Letters to the Editor

Erratum: Measurements of the Sound Absorption Coefficient and the Sound Transmission Loss at the Kobayasi Institute of Physical Research

[J. Acoust. Soc. Am. 32, 376–379 (1960)]

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On page 378, second column, line 6, for "500 cps" read "125 cps.'

Comments on "On the Stability of Random Systems"

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DR. Samuels[1] is to be congratulated on a most interesting paper. It is unfortunate that a number of errors appear in Sec. III which invalidate both that section and Sec. IV. The errors are:

(1) $\overline{W^2}$ in Eq. (55) should read

$$\overline{W^2}=\frac{+i(\varphi^2+2i\beta\varphi-2w_0^2)}{\varphi^3+i6\beta\varphi^2-4(w_0^2+2\beta^2)\varphi-i8\beta w_0^2}. \tag{1}$$

(2) If Eq. (54) is multiplied out, there results the equation

$$1-w_0^2S_0\overline{W^2}-4\beta^2S_1\overline{W^2}=0. \tag{2}$$

If the corrected expressions in Eq. (55) are substituted into (2), the basic frequency equation is

$$S^3+A_2S^2+A_1S+A_0=0, \tag{3}$$

where

$$\left.\begin{aligned}A_2&=2\beta(3-2\beta S_1)\\A_1&=4[w_0^2+2\beta^2(1-\beta S_1)]\\A_0&=2w_0^2[4(\beta-\beta^2S_1)-w_0^2S_0]\end{aligned}\right\}. \tag{4}$$

The frequency equation is thus seen to be a cubic equation and not a sixth-order equation as given by Samuels. Equation (3) above was obtained by Caughey and Dienes[2] by setting up the Fokker Planck equation for the system.

(3) If S_1 is set equal to zero in Eq. (3) herein, the correct equation for (58) of Samuels' paper is

$$S^3+6\beta S^2+4(w_0^2+2\beta^2)S+2w_0^2(4\beta-w_0^2S_0)=0. \tag{5}$$

Analysis of the Routh-Hurwitz stability criteria shows that for stability

$$4\beta>w_0^2S_0. \tag{6}$$

(4) If S_0 is set equal to zero in Eq. (3) herein, the correct equation for (60) of Samuels' paper is

$$S^3+2\beta(3-2\beta S_1)S^2+4[w_0^2+2\beta^2(1-\beta S_1)]S+8\beta w_0^2(1-\beta S_1)=0. \tag{7}$$

Analysis of the Routh-Hurwitz criteria shows that a necessary condition for stability is that

$$\beta S_1<1. \tag{8}$$

Hence the system is not unconditionally stable as Samuels finds.

(5) If S_0 is set equal to zero in Eq. (3) herein, and β is replaced by $-\beta$, Eq. (62) should read

$$S^3-2\beta(3+2\beta S_1)S^2+4[w_0^2+2\beta^2(1+\beta S_1)]S-8\beta w_0^2(1+\beta S_1)=0. \tag{9}$$

Examination of the Routh-Hurwitz stability conditions shows that this system is unstable for all S_1. This means that it is not possible to stabilize an unstable system by means of random noise. Samuels' results are, therefore, shown to be incorrect.

It should also be pointed out that mean squared stability is a necessary—but not a sufficient—condition for the stability of a system. In order to ensure stability of a system, all the moments must be stable.

For example, if we consider the first moment of Q in Eq. (40), it is easily shown that $\langle Q\rangle$ satisfies the differential equation

$$\frac{d^2}{dt^2}\langle Q\rangle+2\beta\frac{d}{dt}\langle Q\rangle+w_0^2\langle Q\rangle=0. \tag{10}$$

The requirements for stability are that

$$\left.\begin{aligned}\beta&>0\\w_0^2&>0\end{aligned}\right\}. \tag{11}$$

Hence, even if Samuels' analysis of the mean squared stability were correct, the stochastic mean of Q would be unstable if either β or w_0^2 were made negative; therefore, the system would be unstable.

[1] J. C. Samuels, J. Acoust. Soc. Am. 32, 594–601 (1960).
[2] T. K. Caughey and J. K. Dienes (unpublished).

Remarks on Dr. Caughey's Comments on My Paper entitled "On the Stability of Random Systems, Etc."

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THE author is grateful to Dr. Caughey for having pointed out some errors in Sec. III of his paper[1] which invalidate certain conclusions concerning the possibility of stabilizing systems with random noise. The corrections to the formulas which he gives are correct. It should be noted, however, that the possibility of stabilizing systems with some form of random noise is not dead. The error in the analysis which led the author to believe that systems could be stabilized with random noise prompted him to try to do it with a second-order system on an analog computer. While the results have been far from conclusive, we have had some encouragement. It is possible, however, that the situation is confused by the entering in of certain nonlinear effects in the computer multipliers.

We should remember that Eq. (10) of Dr. Caughey's comments and his conclusions hold only for *white noise* parameter variations. It would be desirable to extend the stability theory to systems with nonwhite noise parameter variations. The author feels that further theoretical and experimental work is required to clear up the situation.

[1] J. C. Samuels, J. Acoust. Soc. Am. 32, 594–601 (1960).