

Finally, we would like to mention that one can easily write down the analogs of Eqs. (12) and (13) in case of vector and axial vector mesons to obtain

$$m_{K_A}^2 - m_{A_1}^2 = m_{K^*}^2 - m_\rho^2 \quad (14)$$

$$G_{K_A} - G_{A_1} = G_{K^*} - G_\rho \quad (15)$$

It is interesting to note that Eq. (14) leads to  $m_{K_A} \approx 1200$  MeV. Also, Eq. (15) is quite compatible with the Weinberg second sum-rule predictions for  $SW(2)$  symmetry.

Details of this work with more applications will be published elsewhere.

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<sup>1</sup>See, e.g., R. F. Dashen, Phys. Rev. **183**, 1245

(1969); Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961), and **124**, 246 (1961).

<sup>2</sup>M. Gell-Mann, Physics **1**, 63 (1964).

<sup>3</sup>M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968), referred to hereafter as GMOR. Their parameter  $c$  is related to our  $a$  by  $c = \sqrt{2}a$ .

<sup>4</sup>V. S. Mathur, S. Okubo, and J. Subba Rao, to be published, where the value of  $b$  is computed to be  $b \approx -0.13$  rather than  $b \approx -1$ . See also S. Okubo, Phys. Rev. (to be published).

<sup>5</sup>See, e.g., S. Weinberg, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September 1968, edited by J. Prentki and J. Steinberger (CERN Scientific Information Service, Geneva, Switzerland, 1968).

<sup>6</sup>R. E. Marshak, N. Mukunda, and S. Okubo, Phys. Rev. **137**, B698 (1965); Y. Hara, Phys. Rev. **139**, B134 (1965).

<sup>7</sup>The explicit solution obtained in Ref. 4 also gives numerically  $f_\kappa = 0.2f_\pi$ .

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## VERY HIGH-ENERGY COLLISIONS OF HADRONS

Richard P. Feynman

California Institute of Technology, Pasadena, California

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Proposals are made predicting the character of longitudinal-momentum distributions in hadron collisions of extreme energies.

Of the total cross section for very high-energy hadron collisions, perhaps  $\frac{1}{3}$  is elastic and 10% of this is easily interpreted as diffraction dissociation. The rest is inelastic. Collisions involving only a few outgoing particles have been carefully studied, but except for the aforementioned elastic and diffractive phenomena they all fall off (probably as a power of the energy at high energy). The constant part of the total inelastic cross section cannot come from them. And we know that at such energies, the majority of collisions lead to a relatively large number of secondaries (perhaps the multiplicity increases logarithmically with energy). These collisions have not been studied extensively because, with the large number of particles, so many quantities or combinations of quantities can be evaluated that one does not know how to organize the material for analysis and presentation.

It is the purpose of this paper to make suggestions as to how these cross sections might behave so that significant quantities can be extracted from data taken at different energies. These suggestions arose in theoretical studies from several directions and do not represent the result of consideration of any one model. They are

an extraction of those features which relativity and quantum mechanics and some empirical facts<sup>1</sup> imply almost independently of a model. I have difficulty in writing this note because it is not in the nature of a deductive paper, but is the result of an induction. I am more sure of the conclusions than of any single argument which suggested them to me for they have an internal consistency which surprises me and exceeds the consistency of my deductive arguments which hinted at their existence.

Only the barest indications of the logical bases of these suggestions will be indicated here. Perhaps in a future publication I can be more detailed.<sup>2</sup>

Supposing that transverse momenta are limited in a way independent of the large  $z$ -component momentum of each of the two oncoming particles in the center-of-mass system (so  $s = 2W^2$ ), an analysis of field theory in the limit of very large  $W$  suggests the appropriate variables to use for the various outgoing particles in comparing experiments at various values of  $W$  in the c.m. system. They are the longitudinal momentum  $P_z$  in ratio to the total available  $W$ , i.e.,  $x = P_z/W$ , and the transverse momenta  $Q$  in absolute units.

Differential cross sections for  $dx d^2Q$  of the various outgoing particles will then have simple properties as a function of  $W$ . Negative  $x$  means particles with  $P_z$  negative.

First we must distinguish exclusive and inclusive experiments. In exclusive experiments, we ask that certain particles, with given  $x$  and  $Q$ , be formed, and no others. An example is a two-body charge-exchange reaction. A typical exclusive reaction is

$$A + B \rightarrow \sum_1^n C_i + \sum_1^{n'} D_i,$$

where  $A$  is to the right,  $B$  to the left, and  $C_1, C_2, \dots, C_n$  are definite particles with definite  $Q$ 's and  $x$ 's all moving to the right ( $x > 0$ ), whereas the  $D_1, D_2, \dots, D_{n'}$  are moving to the left ( $x < 0$ ). The cross section should then vary, for sufficiently high  $W$ , as  $s^{2\alpha(t)-2}$  or  $(W^2)^{2\alpha(t)-2}$ , where  $\alpha(t)$  is the  $\alpha$  of the highest Regge trajectory capable of converting the quantum numbers of  $A$  into those of the sum of the  $C$ 's, and  $t$  is the transverse momentum difference of  $A$  and the sum of the  $C$ 's.

This is an evident expectation from Regge theory and should be as approximately valid as that theory is for two-body reactions. The additional point here is the clarification of using the variable  $x$  to compare experiments at various energies. If no unitary quantum number is required to be exchanged, so the  $C$  group has the same quantum numbers as  $A$ , this  $C$  group can rise from  $A$  via diffraction dissociation and the cross section should approach a constant ratio to the elastic cross section at this value of  $t$  (i.e., it should be constant if the elastic cross section is).

Next, an inclusive experiment is one in which we look for special particles with  $x, Q$  in the final state but we allow anything else to be produced also. An example is a measurement of the mean number of  $K^+$ 's produced with given  $Q, x$  in a  $pp$  reaction. Such cross sections should approach a constant as  $W \rightarrow \infty$ .

How can these be reconciled? Why does the cross section fall if, for example, in a two-body reaction we must exchange 3-component of isospin? Because under such circumstances, the current of 3-component isospin must suddenly reverse from right moving to left moving. Thus if any fields are connected to such currents as sources, they would be expected to radiate (in a manner analogous to bremsstrahlung). To be an exclusive experiment (say, pure two-body), we require that no such radiation occur, a condition

becoming more and more difficult to satisfy as the energy rises and the current reversal is sharper.

This leads us to expect, in the majority of collisions, many particles over a wide range of  $x$ , but their characteristics for the smaller values of  $x$  are easy to envision. By Lorentz transformation, the fields to be radiated are becoming narrower and narrower in the  $z$  direction as  $W$  rises. The energy in this field is therefore distributed in a  $\delta$  function in  $z$ . Fourier analyzed, this means that the field energy is uniform in momentum,  $dP_z$ . Since each particle of mass  $\mu$  carries energy  $E = (\mu^2 + P_z^2 + Q^2)^{1/2}$ , if we suppose that the field energy is distributed among the various kinds of particles in fixed ratios (independent of energy  $W$ ), we conclude that the mean number of particles of any kind and of fixed  $Q$  is distributed as  $dP_z/E$  for not too large  $x$ . That is, the probability of finding, among all the emitted particles, a particle of kind  $i$ , transverse momentum  $Q$ , and mass  $\mu_i$  is of the form  $f_i(Q, P_z/W)dP_z d^2Q / (\mu_i^2 + Q^2 + P_z^2)^{1/2}$ , where  $f_i(Q, x)$  is ultimately independent of  $W$  and has a limit  $F_i(Q)$  for small  $x$ . As  $W \rightarrow \infty$ , for any finite  $x$ ,  $dP_z/E$  becomes  $dx/x$ , of course.

Because of this  $dx/x$  behavior, the mean total number (or "multiplicity") of any kind of particle rises logarithmically with  $W$ . We need not decide what are "primarily emitted units" and what are secondaries arising from their decay, for the results so far stated are in a form that does not depend on that. If we imagine some primary independently emitted units, however, their number  $\bar{n}$  would also rise logarithmically with energy, and the probability that none of them would be emitted might be  $e^{-\bar{n}}$  (as suggested by a Poisson distribution) which would then fall as a power of the energy, accounting for the Regge expressions which we are supposing are valid for such exclusive collisions.

We can extend this idea to other amplitudes which involve a similar  $\bar{n}$ . In particular, we find that the probability that  $A+B \rightarrow C + \text{anything}$  should vary as  $(1-x_C)^{1-2\alpha(t)} dx_C$ , where  $C$  is moving to the right with almost all the momentum of  $A$  (that is, for  $1-x_C$  small). Here  $\alpha(t)$  is the highest trajectory (excluding the Pomeron chukon) which could carry off the quantum numbers (and squared momentum transfer  $t$ ) needed to change  $A$  to  $C$ .

Thus the Regge trajectory function  $\alpha(t)$  appears not only in an interaction (as  $s^{2\alpha-2}$ ) but also in an emission process, reminiscent of the close re-

relationship of virtual interaction and real emission that Yukawa emphasized.

Finally, for those special reactions which are partially exclusive, in which anything can be emitted except that a hadron must be transferred from the right-moving to the left-moving system (carrying its fermionic and half-integral spin character) the cross section should vary as  $1/s$ . Of this last conclusion, I am less sure than of

the others.

<sup>1</sup>I have taken the approximate existence of Regge poles, the constant total cross sections, and the constant transverse-momentum distributions as empirical facts.

<sup>2</sup>For a somewhat more detailed description, see R. P. Feynman, in Proceedings of the Third Topical Conference on High Energy Collisions of Hadrons, Stony Brook, N. Y., 1969 (to be published).

## GAUGE INVARIANCE, DUALITY, AND $N_\alpha$ - $N_\gamma$ EXCHANGE DEGENERACY IN PHOTOPRODUCTION\*

D. P. Roy

Department of Physics, University of Toronto, Toronto, Ontario, Canada

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From gauge invariance and duality we obtain an exchange degeneracy between  $N_\alpha$  and  $N_\gamma$  contributions to pion photoproduction, which is exact for  $u=M^2$  but approximate for  $u \approx 0$ . This may account for the absence of the wrong-signature dip in backward photoproduction.

Recent data of Anderson *et al.*<sup>1</sup> on backward photoproduction of charged pions clearly show the absence of any dip around  $u \approx -0.15 \text{ GeV}^2$ , which is hard to understand on the basis of a simple  $N_\alpha$ -trajectory exchange. Alternative explanations,<sup>2</sup> based on the assumption of a dominant isospin- $\frac{3}{2}$  exchange, seem to be unsatisfactory on the basis of neutral-pion photoproduction data as well as vector-dominance-model results.<sup>1</sup> However, the above result can be understood in a baryon Regge-trajectory-exchange picture, if the  $N_\alpha$  and  $N_\gamma$  contributions here were approximately exchange degenerate, unlike the case of  $\pi N$  scattering.<sup>3</sup>

We shall see that the requirements of gauge invariance and duality indeed lead to such an approximate exchange degeneracy for the helicity-flip amplitudes, which seem to be the dominant ones. By helicity flip we mean the amplitude with maximum  $u$ -channel helicity projection  $\frac{3}{2}$  ( $A_{1/2,3/2}$  and  $A_{-1/2,3/2}$ ). Of the two independent linear combinations of such amplitudes only one couples to the  $N_\alpha$  (and  $N_\gamma$ ) trajectory. This is the invariant amplitude<sup>4</sup>  $A_2$ . The  $N_\alpha$  residue in  $A_2$  is nonvanishing at the nucleon pole  $u=M^2$ , even though it is a sense-nonsense pole. This is the well-known gauge-invariance requirement which has no analog in purely hadronic processes.

We construct the amplitudes  $A_2^s$  and  $A_2^u$ , corresponding to isospin  $\frac{3}{2}$  in  $s$  and  $u$  channels, from the two isovector photon amplitudes<sup>4</sup>  $A_2^{(+)}$  and  $A_2^{(-)}$ . We have

$$A_2^s = \frac{1}{2}(A_2^{(+)} - A_2^{(-)}). \quad (1)$$

$$A_2^u = \frac{1}{2}(A_2^{(+)} + A_2^{(-)}). \quad (2)$$

Now the  $N_\alpha$  and  $N_\gamma$  contributions to  $A_2^s$  are

$$A_2^s \sim b_\alpha(u) \frac{(s^{\alpha_\alpha(u)-3/2} - t^{\alpha_\alpha(u)-3/2})}{\sin\{\pi[\alpha_\alpha(u) - \frac{1}{2}]\}} + b_\gamma(u) \frac{(s^{\alpha_\gamma(u)-3/2} + t^{\alpha_\gamma(u)-3/2})}{\sin\{\pi[\alpha_\gamma(u) - \frac{1}{2}]\}}, \quad (3)$$

where  $b_\alpha(M^2)$  is related to the gauge-invariant residue function

$$2b_\alpha(M^2)/\pi\alpha' = eg, \quad (4)$$

$$e^2/4\pi = 1/137, \quad g^2/4\pi = 14, \quad \text{and } \alpha' \approx 1 \text{ GeV}^{-2}. \quad (5)$$

Using the duality hypothesis, even in a semilocal sense, it is possible to separate the  $s$ - and  $t$ -channel resonance contributions to the imaginary part of the  $u$ -channel Regge exchange. We get

$$b_\alpha(u) \frac{N^{\alpha_\alpha(u)-1/2}}{\alpha_\alpha(u) - \frac{1}{2}} - b_\gamma(u) \frac{N^{\alpha_\gamma(u)-1/2}}{\alpha_\gamma(u) - \frac{1}{2}} = \sum_i S_i, \quad (6)$$

$$b_\alpha(u) \frac{N^{\alpha_\alpha(u)-1/2}}{\alpha_\alpha(u) - \frac{1}{2}} + b_\gamma(u) \frac{N^{\alpha_\gamma(u)-1/2}}{\alpha_\gamma(u) - \frac{1}{2}} = \frac{\pi eg}{u-M^2} + \sum_j T_j, \quad (7)$$