,\tilde{\varrho}_{\rm int}^{(2)}(\mathbf{z})}{|\mathbf{z}+\mathbf{x}-\mathbf{y}+\mathbf{L}^{(12)}|} \tag{D.6}$$

This describes the tendency of these objects to fall into each other. Similarly, we obtain the second order, which gives (apart from a constant)

$$\bar{V}_{\rm SN}^{(2)} = \frac{\mathcal{E}''_{12}}{2}\left[\left(x_{\rm CM}^{(1)} - \langle x_{\rm CM}^{(2)}\rangle\right)^2 + \left(x_{\rm CM}^{(2)} - \langle x_{\rm CM}^{(1)}\rangle\right)^2\right] , \tag{D.7}$$

which justifies Eq. (27).