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RESONANCE OSCILLATIONS IN A HOT NONUNIFORM PLASMA COLUMN

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This Letter reports new observations of resonances in a low-density plasma column and a quantitative theory of the frequency spectrum which is in excellent agreement with these measurements. Other measurements have recently been reported by Dattner.[1]

A number of attempts have been made to explain the origin of these resonances. The theory of a cold plasma[2] predicts only a single resonance whose frequency, ω, is given by

$$\omega^2 = \omega_p^2/(1+K), \tag{1}$$

where ω_p is electron plasma frequency and K is the effective dielectric constant of the surrounding glass tube and space. Below we refer to this as the main resonance. When electron thermal velocities are taken into account additional resonances are predicted.[3] These are associated with longitudinal plasma waves which are reflected back and forth across the column. However, for a plasma column of uniform density, these resonances are all clustered about the plasma frequency, ω_p, and their frequency separation is about an order of magnitude less than is experimentally observed. Recently Weissglas[4] has suggested that quantitatively better agreement might be obtained by considering a nonuniform electron-density distribution. The theory outlined in this paper makes use of these last two ideas.[3,4] In addition, to get good quantitative agreement with experiment, it is necessary to know the electron-density profile accurately.

We assume that the plasma electrons can be described by the first two moments of the collisionless Boltzmann equation[5]

$$\partial n/\partial t + \nabla\cdot(n\vec{v}) = 0, \tag{2a}$$

$$nm(\partial\vec{v}/\partial t + \vec{v}\cdot\nabla\vec{v}) = -ne\vec{E} - \nabla\cdot\psi; \tag{2b}$$

where n and $\vec{v}$ are the electron density and velocity, respectively, and ψ is the electron stress tensor. The termination of the chain of moment equations is achieved by replacing the tensor pressure in Eq. (2b) with a scalar pressure, i.e., we write $\nabla\cdot\psi = \nabla p = (\gamma kT/m)\nabla n$, where T is the electron temperature. For the high-frequency oscillations discussed here we take γ equal to 3. Only small-amplitude linear perturbations of the steady state are considered. For example, we write the electron density as $n(\vec{r}) + \tilde{n}(\vec{r})\exp(i\omega t)$, where $n(\vec{r})$ is the steady-state density profile described below, and $\tilde{n}(\vec{r})$ is the perturbation. The low-density laboratory plasmas used in this and prior experiments have dimensions small compared to the free-space wavelength; hence it is convenient to assume that the oscillating electric field may be derived from a potential. Using Poisson's equation, together with the above plasma equations, a fourth-order differential equation is obtained for the potential.

In order to solve the potential equation it is necessary to know the steady-state electron density as a function of radius. This has been calculated by Parker[6] from the equations of Langmuir[7] for the collisionless positive column. In this approach one considers a Maxwellian electron gas confined by the self-consistent field of the electrons and ions. The ions are assumed to be generated throughout the column with negligible initial velocity and are accelerated to the insulating wall by the field. This theory predicts the electron-density profiles shown in Fig. 1. The shape of the electron-density profile depends on a single parameter

$$r_w^2/\langle\lambda_D^2\rangle = r_w^2 e^2\bar{n}/\epsilon_0 kT,$$

where $\bar{n}$ is the electron density, averaged over the cross section of the column, and r_w is the radius of the plasma column. For very large

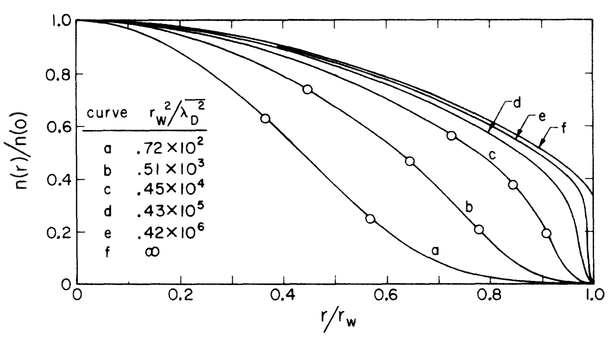

FIG. 1. Electron-density profiles in a cylindrical plasma column.

values of this parameter the sheath at the wall has an almost negligible thickness, whereas for small values of this parameter the sheath occupies most of the column.

Solutions of the potential equation, using the density profiles shown in Fig. 1, were obtained by numerical integration after first putting the equation in dimensionless form and separating out the angular dependence ($e^{i\theta}$ for dipole modes). The two solutions which are regular at $r=0$ are employed. The resonant frequencies are obtained by requiring that the normal component of the velocity $\vec{v}$ vanish at $r=r_w$ and that the logarithmic derivative of the potential at $r=r_w$ be equal to $(-K)$. The values of the four lowest resonant frequencies obtained by this procedure are shown as solid lines in Fig. 2. We have normalized the square of the resonant frequency to the mean-square plasma frequency of the column. The systematic change in the frequency spectrum with the parameter $(r_w^2/\langle\lambda_D^2\rangle)$ is due to the change in the electron-density profile. We have indicated by a circle in Fig. 1 the radius in the plasma at which the local plasma frequency is equal to a resonant frequency. The outermost circle on each curve corresponds to what we have termed the main resonance and the inner points to resonances 1 and 2. Outside this radius plasma waves can propagate; inside they cannot.

Experiments have been conducted using the positive column of a mercury discharge tube of radius 0.51 cm and length 36 cm. The column was contained in a Corning type 3320 glass tube of wall thickness 0.11 cm. The effective dielectric constant of the tube and surrounding space was $K=2.1$. The dipole resonant frequencies were obtained by observing the absorption spectrum of the column when driven by a split

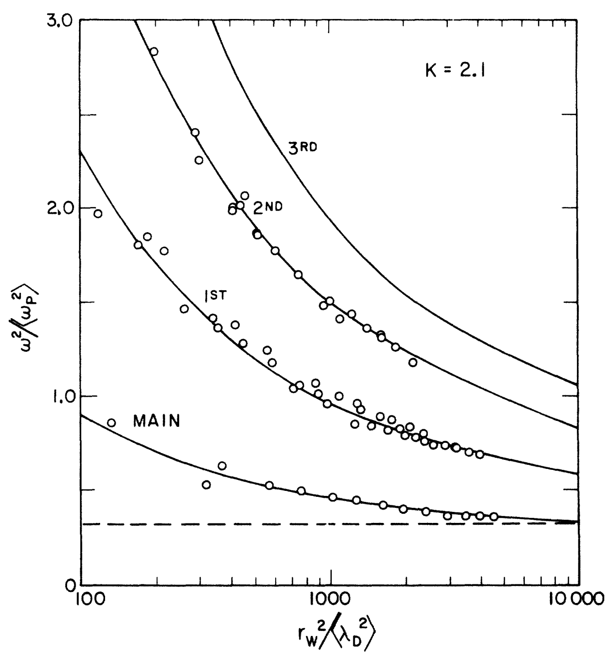

FIG. 2. Dipole resonant frequencies of a cylindrical plasma column. Solid curves are theoretical and circles are experimental.

cylinder capacitor. The first three resonances were observed in a frequency range from 250 to 1200 Mc/sec, corresponding to a current range of 8 to 200 milliamperes. The average electron density $\bar{n}$ was measured simultaneously at each resonance by observing the frequency shift of a cylindrical microwave cavity operating in the TM_{010} mode.[8] The experimental results are shown as circles in Fig. 2. In order to plot them on the same curve we have assumed a constant electron temperature of 3 electron volts. This value is found from Langmuir probe measurements by us and by others in similar discharge tubes. It is interesting to note that with this method of displaying the results the effect of assuming a larger or smaller temperature is simply to shift all experimental points to the left or to the right, respectively.

The dashed line in Fig. 2 indicates the frequency of the main resonance expected from Eq. (1). Since the plasma is not uniform we have used the mean density. It can be seen from Fig. 2 that this is a good approximation only for large values of $r_w^2/\langle\lambda_D^2\rangle$. We would also remark that according to our theory the distinction between the main resonance and the higher plasma wave resonances (1, 2, 3) is somewhat artificial. The lowest resonance has no nodes in the potential whereas the others have 1, 2, 3,$\cdots$ nodes. Experimentally, the lowest mode is least damped, but our theory omits any consideration of dissipative effects.

The excellent agreement between theory and experiment at all densities $(r_w^2/\langle\lambda_D^2\rangle)$ supports the simple model used in calculating electron-density profiles. In addition, measurements of the quadrupole resonant frequencies are in good agreement with calculations for that case.

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TRANSVERSE FIELD INTERACTIONS OF A BEAM AND PLASMA*

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It is well known that strong rf interactions can take place when a cylindrical beam is passed through a plasma. The main interaction of this type, which has been considered in the past, is one in which there is no azimuthal variation of rf field and in which the dominant ac motion of the electrons is in the direction of the motion of the beam. Recently, we have been carrying out an extension of the Gabor, Ash, and Dracott[1] experiment in which they measured rf fields in a plasma sheath by determining the deflection of an electron beam through the sheath parallel to the sheath-plasma interface. It has, therefore, been of interest to know if there is another type of beam-plasma interaction in which an initial rf deflection of the beam increases in amplitude as the beam passes through the plasma. In the presence of such an interaction, not only would the field measurements be unreliable, but there might indeed be the possibility of generation of oscillations because of the introduction of the beam itself.

We came to the conclusion that such an rf hose instability exists. This is equivalent to a mode of interaction with azimuthally varying fields, and had been predicted by Budker[2] several years ago. However, our theory predicts, in addition, that there is a whole class of new interactions with azimuthally varying fields in a magnetic field for which growing waves would occur, not only near the electron plasma frequency, but also near the ion plasma frequency. Experimentally we have demonstrated deflection amplification at frequencies near the electron plasma frequency of the plasma.

Physically, the amplification process may be understood by considering the motion of an ion-neutralized electron beam. With no plasma present, an electron given an initial transverse deflection by an rf field experiences a restoring force toward its unperturbed dc trajectory. This effect is due to the attractive force of the relatively heavy beam-neutralizing ions which suffer virtually no deflection by the rf fields. Thus, the electron would oscillate about its original path, in simple harmonic motion, giving rise to a type of transverse space-charge wave. If, on the other hand, the same beam were passed through a plasma with a frequency of deflection below the plasma frequency, the plasma would behave as a medium with a dielectric constant $\epsilon = \epsilon_0(1 - \omega_p^2/\omega^2)$ which is negative. Consequently, electrons would be repelled from their initial path, and an initial deflection of the electron beam would increase with distance.

In our analysis, we consider a very thin beam of radius a such that $ka \ll 1$, where k is the propagation constant of the wave of interest. It may then be shown that under these conditions, the fields that are present are basically TEM; i.e., the transverse field components are much larger than the longitudinal field components, and that the electric field can be written in terms of a scalar potential, $E = \nabla\phi$, which obeys Laplace's equation

$$\nabla^2\phi = 0 \qquad (1)$$

both in the beam and in the plasma. Conse-