THE JOURNAL OF SYMBOLIC LOGIC
Volume 40, Number 1, March 1975

ON CHARACTERIZING SPECTOR CLASSES

LEO A. HARRINGTON AND ALEXANDER S. KECHRIS[1]

We study in this paper characterizations of various interesting classes of relations arising in recursion theory. We first determine which Spector classes on the structure of arithmetic arise from recursion in normal type 2 objects, giving a partial answer to a problem raised by Moschovakis [8], where the notion of Spector class was first essentially introduced. Our result here was independently discovered by S. G. Simpson (see [3]). We conclude our study of Spector classes by examining two simple relations between them and a natural hierarchy to which they give rise.

The second part of our paper is concerned with finding structural characterizations of classes of relations on the reals in the spirit of Moschovakis [7]. Our main result provides a *single* abstract characterization for the class of Π^1_3 relations on the reals and the 2-envelope of ${}^3E$, the first one being valid if projective determinacy is true, the second if $V = L$ is true.

§1. On Spector classes.

1A. *Characterizing the* 1-*envelope of a type* 2 *object.* A *Spector class* Γ on the structure of arithmetic is a collection of relations on ω which contains all the recursive relations, is closed under the positive propositional connectives, existential and universal number quantification and recursive substitution, is parametrized (i.e. contains a complete relation) and has the pre-well-ordering property. An equivalent definition can be given in terms of computation theories as defined in Moschovakis [8]: A Spector class Γ is just the collection of semirecursive relations of a Spector theory Θ on ω. A systematic study of Spector classes is given in Chapter 9 of Moschovakis [6].

Recursion theory is full of examples of Spector classes. The classical example is Π^1_1. Then we have Σ^1_2 and assuming projective determinacy Σ^1_{2n}, Π^1_{2n+1} for all n (assuming $V = L$ all the Σ^1_n with $n > 1$ are Spector classes). The theory of objects of finite higher type is another basic source of Spector classes: If F is a normal finite type object, then the 1-*envelope* of F, i.e. the class of semirecursive in F relations on ω, is a Spector class (a type n object F is called *normal* if ${}^nE$ is recursive in F, where ${}^nE$ is equality of type $n - 1$ objects).

After some reflection one realizes that although all the above examples (and many others we did not mention here) have a lot in common (and this is revealed by the general theory of Spector classes), they also have many significant structural differences. It thus becomes one of the most interesting and important problems of abstract recursion theory to find structural properties which characterize various

Received September 1, 1973.

[1] During the preparation of this paper the second author was partially supported by NSF grant P-29079.

significant subcollections of Spector classes. We characterize in this paper those Spector classes which are 1-envelopes of type 2 normal objects. The next step in this direction is clearly the characterization of 1-envelopes of higher than type 2 normal objects. Moschovakis [7] has shown that 1-envelopes of normal objects of type >2 are never 1-envelopes of normal type 2 objects. Harrington [2] proved that the 1-envelope of a normal object of type ≥ 3 is also the 1-envelope of a type 3 object and he gave a characterization of the collection of 1-envelopes of normal type 3 objects. It seems nevertheless that there is still room for improvement here.

The key to our result is the idea of using properties of the companion admissible structure of a Spector class Γ to characterize Γ itself, a possibility which already has been foreseen in the last paragraph of Moschovakis [8]. The notion of the companion of Γ is defined in Moschovakis [6]. We summarize below its basic properties.

DEFINITION. Let Γ be a Spector class on ω. A *companion* for Γ is a structure $\mathscr{M} = \langle M, \in, R\rangle$ such that

(a) M is a transitive set, $R \subseteq M$ and M is admissible with respect to R.

(b) M is projectible into ω (i.e. there is a $\Delta_1(\mathscr{M})$ map of a subset of ω onto M) and resolvable (i.e. there is a $\Delta_1(\mathscr{M})$ map τ: Ord $\bigcap M \to M$ such that $M = \bigcup_\xi \tau(\xi)$).

(c) The relations on ω which are $\Sigma_1(\mathscr{M})$ are exactly those in Γ.

The companion theorem of Moschovakis [6] states that each Spector class Γ has a companion $\mathscr{M}$ which moreover is "unique" in the sense that if $\mathscr{M}^1 = \langle M^1, \in, R^1\rangle$ is any other companion then $M = M^1$ and $\Sigma_1(\mathscr{M}) = \Sigma_1(\mathscr{M}^1)$. We shall denote by $\mathscr{M}_\Gamma = \langle M_\Gamma, \in, R\rangle$ "the" companion of Γ.

DEFINITION. A Spector class Γ is called *Mahlo* if $\mathscr{M}_\Gamma$ is Mahlo, i.e., for any $x \in M_\Gamma$ and any $A \subseteq M_\Gamma$, $A \in \Delta_1(\mathscr{M}_\Gamma)$, there is an $N \in M_\Gamma$, N transitive, $x \in N$ such that $\langle N, \in, A \cap N\rangle$ is admissible.

A Spector class Γ is called *inaccessible* iff $\mathscr{M}_\Gamma$ is inaccessible, i.e. for all $x \in M_\Gamma$, there is a transitive $N \in M_\Gamma$ such that $x \in N$ and $\langle N, \in\rangle$ is admissible.

We are now ready to prove the promised characterization of the 1-envelope of a type 2 normal object. The two results below are independently due to S. G. Simpson (see [3]).

THEOREM 1. *Given a Spector class* Γ, *the following are equivalent*:

(a) Γ *is the* 1-*envelope of a normal type* 2 *object.*

(b) Γ *is not Mahlo.*

THEOREM 2. *Given a Spector class* Γ, *the following are equivalent*:

(a) Γ *is the* 1-*envelope of the superjump of a normal type* 2 *object.*

(b) Γ *is inaccessible but not Mahlo.*

(c) Γ *is the* 1-*envelope of a type* 2 *object* F *with* E_1 (*the Tugue object*) *recursive in* F.

PROOF. Let $F \subseteq {}^\omega\omega$ be a normal type 2 object. Let $\mathscr{M}(F) = \mathscr{M}_{1\mathrm{en}(F)}$. It is well known that $\mathscr{M}(F)$ has the nice characterization as the first admissible structure relative to F. That is, for each ordinal σ, define $L_\sigma[F]$ by

$$\begin{aligned} L_0[F] &= HF = \text{hereditarily finite sets},\\ L_{\sigma+1}[F] &= \{x \subseteq L_\sigma[F]\text{: } x \text{ is 1st order definable with parameters}\\ &\qquad\qquad \text{over the structure } \langle L_\sigma[F], \in, F \cap L_\sigma[F]\rangle\}\\ L_\lambda[F] &= \textstyle\bigcup_{\sigma<\lambda} L_\sigma[F]. \end{aligned}$$

Let $\mathcal{M}_\sigma(F)$ denote the structure $\langle L_\sigma[F], \in, F \cap L_\sigma[F]\rangle$. An ordinal will be called *F-admissible* if $\mathcal{M}_\sigma(F)$ is an admissible structure; σ will be called F-inaccessible if σ is F-admissible and the limit of F-admissibles. We denote by ω_1^F the least nonrecursive in F ordinal. Then $\mathcal{M}(F) = \mathcal{M}_{\omega_1^F}(F)$ and ω_1^F is the 1st F-admissible $> \omega$. Also if $S(F) =$ superjump of F, $\mathcal{M}(S(F)) = \mathcal{M}_{\omega_1^{S(F)}}(F)$ and $\omega_1^{S(F)}$ is just the 1st F-inaccessible. This makes clear that (a) $\Rightarrow$ (b) in both Theorems 1 and 2.

To demonstrate the other implication we shall need a few more facts about the $\mathcal{M}_\sigma(F)$'s. For ordinals σ "low" in the $\mathcal{M}(F)$ hierarchy (for our purposes $\sigma < \omega_1^{S(F)}$ is low enough), there is a new real in $L_{\sigma+1}[F]$, that is, ${}^\omega\omega \cap (L_{\sigma+1}[F] - L_\sigma[F]) \neq \varnothing$. Let $I_\sigma(F)$ be the 1st (in the sense of the natural well-ordering of $L_{\sigma+1}[F]$) member of this set.

The above will allow us to define normal type 2 objects F, with certain desirable properties, by induction; note that $L_1[F]$ in no way depends on F and, given $L_\sigma[F]$, $L_{\sigma+1}(F)$ is completely determined given $F \cap L_\sigma[F]$.

Now, let Γ be a Spector class. Using the resolvability of $\mathcal{M}_\Gamma$ it is easy to see that $\mathcal{M}_\Gamma$ has the form $\langle L_\tau[A], \in, A\rangle$ for some ordinal τ and some $A \subseteq \tau$. For $\sigma \leq \tau$, let $\mathcal{M}_\sigma$ denote the structure $\langle L_\sigma[A], \in, A \cap \sigma\rangle$. Assuming Γ is not Mahlo, we may choose A so that for all $\sigma < \tau$, $\mathcal{M}_\sigma$ is not admissible. It should now be fairly easy to find a normal type 2 object F such that ${}_1\mathrm{en}(F) = \Gamma$. Just let $F = {}^2E \vee G$, where G is (inductively) characterized by: for $\alpha \in {}^\omega\omega$, $\alpha \in G$ iff $\exists\sigma(\alpha = I_\sigma(F) = I_\sigma({}^2E \vee G)$ and $\sigma < \omega_1^F = \omega_1^{({}^2E\vee G)}$ and $\sigma \in A)$.

To show that this F has the desired properties: the map, $\sigma \to \mathcal{M}_\sigma(F)$, $\sigma < \tau$, is Δ_1 over $\mathcal{M}_\Gamma = \mathcal{M}_\tau$. Thus τ is F-admissible and so $\tau \geq \omega_1^F$. By the definition of G, $A \cap \omega_1^F$ is Δ_1 over $\mathcal{M}_{\omega_1^F}(F)$, and thus $\mathcal{M}_{\omega_1^F}$ is admissible. So $\omega_1^F = \tau$ and $\mathcal{M}_\tau = \mathcal{M}_\tau(F)$.

If Γ is also inaccessible, then we can similarly find a normal type 2 object F such that $\Gamma = {}_1\mathrm{en}(S(F))$. Let $F = {}^2E \vee G$, where G is (inductively) characterized by: for $\alpha \in {}^\omega\omega$, $\alpha \in G$ iff $\exists\sigma(\alpha = I_\sigma(F) = I_\sigma({}^2E \vee G)$, and $\sigma < \omega_1^{S(F)} = \omega_1^{S({}^2E\vee G)}$, and σ is the ξth F-admissible ordinal, and $\xi \in A)$.

As before, the map $\sigma \to \mathcal{M}_\sigma(F)$, $\sigma < \tau$, is Δ_1 over $\mathcal{M}_\Gamma = \mathcal{M}_\tau$. For any $\sigma < \tau$, since Γ is inaccessible, there is δ, $\sigma < \delta < \tau$, such that $L_\delta(\mathcal{M}_\sigma(F))$ is admissible, and so, by the innocuous definition of G, there is an F-admissible ordinal $> \sigma$ and $\leq \delta$. Thus τ is F-inaccessible and so $\tau \geq \omega_1^{S(F)}$. Since the F-admissible ordinals $< \omega_1^{S(F)}$ have order type $\omega_1^{S(F)}$, $A \cap \omega_1^{S(F)}$ is Δ_1 over $\mathcal{M}_{\omega_1^{S(F)}}(F)$, and therefore, as before, $\omega_1^{S(F)} = \tau$ and $\mathcal{M}_{\omega_1^{S(F)}}(F) = \mathcal{M}_\tau$.

The above two constructions yield that (b) $\Rightarrow$ (a) in Theorems 1 and 2. To complete the proof of Theorem 2, notice that for any normal type 2 object F: $E_1 = S({}^2E) \leq S(F)$, and thus (a) $\Rightarrow$ (c); if $E_1 \leq F$ then for all $\alpha \in {}^\omega\omega \cap L_{\omega_1^F}[F]$ the hyperjump of $\alpha \in L_{\omega_1^F}[F]$, and thus ${}_1\mathrm{en}(F)$ is inaccessible. So (c) $\Rightarrow$ (b). $\square$

[It may be interesting to note that the proof of a result of Moschovakis (in [7]) can be filtered through Theorem 1. Moschovakis has proven: for a 2-pointclass (see §2) Γ with certain closure properties (the most important of which is that $\Pi_1^1 \subseteq \Gamma \cap \check{\Gamma}$), the class of relations on ω in Γ is not the 1-envelope of any normal type 2 object. A 2-pointclass Γ, with the properties Moschovakis assumes, has been shown (see [4]) to have a very strong reflection property, and even the mildest reflection property implies Mahlo-ness.]

1B. *A hierarchy of Spector classes.* Moschovakis [8] defined a partial preordering $\Theta \leq H$ between computation theories which gives rise to the important equivalence relation $\Theta \sim H \Leftrightarrow \Theta \leq H \;\&\; H \leq \Theta$. For any computation theory Θ denote by Γ_Θ the class of semirecursive in Θ relations. We have already mentioned that a Spector class Γ is equal to Γ_Θ for some Spector theory Θ. But a moment's reflection reveals that $\Theta \leq H \Leftrightarrow \Gamma_\Theta \subseteq \Gamma_H$. This leads us to define a natural partial ordering among Spector classes, namely

$$\Gamma \leq \Gamma' \Leftrightarrow \Gamma \subseteq \Gamma'.$$

We shall also consider another kind of relation among Spector classes which we shall call *strong inclusion*, i.e.,

$$\Gamma \prec \Gamma' \Leftrightarrow \Gamma \subseteq \Delta'$$

where, for any Spector class Γ, $\check{\Gamma}$ denotes the class of negations of relations in Γ and $\Delta = \Gamma \cap \check{\Gamma}$. Note that $\Gamma \prec \Gamma' \Rightarrow \Gamma \subsetneqq \Gamma'$, but $\Gamma \subsetneqq \Gamma' \not\Rightarrow \Gamma \prec \Gamma'$ (e.g., $\Gamma = \Pi_1^1$, $\Gamma' = \Pi_1^1(\alpha)$, where $\alpha \in {}^\omega\omega$ is such that $\omega_1^\alpha = \omega_1$ but α is not Δ_1^1).

Let us note now some simple facts concerning the two relations defined above:

(A) If $\mathfrak{C}$ = collection of all Spector classes, then $\langle \mathfrak{C}, \leq \rangle$ has a least element namely Π_1^1. Moreover, for each $\Gamma \in \mathfrak{C}$, $\{\Gamma' : \Gamma \prec \Gamma'\}$ has a $\leq$-least element namely $\Pi_1^1(A)$, where $A \subseteq \omega$ is a complete Γ set. Both of these facts are immediate consequences of a result of Moschovakis [7]. We denote by $j\Gamma$ the $\leq$-least member of $\{\Gamma' : \Gamma \prec \Gamma'\}$ and call it the *jump* of Γ. It is now easy to see that the companion of $j\Gamma$ is just the next admissible structure of the companion of Γ.

(B) To every Spector class Γ there is attached a natural ordinal which we denote by δ_Γ. By definition

$$\delta_\Gamma = \sup\{\xi : \xi \text{ is the length of a pre-well-ordering in } \Delta\}.$$

It is easy then to see that

$$\delta_\Gamma = o(M_\Gamma) = \text{least ordinal not in } M_\Gamma.$$

The so-called "Spector criterion" goes through, namely

$$\Gamma \leq \Gamma' \Rightarrow (\delta_\Gamma < \delta_{\Gamma'} \Leftrightarrow j\Gamma \leq \Gamma').$$

Using the partial ordering on $\mathfrak{C}$ and the jump operation we can now define as usual a *natural hierarchy* of Spector classes as follows:

$$\Gamma_0 = \Pi_1^1, \qquad \Gamma_{\xi+1} = j\Gamma_\xi,$$
$$\Gamma_\lambda = \leq\text{-least upper bound of } \{\Gamma_\xi\}_{\xi<\lambda}, \text{ if it exists.}$$

Let λ_0 be the least ordinal for which this hierarchy stops, i.e. $\leq$-l.u.b. $\{\Gamma_\xi\}_{\xi<\lambda_0}$ does not exist. Let also $\{a_\xi\}_{\xi<\aleph_1}$ be the increasing enumeration of all the countable admissibles.

THEOREM. (a) *The natural hierarchy of Spector classes stops at the least nonprojectible ordinal.*

(b) *For each $\xi < \lambda_0$ = least nonprojectible, $\mathscr{M}_{\Gamma_\xi} = \langle L_{a_\xi}, \in \rangle$.*

PROOF. It is quite simple to see, using the companion theorem of Moschovakis [6] which we summarized above, that $\lambda_0 \geq$ least nonprojectible and for $\xi < \lambda_0$, $\mathscr{M}_{\Gamma_\xi} = \langle L_{a_\xi}, \in \rangle$. To prove that λ_0 = least nonprojectible $=_{\text{def}} \pi_0$, it is enough to show that Γ_{π_0} does not exist. If it did then for every real $\alpha \in {}^\omega\omega$ such that $\omega_1^\alpha = \pi_0$ we would have $\Gamma_{\pi_0} \subseteq \Pi_1^1(\alpha) = \Sigma_1(\langle L_{\omega_1^\alpha}[\alpha], \in\rangle)$. But then by Theorem 3.4 of Sacks [9, p. 297] we must have $\Gamma_{\pi_0} = \Sigma_1(\langle L_{\pi_0}, \in\rangle)$ thus π_0 is projectible, a contradiction. □

Using the results of §1A we can also see that if ρ_0 is the least Mahlo ordinal then $\{\Gamma_\xi\}_{\xi<\rho_0}$ consists of 1-envelopes of normal type 2 objects but Γ_{ρ_0} is of course not the 1-envelope of a normal type 2 object. On the other hand Γ_{ρ_0} is the 1-envelope of the superjump, where 1-envelope is interpreted here in the modified sense introduced by Harrington [2]. Thus in a sense the superjump appears as the "least" non-type 2 object.

§**2. Structural properties of 2-pointclasses.** Let $\mathscr{R} = {}^\omega\omega$ be the set of *reals* and consider the product spaces $\mathscr{X} = X_1 \times X_2 \times \cdots \times X_k$, where $X_i = \omega$ or $\mathscr{R}$. We call subsets of these product spaces *pointsets* and we think of them interchangeably as relations, writing sometimes $A(x)$ instead of $x \in A$, when $A \subseteq \mathscr{X}$. A 2-pointclass (according to the terminology of Moschovakis [7]) is a collection of pointsets. We call a 2-pointclass Γ *weakly Spector* if it contains all recursive relations (in all product spaces), is closed under the positive propositional connectives, number quantification (this means for example that if $R(x, n) \in \Gamma$ then $\exists n R(x, n) \in \Gamma$) and *universal* quantification over $\mathscr{R}$, is ω-parametrized (i.e., for any $\mathscr{X}$ there is a $G \subseteq \mathscr{X} \times \omega$ such that $G \in \Gamma$ and $\{G_n : n \in \omega\}$, where $G_n = \{x : G(x, n)\}$, coincides with the subsets of $\mathscr{X}$ which belong to Γ) and has the pre-well-ordering property. Among weakly Spector classes we can again define $\Gamma \le \Gamma' \Leftrightarrow \Gamma \subseteq \Gamma'$ and $\Gamma \prec \Gamma' \Leftrightarrow \Gamma \subseteq \Delta'$. Clearly Π_1^1 is the $\le$-least weakly Spector class. Jumps can be defined as before but we have not been able to prove they always exist. In fact it seems plausible that under assumptions like the axiom of determinacy one should be able to find Γ's with no jump. Nevertheless assuming $V = L$ everything works smoothly.

THEOREM. *$V = L \Rightarrow$ for any weakly Spector 2-pointclass Γ, $j\Gamma$ exists.*

PROOF. Let Γ be a weakly Spector 2-pointclass. Let $A \subseteq \mathscr{R}$ be a complete set in Γ. We prove that

$$j\Gamma = {}_2\mathrm{en}({}^3E, A),$$

where for a higher type object F, ${}_2\mathrm{en}(F)$ is the 2-pointclass of all semirecursive in F relations. That ${}_2\mathrm{en}({}^3E, A)$ is a weakly Spector 2-pointclass strongly containing Γ is well known. Assume now that $\Gamma \prec \Gamma'$. We have to prove ${}_2\mathrm{en}({}^3E, A) \subseteq \Gamma'$. By Moschovakis' characterization in [7] we have only to prove that if $g : \mathscr{R} \times \mathscr{X} \to \omega$ is a partial function whose graph is in Γ' then the relation

$$R(x) \Leftrightarrow \forall\alpha[g(\alpha, x) \text{ is defined}] \;\&\; \exists\alpha[g(\alpha, x) = 0]$$

is in Γ'. But $Q(\alpha, x) \Leftrightarrow g(\alpha, x)$ is defined $\Leftrightarrow \exists n(g(\alpha, x) = n)$ is in Γ' and thus so is $\forall\alpha(g(\alpha, x)$ is defined). But assuming $\forall\alpha(g(\alpha, x)$ is defined), we have that

$$P(\alpha) \Leftrightarrow g(\alpha, x) = 0 \Leftrightarrow \exists n(g(\alpha, x) = n)$$

is in $\Delta(x) = \Gamma(x) \cap \Gamma(x)$, where $\Gamma(x) = \{P \subseteq \mathscr{X}$: For some $R \in \Gamma$, $y \in P \Leftrightarrow (x, y) \in R\}$. Thus if $P \neq \varnothing$ its least element in the canonical well-ordering of L is a real in $\Delta(x)$. Thus $\exists\alpha(g(\alpha, x) = 0) \Leftrightarrow \exists\alpha \in \Delta(x)(g(\alpha, x) = 0)$, and by a standard pre-well-ordering theorem (see [1, p. 711 (X)]) this last relation is in Γ. □

COROLLARY (OF THE PROOF). $V = L \Rightarrow j\Pi^1_1 = {}_2\mathrm{en}({}^3E)$.

Although there may be some weakly Spector 2-pointclasses with no jumps we can still prove under various determinacy hypotheses that many *interesting* weakly Spector 2-pointclasses have jumps. We give below an important example which ties up nicely (though a bit unexpectedly) with our previous result.

THEOREM. *Projective determinacy* $(PD) \Rightarrow$ *for each odd* n, $j\Pi^1_n = \Pi^1_{n+2}$.

PROOF. Assume projective determinacy and take $n = 1$ for simplicity. Clearly $\Pi^1_1 \prec \Pi^1_3$ and Π^1_3 is a weakly Spector 2-pointclass (see Martin [5], Moschovakis [1]). Let Γ be such that $\Pi^1_1 \prec \Gamma$. Then $\Sigma^1_1 \subseteq \Gamma$ so $\Pi^1_2 \subseteq \Gamma$. If $A \in \Sigma^1_2$, $A \subseteq \mathscr{X}$ then

$$x \in A \Leftrightarrow \exists \alpha B(x, \alpha) \Leftrightarrow \exists \alpha \in \Delta^1_2(x) B(x, \alpha),$$

where the last equivalence follows from the basis theorem. But $\alpha \in \Delta^1_2(x) \Rightarrow \alpha \in \Delta(x)$. So

$$x \in A \Leftrightarrow \exists \alpha \in \Delta(x) B(x, \alpha),$$

thus $A \in \Gamma$. Since $\Sigma^1_2 \subseteq \Gamma$, clearly $\Pi^1_3 \subseteq \Gamma$. $\square$

REFERENCES

[1] J. W. ADDISON and Y. N. MOSCHOVAKIS, *Some consequences of the axiom of definable determinateness*, ***Proceedings of the National Academy of Sciences, U.S.A.***, vol. 59 (1968), pp. 708–712.

[2] L. HARRINGTON, *Contributions to recursion theory on higher types*, Ph.D. Thesis, M.I.T., Cambridge, Massachusetts, 1973.

[3] L. HARRINGTON, A. S. KECHRIS and S. G. SIMPSON, 1-*envelopes of type* 2 *objects*, ***Notices of the American Mathematical Society***, vol. 20 (1973), A-587.

[4] A. S. KECHRIS, *The theory of countable analytical sets*, ***Transactions of the American Mathematical Society*** (to appear).

[5] D. A. MARTIN, *The axiom of determinateness and reduction principles in the analytical hierarchy*, ***Bulletin of the American Mathematical Society***, vol. 74 (1968), pp. 687–689.

[6] Y. N. MOSCHOVAKIS, *Elementary induction on abstract structures*, North-Holland, Amsterdam, 1974.

[7] ———, *Structural characterizations of classes of relations*, ***Generalized recursion theory*** (Oslo, 1972) (J. Fenstad and P. Hinman, Editors), North-Holland, Amsterdam, 1973.

[8] ———, *Axioms for computation theories—first draft*, ***Logic Colloquium '69***, North-Holland, Amsterdam, 1971, pp. 199–255.

[9] G. E. SACKS, *F-recursiveness*, ***Logic Colloquium '69***, North-Holland, Amsterdam, 1971, pp. 289–303.

STATE UNIVERSITY OF NEW YORK AT BUFFALO
AMHERST, NEW YORK 14226

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASSACHUSETTS 02139