Optimal Decentralized Protocol for Electric Vehicle Charging

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Abstract— Motivated by the power-grid-side challenges in the integration of electric vehicles, we propose a decentralized protocol for negotiating day-ahead charging schedules for electric vehicles. The overall goal is to shift the load due to electric vehicles to fill the overnight electricity demand valley. In each iteration of the proposed protocol, electric vehicles choose their own charging profiles for the following day according to the price profile broadcast by the utility, and the utility updates the price profile to guide their behavior. This protocol is guaranteed to converge, irrespective of the specifications (e.g., maximum charging rate and deadline) of electric vehicles. At convergence, the l_2 norm of the aggregated demand is minimized, and the aggregated demand profile is as "flat" as it can possibly be. The proposed protocol needs no coordination among the electric vehicles, hence requires low communication and computation capability. Simulation results demonstrate convergence to optimal collections of charging profiles within few iterations.

I. INTRODUCTION

Electric vehicles (EVs) offer great potential for increasing energy efficiency, reducing greenhouse gas emissions, and relieving reliance on foreign oil for transportation [1]. Currently, several types of EVs are either already in the U.S. market, or about to enter [2], and electrification of transportation is at the forefront of many research and development agendas [3]. On the other hand, the potential comes with a multitude of challenges including those in the integration with the electric power grid. EV charging increases the electric loads, and potentially amplifies the peak demand or creates new peaks [4]. It also increases the demand side uncertainties, and potentially reduces the distribution circuit and transformer lifespan [5]. Moreover, power losses and voltage variations become more likely [6].

The simulation-based study in [7] suggests that, if no regulation on EV charging is implemented, even a 10% penetration of EVs may cause unacceptable variations in the voltage profiles. On the other hand, many studies demonstrate that adopting "smart" charging strategies can mitigate some of the integration challenges, defer infrastructure investment needed otherwise, and even stabilize the grid. For example, scheduling EV charging so that aggregated EV load fills the overnight demand valley may reduce daily cycling of power plants and operational cost of the electricity utility [8]. Furthermore, the energy stored in EVs may be utilized as an alternative ancillary service resource [9] for regulating

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voltage profiles, ride-through support for fault protection, and even compensating fluctuating renewable energy generation [10].

Studies on EV charging control fall into three categories: effect of time-of-use rates [11], coordinated charging scheduling [6], [10], [12], and decentralized scheduling [13] [14]. Reference [11] explores the effect of higher price during peak hours on shifting EV load, but does not provide strategies for setting the price. Reference [6], [10], [12] study centralized control strategies that minimize power losses, load variance, or maximize load factor, allowable EV penetration level. These strategies require a centralized structure to collect all information of all EVs and optimize over the charging profiles of all EVs, hence incur prohibitive communication and computation requirements. Reference [13] demonstrates, through a simulation-based study, that EV load can be smoothed by introducing load-side participation into the power market. They propose a decentralized strategy, but do not provide any analytical optimality guarantees. Reference [14] proposes another decentralized charging strategy, and proves its optimality in the case where all EVs plug in at the same time with the same state-of-charge (SOC), and have the same deadline and allowable charging rates. For future reference, we call this setting *homogeneous*.

In this paper, we partially alleviate some of the restrictions imposed in [14]. First, we define optimal charging profiles of EVs explicitly (this definition generalizes the implicit definition in [14]). Second, we propose a decentralized charging strategy that guarantees optimality in both homogeneous and non-homogeneous cases, where EVs can plug-in at different times with different SOC, have different maximum charging speed and deadlines. Third, we remove the artificial tracking error penalty in EV owners' objective in [14] by introducing another penalty term that vanishes at convergence.

The rest of the paper is organized as follows. Section II formulates EV charging control as a global optimization problem. Section III explores properties of optimal charging profiles, proposes a decentralized algorithm and proves its convergence to optimal charging profiles. Numerical simulations are used to illustrate these results in section IV, and conclusions and future works are presented in section V.

II. PROBLEM FORMULATION

We consider a scenario where an electric utility negotiates with N electric vehicles (indexed $1, \ldots, N$) for a day-ahead charging schedule over T time slots (indexed $1, \ldots, T$), each of length ΔT. Let $D(t)$ denote the base demand (aggregated non-EV demand) at time t, $r_n(t)$ denote the charging rate of EV n at time t, for all $t \in \{1, \ldots, T\}$.

TABLE I

LIST OF SYMBOLS

T	time horizon
t	time, $t \in \{1, \ldots, T\}$
N	number of EVs
n	EV n, $n \in \{1, \ldots, N\}$
$D(t)$	base demand at time t.
$D(\cdot)$	base demand profile.
$r_n(t)$	charging rate of EV n at time t
$r_n(\cdot)$	charging profile of EV n
$r(\cdot)$	charging profile of all EVs
$R_r(\cdot)$	aggregated charging profile corresponding to r
$\underline{r}_n(t)$	charging rate lower bound for EV n at time t
$\overline{r}_n(t)$	charging rate upper bound for EV n at time t
R_n	charging rate sum of EV n
$\mathcal{D}_n$	set of feasible charging profiles for EV n
$\mathcal{D}$	set of feasible charging profiles for all EVs
$\mathcal{S}$	set of optimal charging profiles
$\preceq$	if $a, b \in \mathbb{R}^n$, $a \preceq b \Leftrightarrow a_i \leq b_i$ for all $i \in \{1, \ldots, n\}$.

Let $r_n := (r_n(1), \ldots, r_n(T))$ denote the charging profile of vehicle n, for all $n \in \{1, \ldots, N\}$. Then, roughly speaking, a charging profile $r := (r_1(\cdot), \ldots, r_N(\cdot))$ is "valley-filling", if it minimizes

$$\sum_{t=1}^{T} \left(D(t) + \sum_{n=1}^{N} r_n(t) \right)^2.$$

A more rigorous discussion on valley-filling will be given in section III-A. The overall goal is to achieve a valley-filling charging profile, subject to the following constraints.

For EV n, due to battery specification, its charging rate should be within some interval $[\underline{r}_n, \overline{r}_n]$. Taking deadline into account, we let $\underline{r}_n$ and $\overline{r}_n$ be time-dependent and set $\underline{r}_n(t) = \overline{r}_n(t) = 0$ for t outside the allowable charging period of EV n. Hence

$$\underline{r}_n(t) \leq r_n(t) \leq \overline{r}_n(t)$$

for all $n \in \{1, \ldots, N\}$ and all $t \in \{1, \ldots, T\}$. Let B_n, $s_n(0)$, and η_n denote the battery capacity, initial SOC and charging efficiency of EV n. By its deadline, EV n should be fully charged, this is captured by the total energy stored over the whole time horizon $\eta_n \sum_{t=1}^{T} r_n(t)\Delta T = B_n (1 - s_n(0))$. Define $R_n := B_n (1 - s_n(0)) / \eta_n / \Delta T$, it is equivalent to

$$\sum_{t=1}^{T} r_n(t) = R_n.$$

for all $n \in \{1, \ldots, N\}$. Reference [15] summarizes some of the recently announced EV models, and typical values of $\overline{r}_n$ and R_n can be derived from these models. $\underline{r}_n$ is usually set to be 0.

Definition 1: Let $D \in \mathbb{R}^T$ be the base demand. A charging profile $r = (r_1, \ldots, r_N)$, where $r_n \in \mathbb{R}^T$ for all $n \in$ $\{1, \ldots, N\}$, is *optimal*, or *valley-filling*, if it solves

$$\underset{r_1, \ldots, r_N}{\text{minimize}} \quad \sum_{t=1}^{T} \left(D(t) + \sum_{n=1}^{N} r_n(t) \right)^2 \qquad (1)$$
$$\text{subject to} \quad \underline{r}_n \preceq r_n \preceq \overline{r}_n, \; n = 1, \ldots, N,$$
$$\sum_{t=1}^{T} r_n(t) = R_n, \; n = 1, \ldots, N,$$

where $\underline{r}_n, \overline{r}_n \in \mathbb{R}^T$ and $R_n \in \mathbb{R}$ for all $n \in \{1, \ldots, N\}$.

Definition 2: A charging profile is *feasible* if it satisfies the constraints in (1).

III. MAIN RESULTS

In this section, we first explore properties of optimal charging profiles, and then propose a decentralized algorithm for solving (1). In the end, we prove that the algorithm we propose converges to the set of optimal charging profiles. Given a charging profile $r := (r_1, \ldots, r_N)$, let

$$R_r := \sum_{n=1}^{N} r_n$$

denote the aggregated charging profile corresponding to r.

A. Optimal Charging Profiles

Property 1: Let r be a feasible charging profile. If there exists $A \in \mathbb{R}$, such that $R_r(t) = \max \left\{ \sum_{n=1}^{N} \underline{r}_n(t), A - D(t) \right\}$ for all $t \in \{1, \ldots, T\}$, then r is optimal.

Proof: Note that R_r is the unique solution to the problem

$$\underset{R}{\text{minimize}} \quad \sum_{t=1}^{T} \left(D(t) + R(t) \right)^2 \qquad (2)$$
$$\text{subject to} \quad \sum_{t=1}^{T} R(t) = \sum_{n=1}^{N} R_n$$
$$R \succeq \sum_{n=1}^{N} \underline{r}_n.$$

For any r' feasible for (1), $R_{r'}$ is feasible for (2). Furthermore, the objective function in (2) evaluated at $R_{r'}$ is equal to the objective function in (1) evaluated at r'. Hence, the optimal value d_* of (2) is a lower bound for the optimal value p_* of (1). The aggregated profile R_r attains d_*, so r attains $d_* \leq p_*$. Since r is feasible, it is optimal. $\square$

Let $\mathcal{D}_n := \left\{ r_n | \underline{r}_n \preceq r_n \preceq \overline{r}_n, \sum_{t=1}^{T} r_n(t) = R_n \right\}$ denote the set of feasible charging profiles of EV n. Then $\mathcal{D} := \mathcal{D}_1 \times \cdots \times \mathcal{D}_N$ is the set of feasible charging profiles of all EVs.

Property 2: If the set $\mathcal{D}$ of feasible charging profiles is non-empty, then optimal charging profiles exist.

Proof: $\mathcal{D}$ is the feasible set of (1) by Definition 2. Since $\mathcal{D}_n$ is compact for each n, $\mathcal{D}$ as a product of N compact sets is also compact. Furthermore, $\mathcal{D}$ is non-empty, and the objective function of (1) is continuous. Hence, its optimal value is attained at some $r \in \mathcal{D}$. $\square$
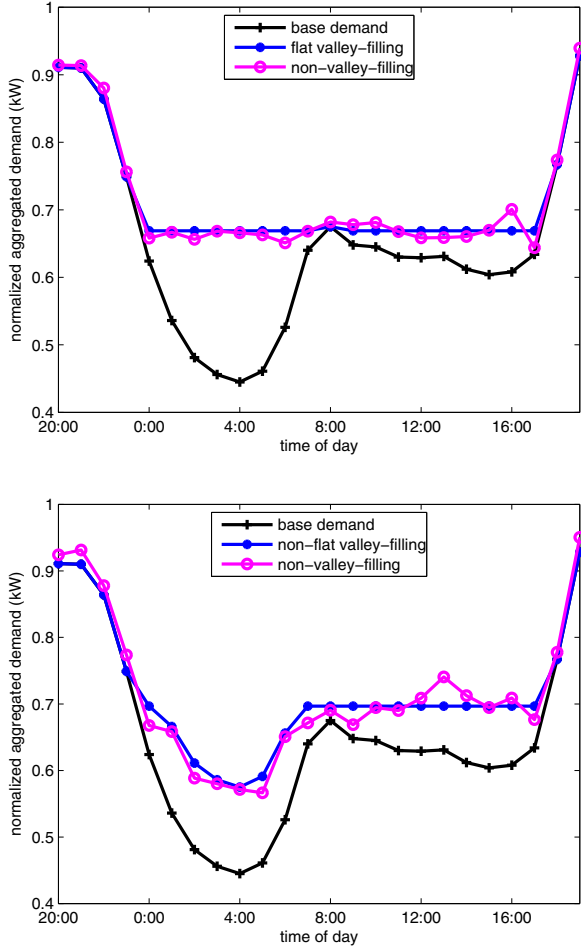
Fig. 1. Base demand curve is the average residential load in the service area of SCE from 20:00 on Feb. 13th to 19:00 on Feb 14th, 2011 [16]. Valley-filling curve corresponds to the outcome of Algorithm ODC, and can be flat or non-flat. A hypothetical non-valley-filling curve is shown with the marker o.

Property 1 is directly related to our intuitive notion of valley-filling. As shown in Figure 1 (top), a valley-filling charging profile has the same aggregated demand during periods in which some EVs charge (0:00-17:00). And the aggregated demand in this period is no higher than that outside this period. We call such aggregated demand profile *completely flat*. However, under certain conditions, complete flatness may not be possible. For example, the "valley" may be so deep at time t that even if every vehicle charges at its maximum rate, the valley may still not be completely filled, e.g., at 4:00 in Figure 1 (bottom). Moreover, vehicles may have such stringent deadline constraints that the flexibility of scheduling is limited, and complete flatness cannot be achieved. Definition 1 relaxes these restrictions as a result of Property 2. In cases where complete flatness is possible, Property 1 guarantees that our constructive definition of valley-filling agrees with complete flatness. In cases where complete flatness is not possible, Property 2 guarantees existence of an optimal charging profile. Furthermore, as

shown in Figure 1 (bottom), a valley-filling charging profile indeed gives a "smoother" aggregated demand, compared to non-valley-filling charging profiles.

We now establish an equivalence relation between charging profiles, and use it to describe an important property of the set of optimal charging profiles.

Definition 3: Feasible charging profiles r and r' are *equivalent*, denoted as $r \sim r'$, if $R_r = R_{r'}$.

That is, r and r' are equivalent if their aggregated charging profiles are the same. It is easy to check that this is indeed an equivalence relation. Now, we can define equivalence classes based on this equivalence relation. Given base demand D, define

$$\mathcal{S} := \{r \in \mathcal{D} | r \text{ optimal w.r.t } D\}$$

as the set of optimal charging profiles.

Theorem 1: If feasible charging profiles exist, then the set $\mathcal{S}$ of optimal charging profiles is non-empty, compact, convex, and an equivalence class.

Proof: Property 2 implies that $\mathcal{S}$ is nonempty when feasible charging profiles exist. Let r be an optimal charging profile, then $r \in \mathcal{S}$. Define $\mathcal{S}' := \{r' \in \mathcal{D} | r' \sim r\}$. It is easy to show that $\mathcal{S}'$ is closed and convex. Since $\mathcal{S}' \subseteq \mathcal{D}$, which is compact, $\mathcal{S}'$ is also compact. Hence $\mathcal{S}'$ is non-empty, compact, convex, and an equivalence class (by construction).

It is easy to see that $\mathcal{S}' \subseteq \mathcal{S}$. In order to prove $\mathcal{S} = \mathcal{S}'$, we now show that $\mathcal{S} \subseteq \mathcal{S}'$. For all $r' \in \mathcal{S}$, from the first order optimality condition for (1),

$$\langle D + R_r, R_{r'} - R_r \rangle = 0,$$
$$\langle D + R_{r'}, R_r - R_{r'} \rangle = 0.$$

Reverse $R_{r'} - R_r$ in the first equation and subtract the second one to obtain $\langle R_r - R_{r'}, R_r - R_{r'} \rangle = 0$. Hence $R_r = R_{r'}$, $r' \sim r$, $r' \in \mathcal{S}'$, $\mathcal{S} \subseteq \mathcal{S}'$. Consequently, $\mathcal{S} = \mathcal{S}'$ is non-empty, compact, convex, and an equivalence class. $\square$

Corollary 1: Optimal charging profile is generally not unique.

Corollary 1 is a direct consequence of the fact that $\mathcal{S}$ is an equivalence class. In general, two EVs can exchange their charging profiles in the opposite direction without affecting the objective value. For instance, EV 1 raises its charging rate at time t_1 by Δr and decreases its charging rate at t_2 by Δr, while not violating the constraints in (1). Conversely, EV 2 decreases its charging rate at time t_1 by Δr and increases its charging rate at t_2 by Δr, while not violating the constraints in (1). If the original charging profile is optimal, the new profile, which is equivalent to the original one, is also optimal, due to Theorem 1.

B. Algorithm ODC

We develop a decentralized algorithm to solve (1). System diagram of the algorithm is shown in Figure 2. Given broadcast electricity price profile, each EV makes an independent decision and chooses its charging profile. The utility guides their behavior by setting the prices.

Algorithm ODC (optimal decentralized charging): Given T, N, K, $D(t)$, R_n, $\underline{r}_n(t)$, $\overline{r}_n(t)$, for all $n \in \{1, \ldots, N\}$ and all $t \in \{1, \ldots, T\}$, define $\alpha := N/2$.

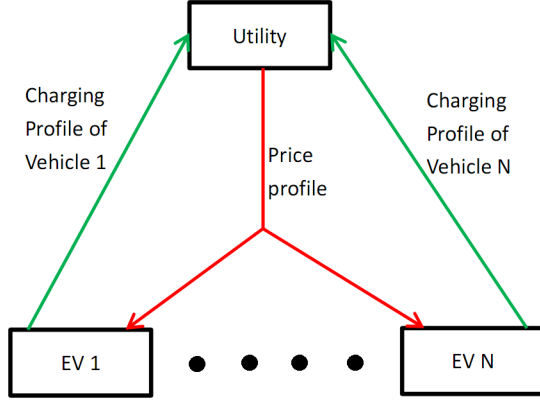|	
$p^k(\cdot)$	price profile in iteration k
$r_n^k(\cdot)$	charging profile of vehicle n in iteration k
K	maximum number of iterations

Fig. 2. Schematic view of the information flows in Algorithm ODC. Given predicted price profile, vehicles make independent decisions and choose their own optimal charging profiles. The utility guides their behavior by setting the price based on the aggregated demand.

1) Initialization:

$$p^0(\cdot) := D(\cdot), \ r_n^0(\cdot) := 0, \ k \leftarrow 0.$$

2) Utility broadcasts $p^k(\cdot)$.

3) EV n calculates a new charging profile $r_n^{k+1}(\cdot)$ as [1]

$$\underset{r_n(\cdot)}{\text{argmin}} \quad \sum_{t=1}^{T} p^k(t)r_n(t) + \alpha \left(r_n(t) - r_n^k(t)\right)^2$$
$$\text{subject to} \quad \underline{r}_n(\cdot) \preceq r_n(\cdot) \preceq \overline{r}_n(\cdot),$$
$$\sum_{t=1}^{T} r_n(t) = R_n.$$

4) Utility collects new charging profiles of all EVs, and updates the price according to

$$p^{k+1}(\cdot) := D(\cdot) + \sum_{n=1}^{N} r_n^{k+1}(\cdot). \qquad (3)$$

5) $k \leftarrow k+1$, go to (2) until $k = K$ or convergence.

In iteration $k+1$, the algorithm can be split into two parts. In the first part, EV n chooses a charging profile that minimizes its objective function

$$C_n(r_n) = \sum_{t=1}^{T} p^k(t)r_n(t) + \alpha \left(r_n(t) - r_n^k(t)\right)^2. \qquad (4)$$

The first term is the electricity cost and the second term penalizes deviations from the charging profile computed in the previous iteration. This extra penalty term ensures convergence of Algorithm ODC. In section III-C, we prove

[1] Scaling factors used to cancel the units are omitted for brevity.

that $r_n^{k+1}(t) - r_n^k(t) \to 0$ for all n and all t as $k \to \infty$. Hence, the objective function of each EV boils down to its electricity cost at convergence.

In the second part of iteration $k+1$, the utility updates the price according to (3). It sets high prices for the periods with high aggregated demand. Intuitively, in the next iteration, vehicles are given the incentive to shift their charging profiles. As a result, aggregated demand may be smoothened.

As proved in section III-C, Algorithm ODC converges to optimal charging profiles. Furthermore, simulation results demonstrate fast convergence (usually within a few iterations) of Algorithm ODC.

C. Analysis of Algorithm ODC

We now analyze the convergence properties of Algorithm ODC. Since optimal charging profile is generally not unique, we need to establish an appropriate notion of convergence to optimal charging profiles. Let superscript k denote the respective value of in iteration k of Algorithm ODC. For example, r_n^k denotes the charging profile of vehicle n in iteration k. Similarly, $R^k := \sum_{n=1}^{N} r_n^k$ denotes the aggregated charging profile in iteration k, $d^k := (D + R^k)/N$ denotes the normalized aggregated demand profile in iteration k.

Lemma 1: If feasible charging profiles exist, then the inequality

$$\left\| d^k + r_n^{k+1} - r_n^k \right\|_2^2 \leq \left\| d^k \right\|_2^2 - \left\| r_n^{k+1} - r_n^k \right\|_2^2 \qquad (5)$$

holds for all $n \in \{1, \dots, N\}$ and all $k \geq 1$.

Proof: Since feasible charging profiles exist, feasible charging profiles for EV n exist, and $r_n^k \in \mathcal{D}_n$ for all $n \in \{1, \dots, N\}$ and all $k \geq 1$. Further,

$$\begin{aligned}
r_n^{k+1} &= \underset{r_n \in \mathcal{D}_n}{\text{argmin}} \sum_{t=1}^{T} p^k(t)r_n(t) + \alpha \left(r_n(t) - r_n^k(t)\right)^2 \\
&= \underset{r_n \in \mathcal{D}_n}{\text{argmin}} \sum_{t=1}^{T} 2d^k(t)r_n(t) + \left(r_n(t) - r_n^k(t)\right)^2 \\
&= \underset{r_n \in \mathcal{D}_n}{\text{argmin}} \sum_{t=1}^{T} \left(d^k(t) + r_n(t) - r_n^k(t)\right)^2 \\
&= \underset{r_n \in \mathcal{D}_n}{\text{argmin}} \left\| d^k + r_n - r_n^k \right\|_2^2. \qquad (6)
\end{aligned}$$

From the first order optimality condition for (6),

$$\langle d^k + r_n^{k+1} - r_n^k, r_n - r_n^{k+1} \rangle \geq 0 \qquad (7)$$

for all $r_n \in \mathcal{D}_n$. Choose $r_n = r_n^k$ to obtain

$$\langle d^k + r_n^{k+1} - r_n^k, r_n^k - r_n^{k+1} \rangle \geq 0.$$

Split the first term to obtain

$$\langle d^k, r_n^{k+1} - r_n^k \rangle \leq - \left\| r_n^{k+1} - r_n^k \right\|_2^2.$$

Hence,

$$\begin{aligned}
& \left\| d^k + r_n^{k+1} - r_n^k \right\|_2^2 \\
&= \left\| d^k \right\|_2^2 + \left\| r_n^{k+1} - r_n^k \right\|_2^2 + 2\langle d^k, r_n^{k+1} - r_n^k \rangle \\
&\leq \left\| d^k \right\|_2^2 - \left\| r_n^{k+1} - r_n^k \right\|_2^2
\end{aligned}$$

for all $n \in \{1, \ldots, N\}$ and all $k \geq 1$. □

Lemma 2: If feasible charging profiles exist, then for all $n \in \{1, \ldots, N\}$ and all $k \geq 1$, $r_n^{k+1} = r_n^k$ if and only if

$$\langle d^k, r_n - r_n^k \rangle \geq 0 \quad (8)$$

for all $r_n \in \mathcal{D}_n$.

Proof: Follows from (7) and the strict convexity of (6).□

Theorem 2: Let $r^k := (r_1^k, \ldots, r_N^k)$ be the charging profile in iteration k of Algorithm ODC. If feasible charging profiles exist, then $r^k \to \mathcal{S}$ as $k \to \infty$.

Proof: The proof is based on a Lyapunov-type argument. Define $L(r) := \sum_{t=1}^{T} \left(D(t) + R_r(t) \right)^2$, then

$$
\begin{aligned}
L(r^{k+1}) &= \sum_{t=1}^{T} \left(D(t) + R^{k+1}(t) \right)^2 \\
&= N^2 \sum_{t=1}^{T} \left(d^k(t) + \frac{1}{N} \sum_{n=1}^{N} \left(r_n^{k+1}(t) - r_n^k(t) \right) \right)^2 \\
&\leq N^2 \sum_{t=1}^{T} \left(\frac{1}{N} \sum_{n=1}^{N} \left(d^k(t) + r_n^{k+1}(t) - r_n^k(t) \right)^2 \right) \\
&= N \sum_{n=1}^{N} \left\| d^k + r_n^{k+1} - r_n^k \right\|_2^2 \\
&\leq N \sum_{n=1}^{N} \left(\left\| d^k \right\|_2^2 - \left\| r_n^{k+1} - r_n^k \right\|_2^2 \right) \quad (9) \\
&\leq N^2 \left\| d^k \right\|_2^2 = L(r^k). \quad (10)
\end{aligned}
$$

The first inequality is due to Jessen's inequality, and the second inequality is due to Lemma 1. It is easy to show that $L(r^{k+1}) = L(r^k)$ if and only if $r_n^{k+1} = r_n^k$ for all n, i.e., $r^{k+1} = r^k$.

If $r^{k+1} = r^k$, it follows from Lemma 2 that for all n, $\langle d^k, r_n - r_n^k \rangle \geq 0$ for all $r_n \in \mathcal{D}_n$. Hence,

$$\langle D + R^k, R_{r'} - R^k \rangle = N \sum_{n=1}^{N} \langle d^k, r_n' - r_n^k \rangle \geq 0 \quad (11)$$

for all $r' = (r_1', \ldots, r_N') \in \mathcal{D}$. This is the first order optimality condition for (1), hence $r^k \in \mathcal{S}$. On the other hand, if $r^k \in \mathcal{S}$, $L(r^k) \leq L(r^{k+1})$, hence $L(r^{k+1}) = L(r^k)$.

It follows that $L(r^{k+1}) = L(r^k) \Leftrightarrow r^{k+1} = r^k \Leftrightarrow r^k \in \mathcal{S}$. Now we have $\mathcal{D}$ is compact; $\mathcal{S}$ minimizes $L(\mathcal{D})$; $L(r^{k+1}) < L(r^k)$ if $r^k \notin \mathcal{S}$. It follows that $r^k \to \mathcal{S}$ as $k \to \infty$. □

Corollary 2: A charging profile r is an equilibrium point of Algorithm ODC if and only if $r \in \mathcal{S}$, the set of optimal charging profiles.

Proof: Since $r^{k+1} = r^k \Leftrightarrow r^k \in \mathcal{S}$ (showed in the proof of Theorem 2), the set of equilibrium points $\mathcal{E} := \{r^k | r^{k+1} = r^k\} = \mathcal{S}$. □

Theorem 3: Let r^* be an optimal charging profile, p^k be the price profile in iteration k of Algorithm ODC. If feasible charging profiles exist, then

- aggregated charging profile converges to that of r^*, i.e.,

$$\lim_{k \to \infty} R^k = R_{r^*};$$

- price profile converges to that of r^*, i.e.,

$$\lim_{k \to \infty} p^k = D + R_{r^*};$$

- for each EV, the change in charging profile between consecutive iterations of Algorithm ODC converges to 0, i.e.,

$$\lim_{k \to \infty} \left\| r_n^{k+1} - r_n^k \right\|_2 = 0$$

for all $n \in \{1, \ldots, N\}$.

Proof: Since the set $\mathcal{S}$ of optimal charging profiles is an equivalence class, for all $r' \in \mathcal{S}$, $R_{r'} = R_r$. Let r^k be the charging profile in iteration k. Theorem 2 implies that we can find a sequence in $\mathcal{S}$, such that the distance between r^k and the kth element of the sequence converges to 0 as $k \to \infty$. It follows that $R^k \to R_r$, as $k \to \infty$. The price profile converges as a direct consequence of (3). Inequality (9) implies

$$L(r^k) - L(r^{k+1}) \geq N \sum_{n=1}^{N} \left\| r_n^{k+1} - r_n^k \right\|_2^2.$$

Since $L(r^k)$ converges, $L(r^k) - L(r^{k+1}) \to 0$. Hence $\|r_n^{k+1} - r_n^k\|_2 \to 0$, for all n, as $k \to \infty$. □

Theorem 3 implies that in Algorithm ODC, the aggregated charging profile and price profile converge to that of an optimal charging profile. Furthermore, change in the charging profile for each EV between consecutive iterations vanishes as $k \to \infty$. Hence, the objective function (4) reduces to the electricity cost after certain number of iterations.

Theorem 4: Algorithm ODC is a gradient projection algorithm for minimizing $L(r)$. That is,

$$r_n^{k+1} = \underset{r_n \in \mathcal{D}_n}{\operatorname{argmin}} \left\| r_n - \left(r_n^k - \gamma \frac{\partial L}{\partial r_n} \right) \right\|_2^2$$

for all $n \in \{1, \ldots, N\}$ and all $k \geq 0$, where $\gamma = 1/(2N)$.

Proof: Since

$$\gamma \frac{\partial L}{\partial r_n}(r^k) = 2\gamma \left(D + R^k \right) = d^k,$$

Theorem 4 directly follows from (6). □

IV. SIMULATIONS

We compare the convergence properties and optimality of Algorithm ODC with the decentralized scheduling algorithm proposed in [14], in homogeneous and non-homogeneous cases. For notational brevity, we call the algorithm in [14] DAP, standing for "Deviation from Average Penalty". Recall that the optimality of DAP is only guaranteed in the homogeneous case. We choose the average residential load profile in the service area of South California Edison from 20:00 on Feb. 13th 2011 to 19:00 on Feb. 14th 2011 as the normalized base demand profile. According to the typical charging characteristics of EVs in [15], we set $\bar{r}_n(t) = 3.3$ kW if EV n can be charged at time t, and 0 kW otherwise. We assume charging rate can change continuously in $[0, \bar{r}_n(t)]$, hence choose $\underline{r}_n(t) = 0$ for all t. We set $R_n = 10, 25, 40$ for three different types (sedan, compact,
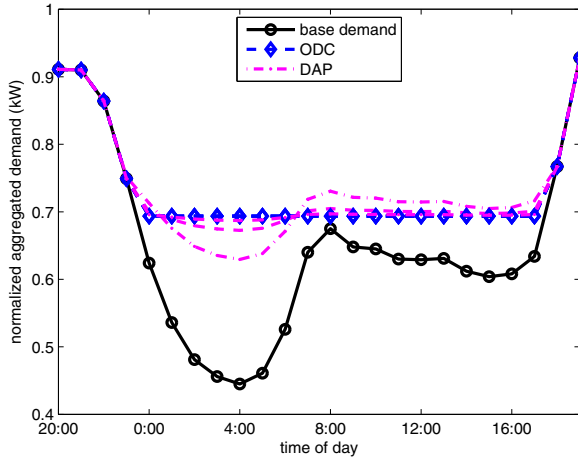
Fig. 3. Normalized aggregated demand profiles in each iteration of Algorithm ODC and DAP in a homogeneous case. All EVs are of type sedan, plug in at 20:00 and have deadline 19:00 the next day.
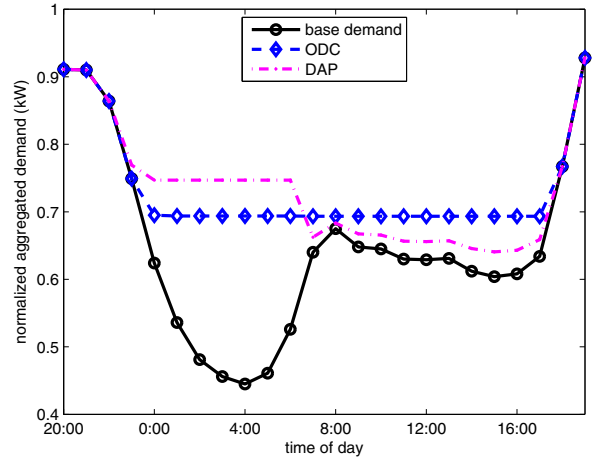

Fig. 5. All EVs are of type sedan and plug in at 20:00. Half of the EVs has a deadline at 6:00 on the next day, and the other half has a deadline at 19:00 on the next day.
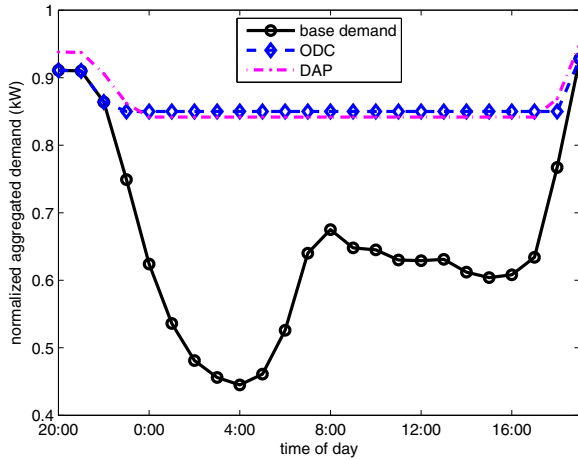

Fig. 4. One third of the EVs belong to each of the three types: sedan, compact, and roadster. All EVs plug in at 20:00 and have deadline 19:00 the next day.

and roadster) of EVs. We consider the penetration level of one EV in every 5 households. Time is divided into 24 slots, each of 1 hour, during which the charging rate of each EV is not changed. As proposed in [14], we choose the price function and parameters for Algorithm DAP as following: $p(d) = 0.15d^2$, $c = 1$, $\delta = 0.15$.

Figure 3 shows the normalized aggregated demand profile in each iteration of Algorithm ODC and DAP. In this case, all EVs are of type sedan, plug in at 20:00, and have deadline 19:00 the next day. Algorithm ODC and DAP both converge to a valley-filling charging profile. Moreover, ODC converges after 1 iteration, while DAP takes several iterations to converge. It can be easily shown that algorithm ODC will always converge after 1 iteration in homogeneous cases.

Figure 4 shows the normalized aggregated demand profiles at convergence in a non-homogeneous case, where one third

of the EVs belong to each of the three types: sedan, compact, and roadster. All EVs plug in at time 20:00 and have deadline 19:00 the next day. ODC still converges to valley-filling charging profiles within few iterations (normalized aggregated demand profile for each iteration is not shown in Figure 4). DAP no longer converges to valley-filling results since aggregated charging rate at peak hour (around 20:00) is non-zero. The optimality proof provided in [14] does not extend straightforwardly to this non-homogeneous case.

Figure 5 shows the normalized aggregated demand profiles in another non-homogeneous case. All EVs are of type sedan, and plug in at 20:00. Half of the EVs has a deadline at 6:00 on the next day, and the other half has a deadline at 19:00 on the next day. Still, ODC converges to valley-filling charging profiles within few iterations. For DAP, aggregated charging profile is flat from 0:00 to 7:00, but lower later. This is because DAP uses a penalty term to prohibit all EV charging profiles' deviation from the average charging profile. Half of the EVs have early deadlines, but have to track the same charging profile as the EVs with late deadlines. Intuitively, those EVs with early deadlines have to take larger charging rates at the beginning hours, leading to lower aggregated demand later. At higher EV penetration levels, the difference becomes more significant. ODC changes "deviation from the average penalty" to "deviation from the last iteration penalty". While preserving convergence property, Algorithm ODC no longer requires different EVs to track a common charging profile; hence, successfully deals with the deadline issue. Meanwhile, by choosing the objective function for EVs, optimality is always guaranteed, in both homogeneous and non-homogeneous cases.

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

In this paper, we studied utilizing decentralized EV charging control to fill the overnight electricity demand valley.

We defined optimality of a charging profile as minimizing the l_2 norm of the aggregated demand, and proposed a decentralized EV charging control algorithm (ODC). In each iteration of ODC, each EV calculates its own charging profile according to the price profile broadcast by the utility, and the utility guides their behavior by updating the price profile. ODC requires low communication and computation capability, and is guaranteed to converge to the set of optimal charging profiles, in both homogeneous and non-homogeneous cases. Simulations support these results, and demonstrate fast convergence of ODC.

B. Future Works

Algorithm ODC deals with day-ahead negotiation of charging profiles. That is, all EVs participate the negotiation at the same time, and implement the schedules they commit to. In a more realistic scenario, EVs may participate the negotiation at different times, not necessarily known to the utility beforehand. We are currently extending the methodology proposed here to study this real-time setting by incorporating predictions on EV arrivals and their energy demand, and accounting for the uncertainties in these predictions.

Distributed energy storage, though currently costly, offers alternative sources for ancillary services to support for the integration of intermittent and uncertain renewable energy generation, as well as for regulating voltage profiles and fault protection. Therefore, the energy storage capacity embedded in EVs may provide a similar flexibility and a long term research goal is to develop probably optimal scheduling protocols to exploit this potential.

VI. ACKNOWLEDGMENT

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