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Introduction

Almost 25 years ago Lorenz published his seminal study on the existence of a strange attractor in the phase space of a severely truncated model system arising from the hydrodynamical equations describing two-dimensional convection. Nearly a century ago Poincaré published his famous treatise Les Methodes Nouvelles de la Mecanique Celeste (1892) in which the possible complexity of behavior in nonintegrable, conservative systems was first envisioned. Both these works address an age old puzzle: How do apparently stochastic outputs arise from an entirely deterministic system subject to non-stochastic inputs?

Several well studied examples exist of simple mechanical systems with a few degrees of freedom that display stochastic behavior, even though the laws of motion are given by well known, regular equations. In fluid mechanics we are quite accustomed to the notion that the time dependence of the output is more complicated than the time dependence of the input: A convection cell with steady heating of the walls can produce time dependent motion. A steady flow impinging on a cylinder can produce vortex-shedding, a time-periodic phenomenon. At sufficiently high flow speeds we can produce a stochastic flow, the turbulent wake. It is thus a natural idea to explore whether the complex behavior seen in simple mechanical models, usually referred to by the term chaos, is not in some way related to the appearance of turbulence in fluid flow.

The basic premise of considering the statistical behavior in a flow field to be a calculable effect, rather than an assumption to be made at the outset, opens up a new realm of experimental and theoretical questions. Considerable progress has been made on flows that may be characterized as "closed" in that the same fluid is continually recycled in the sample volume. Such systems include convection cells and the Taylor-Couette apparatus. Chaotic flow in these systems has already spawned a wide literature.

Of considerable importance to our discussion is the fact that there are regions in shear flows where the dynamics is clearly beyond the initial linear, and weakly nonlinear stages, yet is not fully developed turbulence. The initial portion of a mixing layer, a wake or a jet are familiar examples. In fluid mechanics we often refer to such regimes as transitional flows. Such regions are immensely important from a practical point of view, for it has been found that forcing applied to these regions can seriously influence the development of the turbulence farther downstream. Transitional flow regimes are usually characterized by a limited number of "instability modes" (degrees of freedom) being "excited," yet the flow signal can have an extremely complicated signature. We see these regimes as prime candidates for a description in terms of chaotic motion in simple dynamical systems.

There is another instance in which the idea of chaotic motion enters fluid mechanics with great clarity. If we consider the kinematics of a passive marker advected by a flow, it is easy to see that this problem in general must admit chaotic solutions. This can happen even if the Eulerian flow field doing the advecting is completely "regular," i.e. not chaotic. In particular the Eulerian flow can be a very simple function of space and time. This idea of chaotic advection [1] has surfaced in a variety of topics dealing with the redistribution of material by flows. In the context of shear flows it promises new insights into mechanisms of transport and mixing and into the recurring issue of flow visualization as a diagnostic tool.

Our objectives here are to give a status report on an emerging field, viz the application of the concept of chaos to shear flows. Our position is (1) that the notion of chaotic behavior is evidently a sound, new and exciting phenomenon in the context of simple mechanical systems. (2) It has had some preliminary successes as a predictive tool in very simple, primarily "closed" flow systems. (3) It suggests new ways of taking and looking at data in shear flows, and (4) that it is an intellectually exciting and potentially powerful idea to guide future exploration.

As we proceed it will become clear that the "chaos theory" approach involves a major reassessment of the role of the Navier-Stokes equations in turbulence studies. The chaos viewpoint with its emphasis on stochastic behavior arising from deterministic equations is clearly forced to focus on details of the mechanics of the flow. We see this as a positive development. We shall also argue that the focus on mechanisms and structures in the flow leads naturally to an emphasis on the Lagrangian representation of the equations of motion.

Case Studies

We proceed from a set of specific case studies. We first treat the emergence of chaos in a classical model system of fluid mechanics, the equations of motion for point vortices. This model system has been used frequently to address flows of the transitional type that we mentioned above, in particular we recall the analysis by von Kármán of the point vortex street. Next we turn to a description of experiments on the wake of a cylinder. We give a brief review of the literature on this important problem with particular emphasis on areas where chaos might be a relevant consideration. This leads naturally to our next main topic, the forced wake of an airfoil, a key problem in aeronautics. Again we show how certain flow regimes imply interpretation in terms of chaotic dynamics. Then we discuss the notion of chaotic particle paths in flows. We describe the basic theoretical ideas and some of the experimental verifications.

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Point vortex chaos (2D flow)

Chaos in the few-vortex problem has been studied for several years, and the transition to chaos as the number of identical vortices is increased from three to four is now well known [2]. Some recent results show how chaos manifests itself in the scattering data of vortex pair collisions (Figure 1). Two point vortex pairs with circulations ±1 and ±0.9 come in from "infinity" and are allowed to collide. This experiment is repeated for many values of the impact parameter of the two pairs. For each collision the total scattering time is monitored, i.e the time it takes until two pairs again emerge at some large distance from the scattering site. This scattering time is plotted in Fig.1 versus the impact parameter. It has an immensely complicated structure, which in fact repeats on ever finer scales. It is believed that this function gives an example of what is called a "devil's staircase" [3].

The physical origin of this behavior is not far to seek. Upon collision the components of each pair can be separated. As they approach again, they can either pair up with their original counterpart, or scatter again. Figure 2 gives examples of vortex pair trajectories. In small windows of impact parameter many such collisions take place before the original pairs reform and fly off to inifinity. The height of the peaks in Fig.1 reflects the number of intermediate collisions that take place for a given value of impact parameter.

This is a highly idealized problem with some of the "flavor" of an "open flow." Standard diagnostics of chaos such as the Poincaré section are not useful here. The appearance of chaos in the collisions of vortex pairs has implications for the far field pressure signal as already documented for vortices in confined domains.

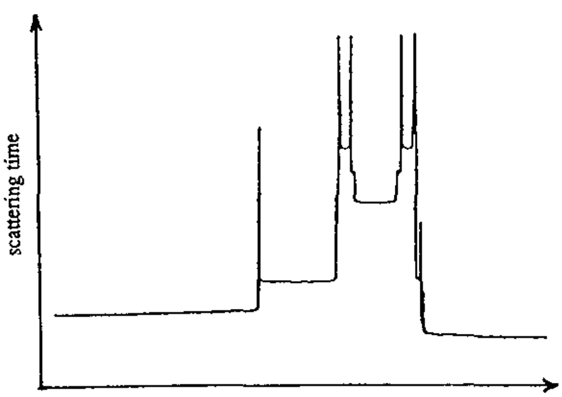

Figure 1: Scattering time versus impact parameter for the scattering of one point vortex pair off another in two-dimensional flow [3].

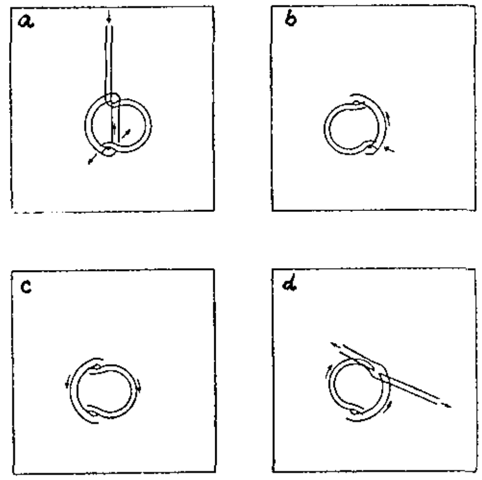

Figure 2: Sample vortex trajectories showing an initial "exchange" (a) followed by two "direct" scatterings (b,c) and finally absolved by an "exchange" [3].

Wake of cylinder

The dynamics of the Kàrmàn vortex street in the wake of a cylinder (wire) has been considered to be a prime candidate for chaotic motion. It has been known for some time that a plot of shedding frequency versus flow Reynolds number often contains discontinuities in slope [4]. Recent proposals that chaotic vortex motion was associated with such discontinuities [5] potentially fit in very nicely with the results discussed above. Reality, however, turned out to be very much richer [6]. Careful experiments in which vibrations of the cylindrical wire were monitored continuously showed that wire vibrations invariably get coupled in with the fluid motion, i.e. the problem is one of aeroelasticity. These results appear to fit in consistently with a large body of data on shedding from cables and wires in the Reynolds number regime 50-200. Extensive tests of the nonlinear coupling of wire vibrations to vortex shedding has been studied using bispectral analysis [7]. Chaotic flow does indeed occur, but it is forced by the vibrations of the cylinder that the shedding itself has induced.

Recent work has considered the spatial structure of the wake behind a vibrating wire. Measurements and flow visualization of the frequency locked wake show the existence of spatially periodic spanwise structure in which the flow is alternately chaotic and regular (Figure 3). These secondary wake eddies, with scales many times the separation of the vortices in the Kàrmàn street are believed to be a new form of collective vortex instability unknown to previous workers.

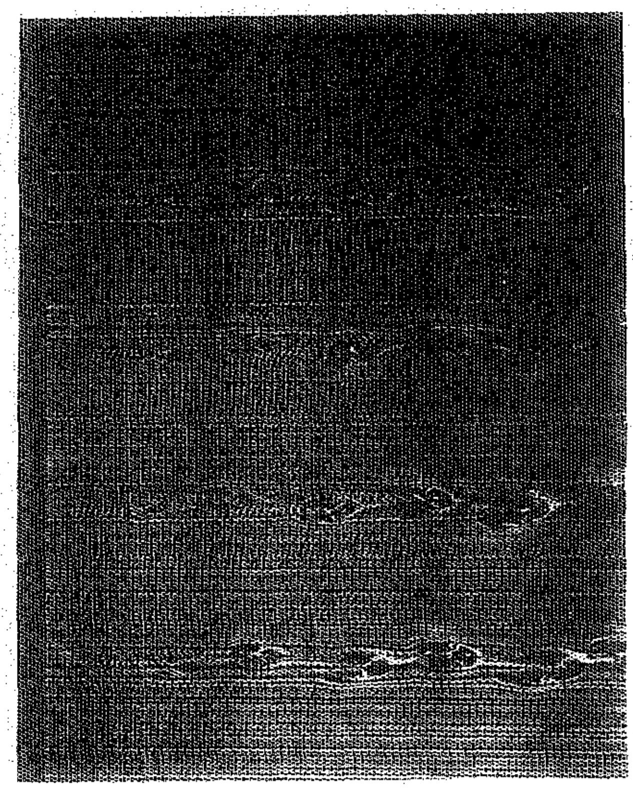

Figure 3: Plan view of the wake of a vibrating wire. Smoke visualization in air starting at x/d=25. Flow from left to right showing alternating bands of chaotic and regular regimes. The large "puffs" mark the collective vortex interactions.

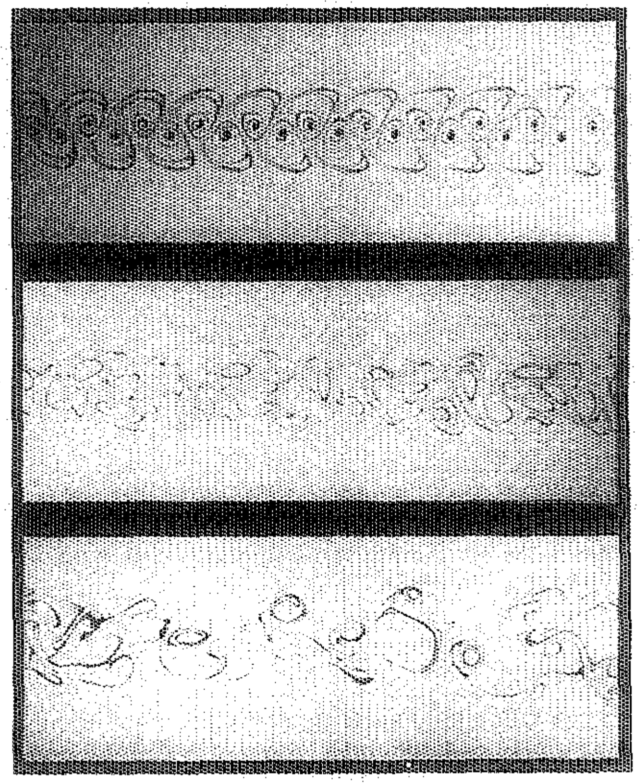

Figure 4: Vortex patterns in forced wakes of an airfoil. Dye visualizations in water. Top: Regular wake. Middle: Quasiperiodic wake. Bottom: Chaotic wake [8].

The wake of an airfoil

By taking active control of the wake forcing using the "strip-heating" technique on an airfoil we have a means for observing the interaction and nonlinear evolution of waves in the wake without being dependent on vibrations that are not under our control. A tremendously rich variety of states is observed. We can only give an indication of the more prominent modes here: In Fig.4 we show (top) frequency locking in which the vortex shedding frequency is the same as the forcing frequency; (middle) quasiperiodic vortex interaction in which periodic clusters of vortices are observed due to the presence of two incommensurate frequencies; (bottom) chaotic vortex interaction in which the vortices in the wake have a random structure due to the interaction of three incommensurate frequencies. All these different states come about by forcing the boundary layers of the airfoil using the strip-heater technique [8]. We believe that some theoretical understanding of these states is possible using recent analytical methods [9].

Chaotic advection

Chaotic advection simulations have been performed for a variety of model flows starting with the "blinking vortex" flow suggested some time ago [1]. Several laboratory experiments on this phenomenon are now available. The experiments have tended to focus on chaotic particle trajectories in regular flows at low Reynolds number. Computations have spanned the entire Reynolds number range.

We report here on a curious special case, a pulsed source/sink system, in which chaotic particle paths lead to efficient mixing of fluid even though there is no circulation anywhere in the flow [10]. The fluid taken out at the sink is reinjected at the source. Streaklines for this flow (Figure 5) show "islands" without vorticity, due to the KAM theorem. There are many implications of this work for scalar transport, fluid mixing and flow visualization.

The phenomenon of chaotic advection provides a mechanistic rationale for the emergence of fractal isosurfaces [11,12] in unsteady flows and a fortiori in turbulent flows.

Acknowledgment

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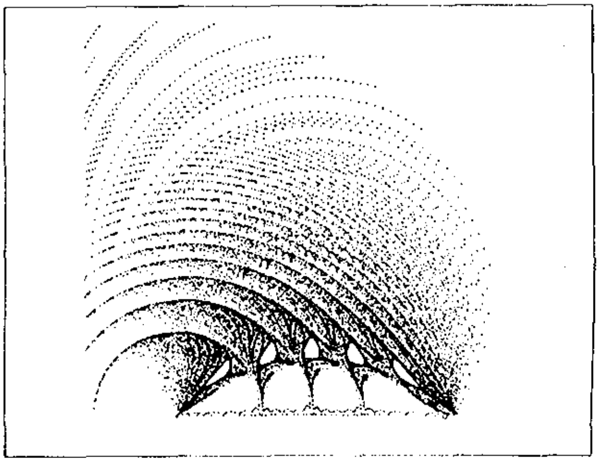

Figure 5: Streakline pattern for a pulsed source/sink system. The flow is irrotational everywhere including the "island" regions seen [9].

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