

**Color breaking in the early universe**Hiren H. Patel,<sup>1,\*</sup> Michael J. Ramsey-Musolf,<sup>1,2,†</sup> and Mark B. Wise<sup>2,‡</sup><sup>1</sup>*Department of Physics, University of Wisconsin-Madison, 1150 University Avenue, Madison, Wisconsin 53706, USA*<sup>2</sup>*California Institute of Technology, Pasadena, California 91125, USA*

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We explore the possibility that  $SU(3)_C$  was not an exact symmetry at all times in the early Universe, using minimal extensions of the standard model that contain a color triplet scalar field and perhaps other fields. We show that, for a range of temperatures, there can exist a phase in which the free energy is minimized when the color triplet scalar has a nonvanishing vacuum expectation value, spontaneously breaking color. At very high temperatures and at lower temperatures, color symmetry is restored. The breaking of color in this phase is accompanied by the spontaneous breaking of  $B - L$  if the color triplet scalar Yukawa couples to quarks and/or leptons. We discuss the requirements on the minimal extensions needed for consistency of this scenario with present collider bounds on new colored scalar particles.

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**I. INTRODUCTION**

The discovery of a boson with mass around 125 GeV at the Large Hadron Collider (LHC) whose properties are so far roughly consistent with those expected for the standard-model (SM) Higgs boson [1,2] adds new credibility to the paradigm of scalar field-driven spontaneous symmetry breaking in the early Universe. In the standard-model Higgs mechanism paradigm, the electroweak  $SU(2)_L \times U(1)_Y$  symmetry breaks to electromagnetic  $U(1)_{EM}$  symmetry in a single step at a temperature  $T \sim \mathcal{O}(100)$  GeV. Monte Carlo studies indicate that if only SM fields are present, the electroweak phase transition (EWPT) at these temperatures would be cross over for a Higgs boson with the mass indicated by the LHC observation [3–6]. In this case, one would expect little else in the way of other cosmological implications or related experimental signatures, such as the production of relic gravity waves were the EWPT to have been first order.

It is interesting, then, to ask whether electroweak symmetry breaking (EWSB) needs to have followed the simple trajectory implied by the SM. For many years, theorists have explored the possibility that the presence of additional scalar fields might change the character of the EWPT, making it strongly first order, as required for electroweak baryogenesis or the generation of potentially observable relic gravity waves (for a recent review and references, see Ref. [7]). In most of these studies, electroweak symmetry breaking (EWSB) still proceeds through a single step, though a few studies of multistep transitions have appeared in the literature [8–11]. In all cases, the focus has been on EWSB while avoiding the loss of  $SU(3)_C$ , which is known to be a good symmetry today. In the minimal supersymmetric standard model (MSSM), for example, the

avoidance of color-breaking (CoB) and charge-breaking (ChB) vacua imposes severe constraints on the parameters of the scalar potential when one invokes the scalar superpartners of the right-handed top quarks to obtain a strong first-order EWPT. In this case, the Universe transitions to a color-conserving EW vacuum that is metastable, and for a sufficiently light Higgs boson, remains there, as the rate for tunneling to a deeper CoB/ChB vacuum is adequately suppressed [12]. It has been proposed in Ref. [13] that the energy of a color-conserving EW false vacuum may be responsible for the cosmic acceleration.

It is possible, however, that the early Universe underwent an epoch when  $SU(3)_C$  was spontaneously broken but later restored. To our knowledge, there exist no theoretical arguments or experimental observations that would preclude this possibility. If realized, it would mean that today's symmetries need not have been present at all times in the early Universe, contrary to the conventional picture in which symmetries are only lost and not restored as the Universe cools.

We are not the first to consider the possibility that part of the the zero-temperature unbroken gauge group of the standard-model or other gauge theory might have been broken in the early Universe. Weinberg first observed that in the context of an  $O(n) \times O(n)$  gauge theory, one may encounter a transition to a state of lower symmetry  $O(n) \times O(n) \rightarrow O(n) \times O(n-1)$  with increasing  $T$  [14]. Langacker and Pi subsequently applied this idea to show how breaking of  $U(1)_{EM}$  and its subsequent restoration could provide a solution to the monopole problem in grand unified theories (GUTs) [15]. The authors of Refs. [16,17] proposed an alternate solution for an  $SU(5)$  GUT that relied on nonrestoration of the symmetry at high  $T$  in the presence of a suitable scalar representation of the gauge group. Similar ideas were explored for spontaneous breaking of  $CP$  invariance in Refs. [18,19]. For the specific case of  $SU(3)_C$ , the authors of Ref. [20] studied the possibility that a CoB phase preceded the color-conserving EW phase

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in the MSSM and found that without the introduction of additional interactions, a Universe that cooled into a CoB phase would always remain there.

In this paper, we demonstrate how  $SU(3)_C$  breaking followed by its restoration might arise, using minimal extensions of the SM scalar sector. We further show that this scenario may be realized without the occurrence of a metastable CoB vacuum at  $T = 0$ . We also outline some of the phenomenological consequences and constraints, focusing on two representative cases corresponding to the SM plus

- (a) a single multiplet of sub-TeV colored scalars  $C$ .
- (b) a single multiplet of colored scalars  $C$ , as well as a gauge singlet  $S$ .

In both cases, the basic mechanism for achieving the novel pattern of symmetry breaking where color is broken in an intermediate phase involves a negative mass-squared term for the colored scalar in the scalar potential. A positive contribution to its  $T = 0$  physical mass squared,  $m_C^2$ , is generated by quartic terms of the form  $C^\dagger C \varphi^\dagger \varphi$ , where  $\varphi$  denotes one of the other scalars in the theory such as the Higgs doublet  $H$  or  $S$ . Provided the product of the associated quartic coupling and square of the vacuum expectation value (VEV) of  $\varphi$  is sufficiently large at  $T = 0$ , the positive contribution to  $m_C^2$  overwhelms the negative bare mass-squared term, leaving at least a local minimum where color is unbroken at  $T = 0$ . If the VEV of  $\varphi$  falls sufficiently rapidly as  $T$  increases, an intermediate phase where the minimum of the free energy breaks  $SU(3)_C$  can exist.

For case (a),  $\varphi$  is the Higgs doublet, and we find that in order to maintain perturbative couplings, the colored scalar  $C$  cannot be too heavy. Constraints from LHC new particle searches then imply that its couplings to first- and second-generation SM fermions must be suppressed. Case (b), wherein  $\varphi$  is the singlet, allows one to avoid this requirement by pushing the mass of  $C$  above present LHC bounds while maintaining perturbativity. This is possible because the VEV of  $S$  can be larger than the Higgs VEV, allowing it to give the dominant positive contribution to the  $m_C^2$ . As we discuss below, for this case the region of parameter space that achieves a CoB phase for a range of temperatures is restricted not so much by present phenomenological constraints but by the behavior of the scalar potential as a function of  $T$ . Nevertheless, both cases illustrate the more general possibility of the novel symmetry-breaking pattern of interest here and the considerations that determine its viability.<sup>1</sup> At the end of this paper, we comment on other variants that may lead to a similar symmetry-breaking pattern.

Before proceeding, we emphasize the exploratory nature of our study. As with any analysis of the phase history of

<sup>1</sup>In analyzing these cases, we have not imposed any requirements on the character of the phase transitions. Instead, we simply concentrate on the possible existence of a CoB phase that is subsequently restored.

the Universe in perturbation theory, one should take the precise numerical results with an appropriate grain of salt. In the case of the nonrestoration of  $SU(5)$  gauge symmetry note above, for example, the work of Ref. [21] indicated that the inclusion of higher-order contributions to the effective potential could modify conclusions based on one-loop results.<sup>2</sup> On the other hand, comparison of lattice results in the standard model and MSSM with perturbative computations suggest that the latter can faithfully reproduce the parametric dependence of phase transition dynamics even if perturbative results for the critical temperatures, latent heat, etc., are not numerically accurate (for a detailed discussion, see Ref. [23]). Thus, we suspect that the viability of a period of color breaking and restoration in the minimal models discussed here—as well as the general behavior of this possibility as a function of the various interactions—should hold after the inclusion of higher-order contributions or completion of nonperturbative computations.

Our discussion of this paradigm is organized as follows: In Sec. II case (a) is analyzed in detail, while in Sec. III we treat case (b). We summarize in Sec. IV.

## II. TWO-FIELD SCENARIO

To set the notation, we first consider case (a) with  $C$  being an  $SU(3)_C$  triplet. In order to avoid the possibility of stable color triplet scalar degrees of freedom, the field  $C$  must be charged under  $SU(2)_L \times U(1)_Y$  to permit couplings to SM fermions. For example the  $C$  could be an  $SU(2)_L$  doublet with hypercharge  $Y = 7/6$ . Then it has the following Yukawa type interactions with the quarks:

$$L_Y = C \bar{u}_R g_{uL} L_L + C \bar{Q}_L g_{Qe} e_R + \text{H.c.} \quad (1)$$

Here we have suppressed the color, weak isospin and flavor indices. The Yukawa coupling matrices  $g_{ue}$  and  $g_{Qe}$  could be such that the dominant decay mode of the colored scalar  $C$  is to a top quark and an anti-tau lepton, thereby alleviating present LHC constraints on new colored scalars. Assigning a baryon number  $1/3$  to  $C$ , the tree-level Lagrangian conserves baryon number.<sup>3</sup>

To make the analysis of the phase structure of the theory as simple as possible, we consider instead the illustrative situation where  $C$  is an electroweak singlet but color triplet. Then it has no renormalizable interactions with quarks and leptons, and the most general gauge invariant renormalizable potential is<sup>4</sup>

<sup>2</sup>For a study of the impact of two-loop effects with colored scalars on the character of an EWPT, see Ref. [22].

<sup>3</sup>Nonperturbative quantum effects violate this symmetry because it is anomalous.

<sup>4</sup>If the  $C$  were also a weak doublet, other terms like  $H^\dagger \tau^a H C^\dagger \tau^a C$  would appear in the scalar potential.

$$V = -\mu_H^2(H^\dagger H) - \mu_C^2(C^\dagger C) + \frac{\lambda_H}{2}(H^\dagger H)^2 + \frac{\lambda_C}{2}(C^\dagger C)^2 + \lambda_{HC}(H^\dagger H)(C^\dagger C). \quad (2)$$

We take all the parameters  $\mu_H^2$ ,  $\mu_C^2$ , and  $\lambda_{H,C,HC}$  to be positive.

The  $T = 0$  vacuum structure follows from the minimization conditions:

$$\partial V / \partial \phi^\dagger = 0 \quad \text{for } \phi = H, C. \quad (3)$$

Four possible stationary points emerge, with the corresponding vacuum energies labeled  $E_a$ , with  $a = \{0, H, C, HC\}$ :

(I)  $H = C = 0$  with energy  $E_0 = 0$ . This is always a local maximum, since very small fluctuations in the fields about this point contribute negatively to the potential energy.

(II) Break  $SU(2)_L$  but not  $SU(3)_C$ :  $\langle H \rangle = (0, v_H)^T$  and  $\langle C \rangle = 0$ . Solving Eq. (3) gives

$$v_H^2 = \frac{\mu_H^2}{\lambda_H} \quad (4)$$

with vacuum energy density

$$E_H = -\frac{\mu_H^4}{2\lambda_H}. \quad (5)$$

(III) Break  $SU(3)_C$  but not  $SU(2)_L \times U(1)_Y$ :  $\langle H \rangle = 0$  and  $\langle C \rangle = (0, 0, v_C)^T$ , where, without loss of generality, we have taken the third component of  $C$  to acquire a VEV. Then  $SU(3)_C$  is spontaneously broken to an  $SU(2)$  color subgroup so that in the broken phase there are still three massless gluons, but five that get mass by the Higgs mechanism. The solution of Eq. (3) then implies

$$v_C^2 = \frac{\mu_C^2}{\lambda_C} \quad (6)$$

with energy at the stationary point

$$E_C = -\frac{\mu_C^4}{2\lambda_C}. \quad (7)$$

(IV) Break both  $SU(3)_C$  and  $SU(2)_L \times U(1)_Y$ :  $\langle H \rangle = (0, v_H)^T$ , and  $\langle C \rangle = (0, 0, v_C)^T$ , with

$$v_H^2 = \frac{\lambda_C \mu_H^2 - \lambda_{HC} \mu_C^2}{\lambda_H \lambda_C - \lambda_{HC}^2}, \quad (8)$$

$$v_C^2 = \frac{\lambda_H \mu_C^2 - \lambda_{HC} \mu_H^2}{\lambda_H \lambda_C - \lambda_{HC}^2},$$

leading to an energy density

$$E_{HC} = -\frac{1}{2} \frac{\lambda_C \mu_H^4 + \lambda_H \mu_C^4 - 2\lambda_{HC} \mu_H^2 \mu_C^2}{\lambda_H \lambda_C - \lambda_{HC}^2}. \quad (9)$$

The absolute minimum must correspond to stationary point II, implying first that the eigenvalues  $m_H^2$  and  $m_C^2$  of the mass-squared matrix, which follow from second derivatives of the potential, must be positive at  $C = 0$  and  $v_H = \mu_H / \sqrt{\lambda_H}$ :

$$m_h^2 = 2\mu_H^2 = 2\lambda_H v_H^2 > 0, \quad (10)$$

$$m_C^2 = -\mu_C^2 + \lambda_{HC} v_H^2 > 0. \quad (11)$$

The requirement in Eq. (10) is trivially satisfied, while Eq. (11) constrains the parameters in the potential to satisfy

$$\lambda_{HC} v_H^2 > \mu_C^2. \quad (12)$$

Equation (11) places the upper bound on  $m_C$  as a function of  $\lambda_{HC}$ :

$$m_C < (\sqrt{\lambda_{HC}}) v_H \approx (174 \text{ GeV}) \sqrt{\lambda_{HC}}. \quad (13)$$

If the color triplet complex scalar has generic decay modes to light quarks and leptons, LHC data implies that its mass must be greater than about 500 GeV. Equation (13) then implies that  $\lambda_{HC}$  is greater than about 9, taking us into the strong coupling regime.

In addition, stationary points III or IV may also be minima, with positivity conditions. For purposes of illustration, we consider the positivity conditions for III, denoting the corresponding scalar masses at this point as  $\tilde{m}_H$  and  $\tilde{m}_C$ :

$$\tilde{m}_H^2 = -\mu_H^2 + \lambda_{HC} v_C^2 = -\lambda_H v_H^2 + \lambda_{HC} v_C^2 > 0, \quad (14)$$

$$\tilde{m}_C^2 = 2\mu_C^2 = 2\lambda_C v_C^2 > 0. \quad (15)$$

Again, Eq. (15) is trivially satisfied, while Eq. (14) implies that

$$m_H^2 < 2\lambda_{HC} v_C^2 = \frac{2\lambda_{HC} \mu_C^2}{\lambda_C} = \frac{2\lambda_{HC} (\lambda_{HC} v_H^2 - m_C^2)}{\lambda_C}. \quad (16)$$

### A. Symmetry breaking at finite $T$

We now determine the conditions under which extremum II is the absolute minimum at  $T = 0$  but either extrema III or IV evolves to an absolute minimum as  $T$  is raised. To that end, we first determine the temperatures  $T^*$  above which extrema disappear along the  $H$  and  $C$  directions. For purposes of both simplicity and intuition, our analytic work is based on the high- $T$  effective theory, wherein the effects of temperature are to replace the coefficients of the quadratic terms in the free energy with their Debye masses.

$$\mu_H^2(T) = \mu_H^2 - T^2 \left( \frac{\lambda_H}{4} + \frac{\lambda_{HC}}{4} + \frac{3g_2^2}{16} + \frac{y_t^2}{4} \right), \quad (17)$$

$$\mu_C^2(T) = \mu_C^2 - T^2 \left( \frac{\lambda_C}{3} + \frac{\lambda_{HC}}{6} + \frac{g_3^2}{3} \right). \quad (18)$$

We have omitted the small hypercharge contribution to  $\mu_H^2(T)$  and replaced the top quark mass with the Higgs VEV using the relation  $m_t = y_t v_H$ . Symmetry will be restored along the  $H$  and  $C$  directions for  $T > T_H^*$  and  $T_C^*$ , respectively:

$$(T_H^*)^2 = \frac{\lambda_H v_H^2}{\left( \frac{\lambda_H}{4} + \frac{\lambda_{HC}}{4} + \frac{3g_2^2}{16} + \frac{y_t^2}{4} \right)}, \quad (19)$$

$$(T_C^*)^2 = \frac{\mu_C^2}{\left( \frac{\lambda_C}{3} + \frac{\lambda_{HC}}{6} + \frac{g_3^2}{3} \right)}. \quad (20)$$

To obtain the pattern of symmetry breaking in which we are interested, we must have the following: (a) at  $T = 0$ , the stationary point of lowest energy has  $SU(2)_L \times (1)_Y$  spontaneously broken, and (b)  $T_H^* < T_C^*$ , so that there is a range of temperature where  $\mu_H(T)^2$  is negative and  $\mu_C^2(T)$  is positive. For temperatures in this range, color is spontaneously broken, but  $SU(2)_L \times U(1)_Y$  is not.

The condition on the temperatures  $T_{H,C}^*$  and Eq. (12) leads to

$$\begin{aligned} \frac{\lambda_H v_H^2}{(\lambda_H + \lambda_{HC} + 3g_2^2/4 + y_t^2)} &< \frac{3\mu_C^2}{(\lambda_C + \lambda_{HC}/2 + g_3^2)} \\ &< \frac{6\lambda_{HC} v_H^2}{(2\lambda_C + \lambda_{HC} + 2g_3^2)}, \end{aligned} \quad (21)$$

or

$$\frac{2\lambda_C + \lambda_{HC} + 2g_3^2}{\lambda_H + \lambda_{HC} + 3g_2^2/4 + y_t^2} < \frac{6\lambda_{HC}}{\lambda_H}. \quad (22)$$

Setting  $\lambda_H \approx 1/4$  as implied by the observed Higgs mass, Eq. (22) then implies

$$\frac{2\lambda_C + \lambda_{HC} + 2g_3^2}{1 + \lambda_{HC} + 3g_2^2 + 4y_t^2} \lesssim 6\lambda_{HC} \quad (23)$$

In short,  $\lambda_{HC}$  cannot be arbitrarily small.

Now consider the requirement that at zero temperature, the energy of the stationary point with only the Higgs getting a vacuum expectation value must be less than the stationary point with only the colored scalar getting an expectation value. This gives

$$|E_H| > |E_C| \quad \text{or} \quad \frac{m_H^4}{2} \gtrsim \frac{(2\lambda_{HC} m_H^2 - m_C^2)^2}{2\lambda_C}. \quad (24)$$

Thus, for a given colored scalar mass  $m_C$  and  $\lambda_{HC}$ , the colored scalar quartic self-coupling  $\lambda_C$  also cannot be arbitrarily small.

To illustrate the range of  $(\lambda_C, \lambda_{HC}, \text{ and } m_C)$  that satisfy the requirements of Eqs. (22)–(24), we plot in Fig. 1 the results of a scan over  $\lambda_C$  and  $m_C$  for two representative values of  $\lambda_{HC}$ . In doing so, we have employed the full one-loop effective potential that includes the Coleman-Weinberg terms at  $T = 0$  and the finite- $T$  contributions without adopting the high-temperature expansion. We have analyzed the evolution of the extrema in a way that

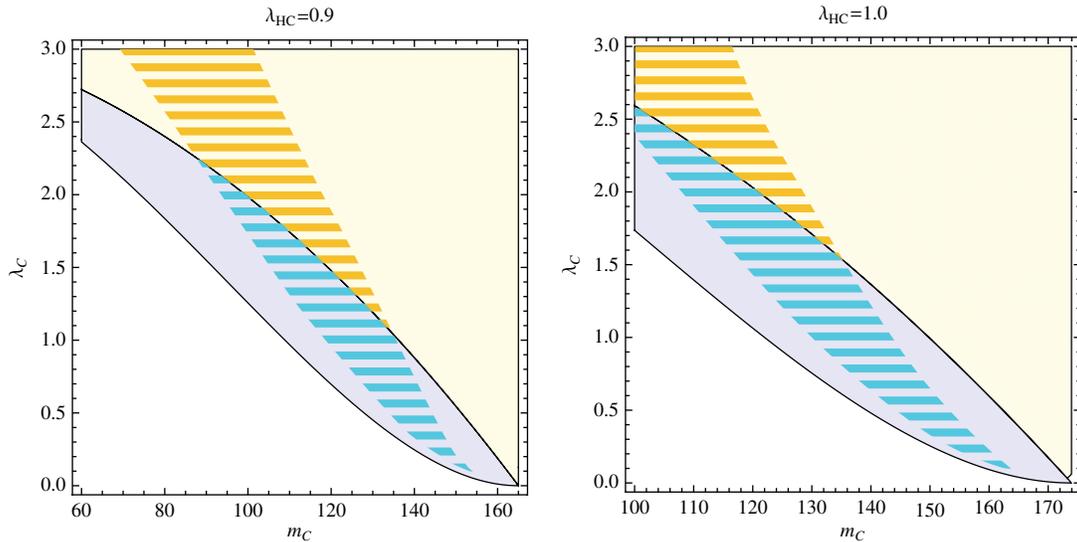


FIG. 1 (color online). Regions in parameter space where color is broken and possibly subsequently restored in the early Universe. Color shading, separated by solid curves, indicates the vacuum structure at zero temperature: *yellow* (upper-right) represents the region where the  $SU(3)_C$ -breaking phase is tachyonic, and *blue* (thinner diagonal region) indicates regions where the  $SU(3)_C$ -breaking phase is metastable. In the *white* (lower-left) regions, the standard electroweak phase is metastable, and the  $SU(3)_C$ -breaking phase is stable. For  $(m_C, \lambda_C)$  inside the yellow hatched area, the Universe first cools into a color-breaking phase, followed by a transition to the color-preserving EW phase. For the blue hatched region, the Universe may remain in the color-breaking phase. Left panel is for  $\lambda_{HC} = 0.9$ , and right panel is for  $\lambda_{HC} = 1.0$ .

maintains gauge invariance, following the approach outlined in Ref. [23]. The yellow areas correspond to regions where the EW phase is absolutely stable, and the CoB is a saddle point. In the blue area, the EW phase is still absolutely stable, but the CoB becomes metastable (no longer a saddle point). Outside these two regions (white), the EW vacuum is metastable, with the CoB phase absolutely stable. For values of the parameters inside the yellow hatched region, the Universe first cools into a CoB minimum, followed by a transition to the EW, color-preserving vacuum at lower temperatures. On the other hand, inside the blue hatched region it is possible that the Universe tunnels from the metastable CoB vacuum to the stable EW phase if the tunneling rate is sufficiently large.

For purposes of illustration, we focus on the yellow shaded region, wherein color-breaking occurs at moderately high temperatures and not at  $T = 0$ . We find that for fixed  $\lambda_{HC}$ , we must look to fairly high  $\lambda_C$  to find phase transitions involving a CoB phase in the early Universe. As  $m_C$  is parametrically increased, the required  $\lambda_C$  to obtain a CoB transition is reduced. As expected from our general discussion above,  $m_C$  must be relatively light, while  $\lambda_C$  is relatively large and may even become nonperturbative. By increasing  $\lambda_{HC}$ , the upper limit on  $m_C$  increases, as we expect from Eq. (13). From Eq. (24), we also anticipate that as one increases  $\lambda_{HC}$ , the lower limit on  $\lambda_C$  increases as well, a trend we see by comparing the left and right panels of Fig. 1. In short, it is not possible to make  $m_C$  arbitrarily large without entering the realm of nonperturbative quartic couplings.

It is conceivable that a heavier colored scalar with nonperturbative couplings can lead to the pattern of symmetry breaking we are interested in, though we cannot say so with confidence based on our perturbative analysis. For  $m_C$  on the order of a few times the weak scale, one must confront the present LHC limits on the existence of colored scalars, which are generally precluded for  $m_C \lesssim 600\text{--}900$  GeV, assuming they are pair produced (strongly) and decay semileptonically (e.g., first- and second-generation leptosquarks [24–26]). An exception occurs when the scalar couples strongly only to third-generation fermions, as the corresponding mass limits are considerably weaker (e.g. see Refs. [27,28] for third-generation leptosquarks). We defer the corresponding phenomenological analysis to a future study.

### III. ADDING A SINGLET

To avoid a light colored scalar or nonperturbative couplings, we augment the field content by a real scalar singlet  $S$ , whose nonzero VEV  $v_S$  at  $T = 0$  generates an additional contribution to colored scalar mass  $m_C$ . Since the singlet VEV is not connected with the weak scale, it can be as large as needed to increase  $m_C$  without strong coupling. The most general extension of the potential in Eq. (2) is obtained by adding the potential

$$\begin{aligned} \Delta V = & -\frac{\mu_S^2}{2}S^2 + \frac{\lambda_S}{4}S^4 + \lambda_{HC}(H^\dagger H)(C^\dagger C) \\ & + \frac{\lambda_{HS}}{2}(H^\dagger H)S^2 + \frac{\lambda_{CS}}{2}(C^\dagger C)S^2 + \frac{e_S}{3}S^3 \\ & + e_C C^\dagger C S + e_H H^\dagger H S. \end{aligned} \quad (25)$$

Again we take the  $\mu^2$ 's and the  $\lambda$ 's to be positive. The analysis of the vacuum structure at zero and finite temperature is considerably more complicated due to the addition of a new scalar degree of freedom. We now highlight key points, while imposing a few simplifying assumptions.

In the color unbroken phase, with both  $v_S$  and  $v_H$  nonzero at zero temperature, the mass of the colored scalar is given by

$$m_C^2 = -\mu_C^2 + \lambda_{HC}v_H^2 + \frac{\lambda_{CS}}{2}v_S^2 + e_C v_S. \quad (26)$$

By setting  $e_C = e_S = e_H = 0$  (corresponding to a discrete  $Z_2: S \rightarrow -S$  symmetry), it is straightforward to obtain expressions for the singlet and Higgs VEVs in terms of the model parameters:

$$v_S^2 = \frac{\mu_H^2 \lambda_{HS} - \mu_S^2 \lambda_H}{\lambda_{HS}^2/2 - \lambda_H \lambda_S}, \quad (27)$$

$$v_H^2 = \frac{\mu_S^2 \lambda_{HS}/2 - \mu_H^2 \lambda_S}{\lambda_{HS}^2/2 - \lambda_H \lambda_S}. \quad (28)$$

We can simplify the analysis further by decoupling the Higgs doublet from the other scalars by setting  $\lambda_{HS} = \lambda_{HC} = 0$ . Then the problem of studying the vacuum structure resembles that of Sec. II, where the fields were just  $H$  and  $C$ , but in this case, with the singlet  $S$  now playing the role of  $H$ . Since  $v_S$  is not constrained by weak interaction phenomenology, it may be much larger than  $v_H$ , allowing  $m_C$  to be greater than  $\sim 1$  TeV [via Eq. (26)] without the coupling  $\lambda_{CS}$  becoming nonperturbatively large.

The presence of the additional couplings in Eq. (25) will modify the finite- $T$  masses of the scalar fields. Again, for simplicity, we first set the cubic couplings to zero. One then has (restoring dependence on  $\lambda_{HS}$  and  $\lambda_{HC}$ )

$$\mu_H^2(T) = \mu_H^2 - T^2 \left( \frac{\lambda_H}{4} + \frac{\lambda_{HC}}{4} + \frac{\lambda_{HS}}{24} + \frac{3g_2^2}{16} + \frac{y_t^2}{4} \right), \quad (29)$$

$$\mu_C^2(T) = \mu_C^2 - T^2 \left( \frac{\lambda_C}{3} + \frac{\lambda_{HC}}{6} + \frac{\lambda_{CS}}{24} + \frac{g_3^2}{3} \right), \quad (30)$$

$$\mu_S^2(T) = \mu_S^2 - T^2 \left( \frac{\lambda_S}{8} + \frac{\lambda_{HS}}{12} + \frac{\lambda_{CS}}{8} \right). \quad (31)$$

From these results, we observe that it is generally difficult to obtain the pattern of symmetry breaking that we seek. In general, there can exist several vacua at  $T = 0$ , some of which will break  $SU(3)_C$  but not  $SU(2)_L$ , others breaking  $SU(2)_L$  and not  $SU(3)_C$ , and still others breaking both

( $v_S$  may be nonzero for one or more of these cases). On the one hand, the color-preserving EW vacuum must have the lowest energy at  $T = 0$ . In order for a CoB vacuum to be the state of lowest energy at some finite  $T$ , the energy of the color-preserving vacua must rise more quickly than that of the CoB vacuum, as  $T$  is parametrically increased. On the other hand, we see that  $\mu_{C,H}^2(T)$  will typically decrease compared to  $\mu_S^2(T)$ , partly because of its dependence on the gauge couplings. Consequently, as EW symmetry is restored, the color-preserving vacua with  $v_s \neq 0$  are more likely to emerge as the state of lowest energy than a CoB vacuum.

This situation is illustrated in Fig. 2, where we show the temperature evolution of the CoB and EW extrema for a representative choice of parameters. While the EW vacuum is the absolute minimum at  $T = 0$ , we observe that as  $T$  increases,  $SU(3)_C$  is restored long before EW symmetry is restored, so that there is never a period when color is broken.

The inclusion of nonzero cubic terms in the potential [Eq. (25)]—thereby explicitly breaking the  $Z_2$  symmetry—can modify this situation by raising the critical temperature  $T_C^*$  relative to that of the EW and singlet vacua (For an analogous study of the impact of tree-level cubic terms, see e.g. Ref. [10]). This happens for  $\lambda_{CS}$  positive as we have chosen it to be. In this case, the energy of the EW minimum becomes larger than that of the CoB minimum for  $T_{\text{crit}}$ . For a range of temperatures satisfying  $T_{\text{crit}} < T < T_C^*$ , the CoB extremum becomes the state of lowest energy, while for  $T > T_C^*$ ,  $SU(3)_C$  symmetry is restored.

This situation is illustrated in Fig. 3. Here, we show the temperature evolution of the free energy for the phases involved in the transition for two illustrative parameter choices. The corresponding scalar masses are  $(m_C, m_S) \simeq (1660, 730)$  GeV. Since we have set the terms that couple the Higgs doublet to the fields  $S$  and  $C$  to zero in this example, we can increase the values of  $m_C$  and  $m_S$  by

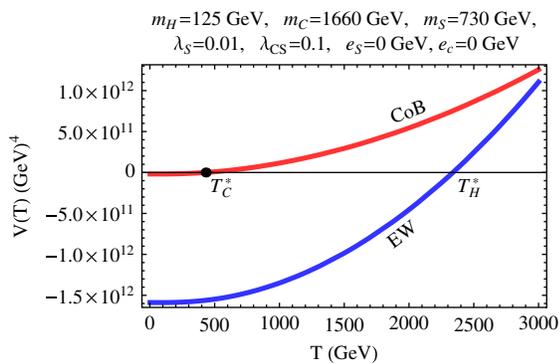


FIG. 2 (color online). The free energy for the color-breaking (CoB) and SM electroweak (EW) phases in the  $Z_2$ -symmetric limit, as a function of temperature  $T$ . Within this class of parameter choice, the desired pattern of parameter breaking does not happen.

$m_H=125$  GeV,  $m_C=1660$  GeV,  $m_S=730$  GeV,  
 $\lambda_S=0.01$ ,  $\lambda_{CS}=0.1$ ,  $e_S=6.1$  GeV,  $e_C=-300$  GeV

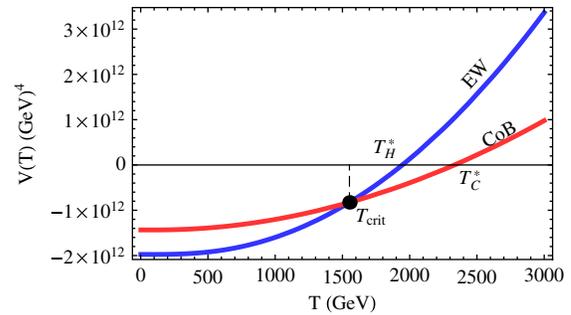


FIG. 3 (color online). The free energy for the color-breaking (CoB) and SM electroweak (EW) phases as a function of  $T$  for nonzero  $Z_2$ -breaking  $e$  parameters. For the choice of parameters labeled above the figure, the critical temperature is  $T_{\text{crit}} \simeq 1550$  GeV.

scaling up the dimensionful parameters  $\mu_S^2$  and  $\mu_C^2$ ,  $e_S$  and  $e_C$ . The evolution of the EW and CoB minima are indicated by the blue and red curves, respectively. In both cases, we have verified that at zero temperature, the color-breaking phase is a saddle point, so that one encounters no possibility that the EW minimum is metastable. The pattern of symmetry breaking can be seen by following the curves from right (high  $T$ ) to left (low  $T$ ). At high  $T$ , the symmetric phase, corresponding to  $V = 0$ , is the absolute minimum. Below  $T_C^*$ , the CoB minimum takes on negative energy and a transition occurs to the CoB phase. Below the intersection point of the blue and red curves ( $T = T_{\text{crit}}$ ), the EW minimum has lower energy and a transition to this phase occurs.

A comprehensive study of the parameter space for this multifield scenario goes beyond the scope of the present work, where we seek to illustrate the basic mechanism. Nonetheless, we have studied the dependence of this pattern on some of the parameters in the potential. Consider changing the value of  $\lambda_{CS}$ . The plots in Fig. 4 show the impact of increasing or decreasing this coupling by 20%. Over this range  $T_{\text{crit}}$  increases as  $\lambda_{CS}$  does. Note also that the color-breaking phase exists over a wider range of temperatures as  $\lambda_{CS}$  is increased.

With the inclusion of new scalars that are considerably heavier than the electroweak scale, one has to contend with the hierarchy problem in this scenario. We note that the smallness of cubic parameters  $e_H$ ,  $e_C$ ,  $e_S$  compared to the larger mass scales is stable against radiative corrections thanks to the associated  $Z_2$  symmetry. However, to maintain a light Higgs, the Higgs mass must be fine-tuned to a certain degree. In our case we have implemented this tuning by decoupling the fields. Moreover, even a modest coupling between the Higgs and the new fields completely changes the pattern of symmetry breaking. For example, if we leave the parameters the same as in Fig. 3, but take  $\lambda_{HC} = \lambda_{HS} = 0.00012$ , the  $T = 0$  vacuum no longer has electroweak symmetry breaking (i.e. only the singlet has a VEV). On the other hand, at  $\lambda_{HC} = \lambda_{HS} = 0.0001$ , the

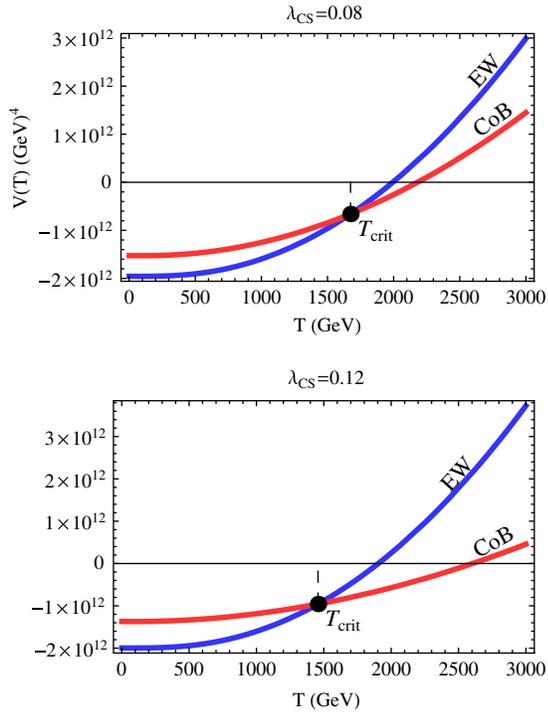


FIG. 4 (color online). The free energy for the CoB and EW phases as a function of  $T$  for the same choice of parameters as in Fig. 3, but with coupling constant, *top panel*:  $\lambda_{CS} = 0.08$  and *bottom panel*:  $\lambda_{CS} = 0.12$ . Raising  $\lambda_{CS}$  has the effect of lowering  $T_{\text{crit}}$  and lengthening (in temperature) the duration of spontaneous color breaking in the early Universe.

pattern of symmetry breaking is similar to what is shown in Fig. 3.

We have also studied the dependence of the pattern of symmetry breaking on the cubic couplings  $e_C$  and  $e_S$  that are essential for the viability of case (b). We find that decreasing the magnitude of  $e_S$  (for  $e_S > 0$ ) tends to lower the  $T = 0$  energy of the CoB extremum, and for sufficiently small  $e_S$  the state of minimum energy breaks color. Thus, for a given set of values for the remaining parameters, we expect a lower bound on  $e_S$ . Conversely, increasing  $|e_C|$  for  $e_C < 0$  decreases the energy splitting between the  $T = 0$  EW minimum and the CoB extremum, so we anticipate that in general there will be an upper bound on  $|e_C|$  for  $e_C < 0$  for a given set of the other parameters.

#### IV. DISCUSSION

We have shown that in extensions of the SM with an additional color triplet scalar or color triplet scalar and singlet scalar it is possible to have a cosmological history where there is a region of temperature with color spontaneously broken. At temperatures both above and below this region, region color is restored. In the case of just adding a color triplet to the SM, it is not possible (in regions of parameter space where perturbation theory is valid) for the additional color triplet scalars to be at the TeV scale.

Rather, they are necessarily lighter, and unless one evokes a particular flavor structure in their coupling to fermions, such models are ruled out by constraints from the LHC. However, by adding another singlet scalar, we were able to obtain a region in temperature where color is broken and where the additional colored scalars are sufficiently heavy to avoid LHC constraints. This is the minimal extension of the SM with the novel color-breaking cosmological history we are interested in that has the color triplet scalars at the TeV scale (or heavier). In this model we found regions of parameter space with a high-temperature color-breaking phase but where at zero temperature the color-breaking stationary points are not local minima. However, tuning is required to keep the Higgs doublet light compared to the singlet. It would be interesting if a scenario for color breaking at high temperature could be found without fine tuning.

Apart from intrinsic interest in understanding the cosmological evolution of our Universe, the existence of such a color-breaking phase may open up new avenues for generating the baryon excess of the Universe. Neglecting Majorana right-handed neutrino masses, the standard model has a global  $B - L$  symmetry. If the new colored scalar is a triplet (under color) and has the right hypercharge to couple to a quark-lepton fermion bilinear, then the color-breaking phase also breaks the global  $B - L$  symmetry. As a result, it is possible that a net  $B - L$  charge could have been created either dynamically during the color-breaking or color-restoring phase transitions, or through out-of-equilibrium decays of the colored scalars during the CoB phase. In principle, this  $B - L$  asymmetry need not have been washed out by weak sphalerons.

Finally, we would like to mention that adding other degrees of freedom may enlarge the region of parameter space of the scalar potential where the unusual color-breaking phase in the early Universe occurs. For example, if the singlet scalar couples to additional fermions then at finite temperature, these fermions contribute to the singlet scalar mass squared positively, reducing the magnitude of its vacuum expectation value at high temperature compared with the case in which the fermions are absent. This could assist the formation of the color-breaking phase.

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