HIGH MASS PLANETS AND LOW MASS STARS*

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ABSTRACT. There are three main ingredients in brown dwarf theoretical models: (i) The equation of state; (ii) The entropy equation, which relates the internal and atmospheric thermodynamic states; and (iii) The atmospheric boundary condition (the infrared opacity). It is argued that the first two ingredients are very well understood. The opacity is less well understood and the major unresolved problem. Simple scaling laws are described and discussed for the relationships between luminosity (L), mass, opacity and age (t), assuming no thermonuclear energy sources. In the limit of extreme degeneracy, $L \propto t^{-1.25}$. However, detectable brown dwarfs (including VB8B) are still significantly contracting (i.e. actual radius ~10% larger than the zero temperature limit). As a consequence, dlnL/dlnt ~ -1.0 to -1.1 at this epoch for VB8B. Large opacity increases the effect of non-degeneracy. Complicating factors in brown dwarf evolution (super- or sub-adiabaticity, Debye cooling, freezing, differentiation, variable opacity) are discussed but only the latter two seem like to be important.

INTRODUCTION

This contribution deals with bodies which never had significant thermonuclear energy sources. Since deuterium burning is only a minor delaying tactic on the path toward degenerate cooling if the mass is less than 0.08 $M_\odot$, we consider all masses up to this limit. These ''brown dwarfs'' are like Jupiter only simpler. We understand Jupiter well (c.f. Stevenson, 1982) so we should understand brown dwarfs even better. A contrary impression has arisen, primarily because of the

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large discrepancies between the results quoted by Tarter (1975) and the derived scaling laws of Stevenson (1978). In fact, these discrepancies arose not because of inadequacies in the basic physics, but because Tarter constructed purely empirical scaling relations (to satisfy the numerical results of Graboske and co-workers) which did not describe fully degenerate configurations, whereas Stevenson developed an analytical theory which is designed to work best for the degenerate phase.

In this contribution, the ingredients of the theory are described and discussed, so that the reader can judge for him or herself where the uncertainties lie. An analytical model, based on a polytrope (n = 3/2) is developed which encompasses both degenerate and non-degenerate regimes and forms the basis of improved (but more complicated) scaling laws. More complete modeling efforts are also discussed briefly.

The beginning assumptions are:

(i) Brown dwarfs are homogeneous and close to cosmic composition.

(ii) These bodies are fully convective, except for the outermost radiative layer.

(iii) Convective efficiency is high so that the entire body is very close to isentropic.

(iv) The atmospheric radiative boundary condition can be characterized by a single (broad band) opacity.

In fact, none of these assumptions is likely to be strictly correct and each will be challenged as this paper develops. Nevertheless, the uncertainties turn out to be small, except in the opacity which remains a problem, especially for relating what the theorist calculates to what the observer measures.

EQUATION OF STATE

One of the reasons why brown dwarfs are simpler than giant planets is the fact that their equation of state is within 10% of $p \propto \rho^{5/3}$, almost everywhere and irrespective of temperature (i.e. evolutionary age). The total pressure can be approximated as a sum of Fermi, exchange, Coulomb, thermal electronic, and thermal ionic terms:

$$p = p_{Fermi} + p_{exch} + p_{coul} + p_{th,el} + p_{th,ion} \quad (1)$$

$$p_{Fermi} = 9.92 \times 10^{12} (\rho/\mu_e)^{5/3} \text{ dyne.cm}^{-2} \quad (2)$$

$$p_{exch} = -2.06 \times 10^{12} (\rho/\mu_e)^{4/3} f_{ex} \text{ dyne.cm}^{-2} \quad (3)$$

$$p_{coul} = -4.04 \times 10^{12} (\rho/\mu_e)^{4/3} \bar{z}^{5/3} f_c \text{ dyne.cm}^{-2} \quad (4)$$

$$p_{th,el} = f_e n_{el} k_B T = f_{el} (\rho/\mu_e m_p) k_B T \quad (5)$$

$$p_{th,ion} = f_i n_i k_B T = f_i (\rho/\bar{A} m_p) k_B T \quad (6)$$

$$f_{ex} \cong \min (1, 2.4/\Psi)$$

$$f_{el} \cong \min (1, \Psi/2.4) \quad (7)$$

$$f_c \cong \min (1, \Gamma^{1/2}) \quad (8)$$

$$f_i \cong 1 + (\Gamma/300) \quad (9)$$

$$\Psi \equiv 1.594 \times 10^{-5} T/(\rho/\mu_e)^{2/3} \quad (10)$$

$$\Gamma \equiv 2.27 \times 10^5 (\rho/\mu_e)^{1/3} \bar{z}^{5/3}/T \quad (11)$$

where ρ is the density in $g.cm^{-3}$, μ_e is the mean molecular weight per electron ($\mu_e \cong 1.15$ for cosmic composition), $\bar{z}$ is the mean nuclear charge (~1.08), k_B is Boltzmann's constant, T is the temperature in Kelvin, m_p is the mass of the proton, $\bar{A}$ is the mean atomic mass, Ψ is a degeneracy parameter ($\Psi \ll 1$ implies degeneracy), and Γ is a plasma parameter ($\Gamma \ll 1$ implies the Debye-Huckel regime, $\Gamma \gg 1$ implies a strongly coupled Coulomb plasma, $\Gamma \gtrsim 180$ implies a solid lattice). The fudge parameters f_e, f_c, and f_i are given approximate but adequate forms here; for a lengthy discussion, see, for example, DeWitt (1969).

In an isentropic, nearly ideal electron gas, $\gamma \equiv (d\ln T/d\ln\rho) \simeq 2/3$ and all the terms in equation (1) except p_{exch} and p_{coul} scale approximately as $\rho^{5/3}$. As we discuss below, even dense Coulomb plasmas have $\gamma \sim 0.6$ and this scaling still applies. Under the conditions of interest ($\rho \sim 10^2$–10^3 g.cm^{-3}), $p_{exch} + p_{coul}$ is over a factor of ten smaller than the sum of all the other terms in equation (1), irrespective of T.

Equation (1) breaks down in a thin layer which underlies the radiative layer of the atmosphere and overlies the fully ionized interior. In this region, molecules and atoms undergo dissociation and ionization. This layer is typically less than 10^{-3} of the total mass and large variations in its treatment have negligible effect on the resulting static and evolutionary properties of the brown dwarf.

THE ENTROPY

Except in the ideal gas limit, the internal entropy of a brown dwarf is not analytically calculable and one must rely on Monte Carlo simulations (e.g. Hubbard and DeWitt, 1976) or variational models of the fluid state (Stevenson, 1975). In general, the internal entropy can be expressed in the form

$$S_{int} = A \ln (T/\rho^{\gamma}) + B \tag{12}$$

where A, B, and γ are approximately constant. If the entropy is in units of Boltzmann's constant per nucleus, then $A \simeq 2.2$, $\gamma \simeq 0.63$, and $B \simeq -11.6$ (chosen to fit Stevenson, 1975, for a cosmic H/He). This formula includes the electronic contribution, but fails at low temperatures. However, this failure occurs only if hydrogen freezes, a situation never encountered in models of interest. The uncertainties in S_{int} translate to $\lesssim 30\%$ uncertainty in temperature.

For $T \lesssim 2000$ K, the atmospheric entropy is dominated by the translation and rotation of hydrogen molecules:

$$S_{atm} \simeq 1.27 \ln (T/\rho^{0.42}) - 3.0 \tag{13}$$

for cosmic H/He. Above T ~ 2000 K, dissociation of hydrogen molecules becomes increasingly important.

THE OPACITY

If a single, broad-band opacity, κ, characterizes the IR emission of the atmosphere, then

$$p_e \simeq g/\kappa \tag{14}$$

where p_e is the pressure at optical depth unity and g is the local gravitational acceleration. A numerical factor somewhat different from unity could be included on the R.H.S. but in practice, κ is too uncertain to justify bothering about this factor. For the same reason, we will assume that the atmospheric entropy (equation 13) can be evaluated using the thermodynamic state at optical depth unity, even though this level is in a mildly stable part of the atmosphere. (For Jupiter, this assumption is equivalent to an error of only a few percent in κ.)

Figure 1 illustrates the problem of deciding what value of to use. If grain opacity were ignored, $\kappa \sim 10^{-2}$-10^{-1} $cm^2.g^{-1}$ but then a grey atmosphere assumption may be invalid (Lunine et al., this conference). In fact, as they argue, grain opacity seems to be required by the data for VB8B. The problem with grain opacity is that its magnitude is difficult to estimate because of large uncertainties in the grain size. It is certainly not correct to use tables such as those provided by Alexander (1975) for 0.1 μ particles, since the grains or droplets in brown dwarf atmospheres are expected to grow much larger, on average. However, one does not know the size spectrum, so the only safe statement is that the opacity is bounded above by calculations based on very small grain size. Furthermore, the grain opacity will diminish at low temperature (rather than be ''frozen in'', as Alexander assumed) because a cloud deck forms and the radiating level will eventually be above the clouds. Fortunately, as the simple model below illustrates, the opacity enters rather weakly in the determination of brown dwarf evolution, unless it changes very rapidly with temperature.

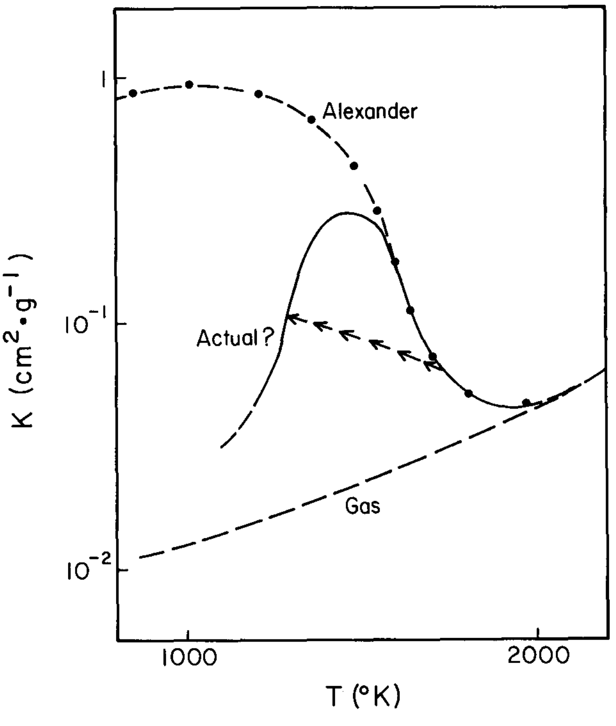

Figure 1. Approximate opacity curves (actually dependent on pressure as well as temperature) for an atmosphere that has no grains (gas only), has 0.1 μ grains ''frozen in'' (Alexander, 1975) or allows for large grains and development of a cloud deck (curve labeled realistic).

ANALYTICAL MODEL

The analytical model neglects thermonuclear reactions and is constructed from three ingredients: (i) the relationship between radius, mass, and entropy for an adiabatic $n = 3/2$ polytrope; (ii) the entropy equation linking the effective temperature to the central temperature; (iii) the first law of thermodynamics, linking the contraction to the luminosity.

Consider, first, the radius-mass-entropy relationship. If we neglect exchange and Coulomb pressure contributions then

$$p = K\rho^{5/3}$$

$$K = K_o \, [\, 1 + aT/\rho^{2/3} \,] \qquad \text{(non-degenerate)}$$

$$K = K_o \, [\, 1 + aT/2\rho^{2/3} + b(\, T/\rho^{2/3})^2 \,] \qquad \text{(degenerate)} \quad (15)$$

where a and b are constants, K_o is the zero temperature limit of K, and $T/\rho^{2/3}$ is assumed constant, in accordance with the discussion above. Although the corrections due to non-degeneracy are important, it turns out that the ''degenerate'' limit of equation (15) is satisfactory for $R \lesssim 3R_o$, where R is the actual radius and R_o is the zero temperature radius for the same mass. It then follows from the solution for an n = 3/2 polytrope (Clayton, 1968) that

$$R = R_o \left[1 + \frac{a}{2} (T/\rho^{2/3}) + b (T/\rho^{2/3})^2 \right] \tag{16}$$

$$R_o = 2.2 \times 10^9 (M_\odot/M)^{1/3} \tag{17}$$

Since the shape of the density profile is invariant, this can be rewritten in the form

$$\frac{R}{R_o} = 1 + \frac{T_c}{T_o} \left(\frac{R}{R_o} \right)^2 + 0.4 \left(\frac{T_c}{T_o} \right)^2 \left(\frac{R}{R_o} \right)^4 \tag{18}$$

$$T_o \simeq (4.9 \times 10^8 \text{ K}) \left(\frac{M}{M_\odot} \right)^{4/3} \tag{19}$$

where T_c is the central temperature. Equation (18) predicts that as the star contracts (R/R_o decreasing), the temperature first rises to reach a maximum ~0.18 T_o and then decreases. The requirement that the maximum temperature exceed ~3 x 10^6 K for hydrogen burning implies a minimum main sequence mass of ~0.08 $M_\odot$. For our purposes, it is sufficient to approximate equation (18) by $T_c/T_o = R_o/R - (R_o/R)^2$.

Turning now to the implications of isentropy, equations (12), (13), and (14) can be combined, together with $\rho_c = \rho_o(R_o/R)^3$, $g = (GM/R_o^2)(R_o/R)^2$, and $p_e = \rho_e kT_e/\mu$ ($\mu \simeq 3.75 \times 10^{-24}$ g) to give

$$T_e = 3.8 \times 10^{-6} \, T_c^{1.22} (M/M_\odot)^{-1.05} (R/R_o)^{1.7} (K/10^{-2})^{-0.29} \tag{20}$$

Notice that a given fractional change in T_e must be matched by a comparable fractional change in T_c, if is constant. This is important for a qualitative understanding of the luminosity-time relation derived below.

From the first law of thermodynamics, and the assumption of hydrostatic equilibrium, the luminosity L is given by

$$L = -\frac{d}{dt}\int_o^M \left(E + \frac{p}{\rho^2}\frac{d\rho}{dt} \right) dm \qquad (21)$$

where E is the internal energy and the second term represents the rate of release of gravitational energy. If we subdivide the pressure and internal energy into zero temperature components plus thermal corrections then

$$L = -\frac{d}{dt}\left[\int_o^M \left(C_v T + \frac{p_{th}}{\rho^2}\frac{d\rho}{dt} \right) dm \right] \qquad (22)$$

where C_v is the specific heat at constant volume and p_{th} is the thermal contribution to the pressure. For $C_v \simeq 2k_B/\mu$ ($\mu \simeq 1.08\ m_p$) and $p_{th} \simeq \rho k_B T/\mu$, we find

$$L \simeq -\frac{0.6\ M\ k_B\ T_o\ R_o}{R^3 \mu}(R + R_o)\frac{dR}{dt} \qquad (23)$$

where the coefficient (0.6) is specific to the n = 3/2 polytrope. Since $L = 4\pi R^2 \sigma T_e^4$, equations (18), (20), and (23) lead to a differential equation of the form

$$x^2 (x - 1)^{4.88} = -\tau (1 + x)\ dx/dt \qquad (24)$$

where $x = R/R_o$ and τ is a time constant (dependent on M and κ). The asymptotic solution (x -> 1) of this equation is

$$x = 1 + \varepsilon - 0.52\,\varepsilon^2 \ldots.$$

$$\varepsilon \equiv \left(\frac{3.88\,t}{\tau}\right)^{-0.258}$$

$$\simeq 0.06 \left(\frac{5 \times 10^9\ \mathrm{yr}}{t}\right)^{0.258} \left(\frac{M}{0.08\,M_\odot}\right)^{0.17} \left(\frac{\kappa}{10^{-2}}\right)^{0.3} \quad (25)$$

from which the following scaling laws immediately follow:

$$\frac{L}{L_\odot} \simeq 1.5 \times 10^{-5} \left(\frac{5 \times 10^9\ \mathrm{yr}}{t}\right)^{5/4} \left(\frac{M}{0.08\,M_\odot}\right)^{5/2} \left(\frac{\kappa}{10^{-2}}\right)^{0.3} \left(1 - \frac{7}{2}\varepsilon\right)$$

$$T_e \simeq 1420 \left(\frac{5 \times 10^9}{t}\right)^{5/16} \left(\frac{M}{0.08\,M_\odot}\right)^{0.79} \left(\frac{\kappa}{10^{-2}}\right)^{0.075} \left(1 - \frac{11}{8}\varepsilon\right) \quad (26)$$

where some irrational exponents have been approximated by nearby fractions.

Several interesting features emerge from this scaling law. Consider, first, the asymptotic, fully degenerate limit (ε negligible). The luminosity ($\sim T_e^4$) is then balanced by the decrease of internal thermal energy ($\sim dT_c/dt$). Using equation (20) ($T_e \sim T_c^{1.22}$), it follows immediately that $L \sim t^{-1.25}$. As required by the finiteness of the energy supply, $\int^\infty L dt$ is finite. Tarter's scaling law ($L \sim t^{-0.84}$) is invalid because it violates this fundamental constraint. (This difficulty cannot be avoided by appealing to Debye cooling or freezing because they do not happen for any effective temperature that can be attained in the age of the Universe.) However, corrections due to finite temperature (finite ε) are non-negligible in general. From equation (26), $-d\ln L/d\ln t \simeq (5/4) - (7/4)\varepsilon$, indicating that even at $\varepsilon \sim 0.1$, there is a substantial deviation from the asymptotic behavior. In fact, detectable bodies (including VB8B) are likely to have $\varepsilon \sim 0.1$ or larger; this casts doubt on the usefulness of the scaling laws. Another way to appreciate this difficulty is to demand that T_e decrease monoptically with time; accordingly the above approximation must fail for $\varepsilon \gtrsim 0.26$. Other factors being equal, the biggest influence on ε is the opacity; large opacity implies that the approach to degeneracy is greatly delayed. Notice that the formula for L involves K directly and also indirectly (through ε); these two dependences tend to counterbalance, as expected since the time-integral of the luminosity should be independent of K and determined only by the Virial theorem and the first law of thermodynamics.

Application to VB8B yields satisfactory agreement for a wide range of masses, provided no constraint is imposed on the evolution time. If we require 1×10^9 yr $\lesssim t \lesssim 5 \times 10^9$ yr then the mass of VB8B is in the range $0.04 \lesssim M/M_0 \lesssim 0.08$, with the middle of this range being most plausible. Figure 2 shows theoretical evolution curves and a comparison with VB8B.

COMPLICATIONS

(a) Deviations from an n = 3/2 polytrope: This is increasingly important as the mass becomes lower, but the main effect

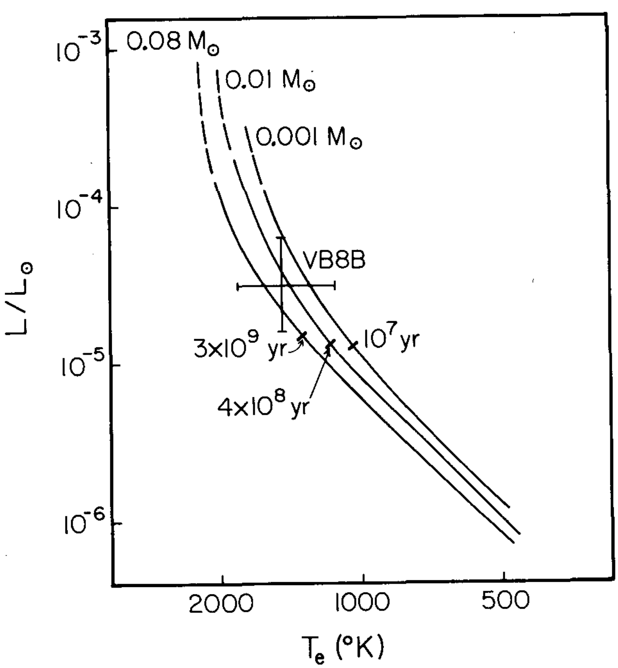

Figure 2. Theoretical Hertzsprung-Russell diagram for three masses, with the VB8B data point superimposed. Times indicate elapsed age for that mass and luminosity.

is on the radius-mass relationship at T = 0; the general properties of the scaling laws above are preserved. Finite temperature corrections become less important at lower mass (even at early times) because if $P \; K\rho^{1+(1/n)}$ then $R \sim K^{n/(3-n)}$ which is a weaker dependence of R on T when $n < 3/2$. At the mass of Jupiter (0.001 $M_\odot$), the best approximation to the polytropic index is $n \approx 1.0$.

(b) Superadiabatic Convection: According to mixing length theory, the heat flux transported by convection is about

$$F \simeq 0.1 \; (\alpha T)\rho \; C_p \; v_s \; T \; \eta^{3/2} \tag{27}$$

where α is the coefficient of thermal expansion, v_s is the local sound speed, η is the fractional superadiabaticity and it is assumed that mixing length ~ pressure scaleheight. For an object like VB8B, is largest just below optical depth unity and $\eta \lesssim 0.03$. This is a negligible effect.

(c) Subadiabatic, Conductive Core: In the deep interior, degenerate electrons provide a high thermal conductivity $k \simeq 10^8 \rho^{2/3}$ cgs (Stevenson and Ashcroft, 1974). As a result, conduction becomes important, especially in the more massive bodies (Stevenson, 1978). However, this has only a small effect on the luminosity-time relation, at least for $t \lesssim 5 \times 10^9$ yr.

(d) Debye Cooling: If $T \lesssim \theta_D$, the Debye temperature, in the deep interior then the specific heat drops dramatically ($C_v \sim (T/\theta_D)^3$, $T \lesssim \theta_D$) and the luminosity would be greatly affected. This phenomenon has been extensively studied for white dwarfs (see discussion in Shapiro and Teukolsky, 1983, Ch. 4). However, $\theta_D \propto \rho^{1/2}$ whereas $T_c/\rho_c^{2/3}$ is comparable for brown dwarfs of comparable age. As a result, Debye cooling should be most important at low masses. Since it has not yet happened in Jupiter (Stevenson, 1982), it clearly has not happened in more massive bodies. In fact,

$$\frac{T_c}{\theta_D} \simeq 20 \left(\frac{M}{0.08 \; M_\odot} \right)^{0.3} \tag{28}$$

at $t \sim 5 \times 10^9$ yr, $\mathcal{K} \sim 10^{-2}$ cm^2/g.

(e) Freezing: This is even less likely because the melting temperature $T_M \propto \rho^{1/3}$ for a Coulomb plasma. In fact, $T_M/\theta_D \simeq \rho^{1/6}$ (so $T_M \sim \theta_D$ at $\rho \sim 1$, and $T_M < \theta_D$ at higher densities).

(f) Differentiation: The pressure at the center of a 0.06 $M_\odot$ brown dwarf is comparable to the pressure at the center of the Sun; the temperature is about one order of magnitude lower. 'Bare' ions of charge $Z \gg 1$ have a solubility in metallic hydrogen of order $\exp(-T_z/T)$, where

$$T_z \simeq 10^4 Z^{7/3} \qquad (29)$$

(Stevenson, 1976). For example, the critical temperature for the unmixing of Fe^{24+} is $\sim 6 \times 10^6$ K. For an ion with atomic abundance $\sim 10^{-4}$, insolubility and core formation occurs in a brown dwarf ($T_c \sim 1 \times 10^6$) provided $Z \gtrsim 20$. This is probably achieved for iron, although the situation is complicated by the presence of at least a few bound states. Actually, bound states can increase the likelihood of insolubility in some circumstances (Stevenson, in preparation). Although brown dwarfs may form iron cores (and the iron may possibly extract some other high Z elements also), this is unlikely to affect brown dwarf evolution, except indirectly by reducing the abundance of elements needed to provide grain opacity in the atmosphere. Unless the body has a greatly enhanced abundance of high Z elements relative to cosmic abundance, the resulting core will be small and the gravitational energy of its formation will be negligible (unlike the situation in Jupiter or Saturn, see Salpeter, 1973).

(g) Variation of Opacity: The scaling relations are only applicable for constant κ. If the dependence of κ on temperature is strong enough then very different evolutions can occur. Consider equation (20), $T_e \sim T_c^{1.22} \kappa^{-0.29}$. If $d\ln \kappa / d\ln T_e < -3.5$, then $d\ln T_e / d\ln T_c < 0$, an impossible situation to sustain (central temperature goes up as the effective temperature drops). This behavior was previously noted in a somewhat different context by Rappaport et al. (1982). The star responds by dropping to a much lower T_e on the other side of the opacity peak (Fig. 1). Since the energy stored within the star is unaltered, it can then radiate for a longer period of time at this reduced T_e. This may be relevant to the interpretation of VB8B.

DETAILED MODELS

Aside from the early and somewhat incomplete modeling of Grossman and Graboske (1973; see also Tarter, 1975), the only detailed numerical models of brown dwarfs are those of D'Antona and Mazzitelli (1985), Nelson et al. (1985), and the work described at this conference. Nelson and co-workers were the first to make a careful comparison between the theory and the observational characteristics of VB8B. Although there are some significant differences among these models (mainly in the entropy equation and opacity) the essential features of the scaling laws described above are confirmed. A more careful comparison between simple analytical models and the detailed numerical models of Hofmeister and Stevenson (in preparation) reveals that the fully degenerate limit is rather slowly attained. For this reason, analyses of the luminosity function and detectability of brown dwarfs (c.f. Probst, 1983) may require more accurate (non-power law) descriptions of the luminosity-time relationship. The calculations of Hofmeister and Stevenson also indicate the possibility of a ''jump'' in T_e and L because of the strong inverse dependence of opacity on temperature at $T \lesssim 2000$ K.

CONCLUDING COMMENTS

Basically, brown dwarf theory is in good shape. There are admittedly quite large uncertainties in the opacity and this translates into significant uncertainties in the comparison of observables and theoretical predictions, especially if brown dwarfs are not grey bodies. These uncertainties in opacity also imply substantial uncertainties in the luminosity (but not the mass) at the bottom end of the main sequence (D'Antona and Mazzitelli, 1985). However, the <u>interiors</u> of brown dwarfs offer no major theoretical challenges except, possibly, the question of differentiation. The identification of VB8B as a brown dwarf of mass $0.06(\pm 0.02)M_\odot$ seems reasonable although a residual doubt persists because of the absence of adequate data to characterize how ''non-grey'' the atmosphere is.

The biggest challenges for the future lie not in theory but in observation: we need more than two colors. We need spectra. We need more bodies. Refinement of the theory does not seem to be a compelling task until this happens.

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DISCUSSION

HUBBARD: The predicted unmixing temperature which you present for Fe^{24+} is ~6 x 10^6 K. This is very much higher than the calculated central temperatures in these bodies. Is your assumed ionization state consistent with the actual temperatures? What is the ionization state of the pure (or almost pure) Fe phase?

STEVENSON: At a hydrogen density of 10^3 $g.cm^{-3}$, a dilute solution of iron in hydrogen will pressure-ionize to Z ~ 20-24, based on pseudopotential calculations. Pure iron may be slightly less highly ionized (although its density is ~1 x 10^4 $g.cm^{-3}$, close to the pressure ionization estimate in Clayton [1968], p. 154). In general, my calculations indicate greater unmixing as the temperature is decreased, even when some of the ionization is temperature induced. The models of Stevenson [1976] should not be used when bound states are present; more recent work (in progress) suggest that bound states can sometimes enhance the insolubility.