Rate-Distortion With Mixed Types of Side Information

Michael Fleming and Michelle Effros[1]
California Institute of Technology, MC 136-93, Pasadena, CA 91125, USA
e-mail: {mike,effros}@z.caltech.edu

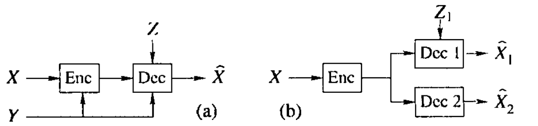

Figure 1: (a) The mixed s.i. system. (b) The HB system.

Abstract — **We present the rate-distortion function and bound the rate loss for a system with some side information (s.i.) known at both the encoder and decoder, and some known only at the decoder. We extend the corresponding Wyner-Ziv rate-distortion results to give a lower bound for jointly Gaussian sources and upper and lower bounds for binary symmetric sources. Applying the construction from our binary upper bound to the Heegard and Berger (HB) problem of decoding when s.i. may be present improves the best upper bound for that problem. Applying it to the two-receiver system with different s.i. at each decoder provides a new upper bound.**

I. $R(D)$ for the Mixed Side Information System

Figure 1(a) shows the mixed s.i. system. Let $\rho : \mathcal{X} \times \hat{\mathcal{X}} \rightarrow [0, \infty)$ be a single-letter distortion measure. We assume that there exists an $\hat{x}_0 \in \hat{\mathcal{X}}$ such that $E\rho(X, \hat{x}_0) \leq \rho_{max} < \infty$. Let $\mathcal{M}(D)$ be the set of all test channels $\mu(w|x, y)$ defining an auxiliary random variable W that satisfies $W \rightarrow (X, Y) \rightarrow Z$ and for which there exists a function $\hat{X} = \hat{X}(W, Y, Z)$ such that $E\rho(X, \hat{X}) \leq D$. Then, when (X, Y, Z) are discrete sources, the mixed s.i. rate-distortion (R-D) function is given by

$$R_{X|Y\{Z\}}(D) = \inf_{\mu(w|x,y) \in \mathcal{M}(D)} [I(X; W|Y) - I(W; Z|Y)].$$

The right hand side above is a lower bound for $R_{X|Y\{Z\}}(D)$ when (X, Y, Z) are continuous sources.

The rate loss [1] is $L(D) \triangleq R_{X|Y\{Z\}}(D) - R_{X|YZ}(D)$, where $R_{X|YZ}(D)$ refers to the system in which Z, like Y, is available at both encoder and decoder. We show that for discrete sources and difference distortion measures, $L(D)$ satisfies the same bound as in [1, Sec. II] for the Wyner-Ziv system. For Hamming distortion and binary sources, $L(D) \leq 0.22$ bits.

II. Mixed Side Information Examples

Gaussian: Let $\rho(x, \hat{x}) = |x - \hat{x}|^2$ and let (X, Y, Z) be zero-mean, jointly Gaussian with covariance matrix K. Let $L = (l_{ij}) = K^{-1}$, and assume $I(X; YZ) < \infty$. Generalizing the Gaussian Wyner-Ziv lower bound in [2], we show that for all $D > 0$, $R_{X|Y\{Z\}}(D) \geq \max\left\{-\frac{1}{2}\log(l_{11}D),\ 0\right\}$.

Binary: Assume Hamming distortion, let X be uniform and binary, and let Y and Z be related to X via independent binary symmetric channels with crossover probabilities $a < \frac{1}{2}$ and $b < \frac{1}{2}$ respectively. We aim to bound $R_{X|Y\{Z\}}(D)$.

We consider the case $Y = 0$; by symmetry, the case $Y = 1$ is identical. Given $Y = 0$, the problem is identical to the Wyner-Ziv binary symmetric example in [3], except that the marginals of X and Z are now skewed. For that example, the optimal test channel timeshares between two different channels. The first relates W to X via a binary symmetric channel. The second is the zero-rate channel. That solution is not optimal for this skewed problem; we do better using an asymmetric first channel. If W is constructed via a symmetric channel, the optimal reproduction is either always equal to W or always equal to Z. An asymmetric channel allows the reproduction to depend on both W and Z and does everywhere at least as well, reducing the average distance to our lower bound, over all (a, b, D) of interest, by 61.5%.

Heegard and Berger's system (Figure 1(b)) considers source coding with unreliable s.i. at the decoder. It splits the decoder into two; decoder 1 for when s.i. is present and decoder 2 for when it is absent. We here improve the R-D upper bound for the example in [4] where Z_1 is related to X via a binary symmetric channel. The form of the R-D function [4, 5] suggests a coding strategy of two parts. The first can be decoded without Z_1 and ensures a minimum reproduction fidelity at both decoders. The second requires Z_1 for its decoding, and allows refinement at decoder 1. Once the first part is chosen, it serves as s.i. (known to both encoder and decoder 1) for coding the second part. Thus, coding the second part is a mixed s.i. problem. We create the first part using a symmetric binary channel and the second part using an asymmetric channel as above. We timeshare this solution with one in which the second part is constant and with the zero-rate solution. Our bound is never looser than the existing bounds (by Heegard and Berger [4] and Kerpez [6]) by more than numerical accuracy, and reduces the average distance to Kerpez's lower bound by 19%. Our construction can also be extended to upper bound $R(D_1, D_2)$ for a generalization of the HB system in which s.i. Z_2 is provided to decoder 2. Previous constructions were not immediately generalizable.

References

[1] R. Zamir. The rate loss in the Wyner-Ziv problem. *IEEE Trans. Inform. Theory*, 42:2073–2084, Nov 1996.

[2] A. D. Wyner. The rate-distortion function for source coding with side information at the decoder–ii: General sources. *Information and Control*, 38:60–80, 1978.

[3] A. D. Wyner and J. Ziv. The rate-distortion function for source coding with side information at the decoder. *IEEE Trans. Inform. Theory*, 22:1–10, Jan 1976.

[4] C. Heegard and T. Berger. Rate distortion when side information may be absent. *IEEE Trans. Inform. Theory*, 31(6):727–734, Nov 1985.

[5] A. H. Kaspi. Rate-distortion function when side-information may be present at the decoder. *IEEE Trans. Inform. Theory*, 40(6):2031–4, Nov 1994.

[6] K. J. Kerpez. The rate-distortion function of a binary symmetric source when side-information may be absent. *IEEE Trans. Inform. Theory*, 33(3):448–52, May 1987.

[1]This work was supported by NSF Award No. CCR-0220039, the F.W.W. Rhodes Memorial Scholarship, a Redshaw Award, and a grant from the Lee Center for Advanced Networking at Caltech.