

ON THE ROLE OF SURFACE TENSION IN THE MIGRATION OF MELTS AND FLUIDS

D.J. Stevenson

Division of Geological and Planetary Sciences, California Institute of Technology,  
Pasadena, California 91125

**Abstract.** It is shown that surface tension can cause redistribution of melt in a partially molten medium, in accordance with a non-linear diffusion equation for the melt fraction. The associated diffusivity  $D$  depends on the surface energy and is positive (stable) for dihedral angle  $< 60^\circ$  and negative (unstable) for dihedral angle  $> 60^\circ$ . In the more likely stable case,  $D \sim 10^{-7} \text{ cm}^2 \text{ s}^{-1}$  is typical for mantle melts, but a value as high as  $10^{-1} \text{ cm}^2 \text{ s}^{-1}$  is conceivable for volatile-rich fluids. Surface tension may play an important role in creating pathways for metasomatizing fluids in the Earth, but does not appear likely to affect substantially the existing estimates of large scale melt migration (e.g., beneath mid-ocean ridges).

Introduction

Melt migration through partially molten rock is the essential first step in igneous activity and has been the subject of considerable theoretical development and modeling, especially recently (McKenzie, 1984, 1985; Scott and Stevenson, 1986, and other references therein). However, one aspect of melt and fluid migration has remained poorly understood previously: the role of surface tension. There are conflicting views, ranging all the way from Turcotte (1982) who asserts (p. 401) that "surface tension can only play a role on the scale of the individual grains" to Waff (1980) who has argued that surface tension can sometimes prevent melt from migrating geologically relevant distances. In fact, neither of these extreme views is correct. I show here that surface tension effects can, by themselves, drive migration of melt or fluid over geologically interesting distances. The effect can be particularly large for a low viscosity fluid, and could be very important for metasomatism, as Watson (1982) has stressed. However, the model developed here predicts that surface tension does not affect melt migration in a sufficiently strong way that existing work on, for example, pressure-release melting beneath mid-ocean ridges is invalidated.

There is no previous theoretical work in this area, so the model presented here is kept to the barest essentials. For convenience, the descriptions below refer to partial melts, but there is usually no difference in principle if it were a rock permeated by a volatile-rich fluid, except where explicitly noted otherwise. The basic ideas of the model are simple: in a partially molten, permeable medium the melt tends to redistribute so as to minimize the surface energy per unit volume. The driving force for

the redistribution is the variation with melt fraction of the pressure difference between the solid and liquid, caused by surface tension. This causes a pressure gradient within the liquid even at constant average pressure for the two phase medium. This pressure gradient drives melt migration in accordance with Darcy's law. I show below how these simple ideas lead to a non-linear diffusion equation for the melt fraction.

The Model

Here are the assumptions:

**1. Textural Equilibrium.** The melt is assumed to distribute itself locally (i.e. on the grain size lengthscale) in accordance with surface tension equilibrium. This implies constant mean curvature of all liquid-solid interfaces if (as I assume throughout) only one solid phase is present (Bulau et al., 1979; von Barga and Waff, 1986). This equilibrium takes no longer to achieve than solid state diffusion across a single grain and is therefore expected to be quick, geologically speaking (McKenzie, 1984). It usually implies a unique local melt topology for a given melt fraction and a well-defined pressure difference between liquid and solid (Landau and Lifshitz, 1969).

**2. Chemical Equilibrium.** The melt and solid are in chemical equilibrium and maintain uniform composition. Changes in melt fraction occur only through redistribution of melt.

**3. No Compaction or Shear.** For simplicity, it is assumed that the only stress differences present are those due to surface tension. Actually, deviatoric stresses and compaction can be easily included, as mentioned below, except for one difficulty: the permeability may become a tensor.

**4. No Ripening.** Over a long period of time, grains grow. This process of Ostwald ripening (Marder, 1985; Glicksman and Marsh, 1986) is assumed to be slow compared to surface tension-driven melt redistribution. I assume constant grain size.

**5. Low Melt Fraction.** This simplifies the theory but can be dispensed within a more complete development.

Figure 1 shows a cross-section through a melt tubule at a grain edge intersection (i.e. triple junction). At low melt fraction and dihedral angle  $\theta < 60^\circ$  (defined in the figure), this is where most of the melt resides (Bulau et al., 1979; Waff and Bulau, 1979; von Barga and Waff, 1986). Since the radius of curvature along the tubule (perpendicular to the plane of the figure) is large, the surface tension pressure difference between solid and liquid phases is

$$P_s - P_l \equiv \Delta P_\sigma = \frac{\gamma_{sl}}{r} \quad (1)$$

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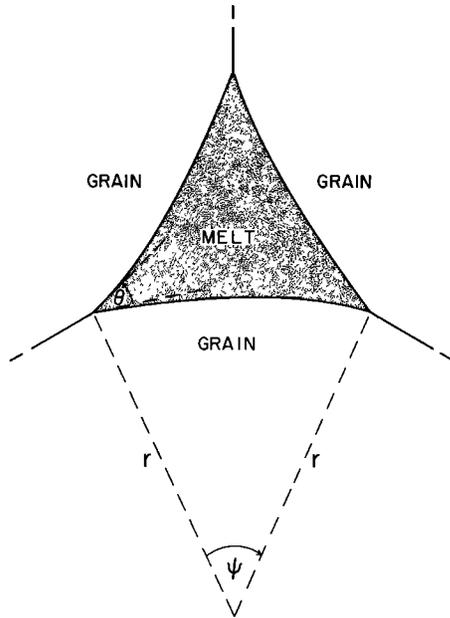


Fig. 1. Cross-section of a tubule at a grain-edge intersection (triple junction). The radius of curvature of the grain-melt interface is  $r$ , and  $\theta$  is the dihedral angle.

where  $\gamma_{sl} > 0$  is the solid-liquid interfacial energy and  $r \geq 0$  for  $\theta \geq 60^\circ$ . After some tedious trigonometry, the cross-sectional area of the tubule,  $A_t$ , is found to be

$$A_t = \frac{3r^2}{2} \left( \frac{2\sqrt{3}}{3} \sin^2 \frac{\psi}{2} + \sin\psi - \psi \right)$$

$$\psi \equiv \frac{\pi}{3} - \theta \quad (2)$$

Since the melt fraction  $f$  is proportional to the ratio of  $A_t$  to the cross-sectional area of a grain, it follows that  $f \propto r^2$  and

$$\Delta P_\sigma = \frac{\alpha}{r^{1/2}} \quad (3)$$

where  $\alpha \geq 0$  for  $\theta \geq 60^\circ$  and is of order  $\gamma_{sl}/R$ , where  $R$  is the grain size. However,  $|\alpha|$  is small if  $\theta$  is very near  $60^\circ$ . These results agree with detailed numerical models, provided  $f < 0.05$  (von Bargen and Waff, 1986). It has only limited applicability at  $\theta > 60^\circ$ , where pockets rather than tubules develop at low  $f$ , but most systems of geological interest have  $\theta < 60^\circ$  (Waff and Bulau, 1979; Vaughan and Kohlstedt, 1982; Jurewicz and Watson, 1984).

Consider a two-phase medium with constant mean pressure  $(1-f)p_s + fp_l$  but variable  $f \ll 1$ . To a good approximation,  $p_s$  is then constant ( $P_0$ ) and

$$P_l = P_0 - \Delta P_\sigma \quad (4)$$

Darcy's law states that the melt flux  $u$  (volume of liquid per unit area per unit time) is given

by

$$u = \frac{-k(f) \partial P_l}{\eta_l \partial x} \quad (5)$$

assuming that the pressure gradient is in the  $x$ -direction. Here,  $k$  is the permeability and  $\eta_l$  is the liquid dynamic viscosity. Each phase is incompressible, so there is also a continuity equation:

$$\frac{\partial u}{\partial x} = - \frac{\partial f}{\partial t} \quad (6)$$

where  $t$  is time. Combining equations (3)-(6):

$$\frac{\partial f}{\partial t} = - \frac{\partial}{\partial x} \left[ D(f) \frac{\partial f}{\partial x} \right] \quad (7)$$

$$D(f) \equiv \frac{\alpha k(f)}{2\eta_l f^{3/2}} \quad (8)$$

-- a nonlinear diffusion equation for surface tension-driven melt migration. Notice that it is the melt fraction that diffuses, not the melt itself. The diffusivity  $D$  is positive for  $\theta < 60^\circ$  (i.e. fluctuations in melt fraction are spontaneously smoothed) and negative for  $\theta > 60^\circ$  (fluctuations are spontaneously amplified). Equation (7) can also be derived and understood by an energy approach (extension of Scott and Stevenson, 1984, 1986) which leads to the identification

$$\Delta P_\sigma = - \frac{\partial \Delta E}{\partial f} \quad (9)$$

where  $\Delta E$  is the surface energy per unit volume of the two phase medium. This result can also be derived explicitly for the geometry of Figure 1, and agrees with numerical modeling (von Bargen and Waff, 1986). Equation (7) can be generalized to larger  $f$  and then  $D = -\eta_l^{-1} (1-f)k(f) d/df [(1-f)\Delta P_\sigma]$ . It is also straightforward to modify equation (5) to include gravity and compaction. For example, the equivalent of nondimensionalized equations (3) and (4) in Scott and Stevenson (1984) now become

$$u_z = -f_t$$

$$u = f^n \left[ 1 - \frac{\Sigma}{2f^{3/2}} f_z + u_{zz} \right] \quad (10)$$

$$\Sigma = \alpha / \Delta \rho g L$$

where  $L$  is the 'compaction length'  $(k \eta_l / \eta_s)^{1/2}$ ,  $k \equiv k_0 f^n$ ,  $\eta_s$  is the bulk viscosity of matrix (assumed constant), subscripts are partial derivatives,  $\Sigma$  is a nondimensional surface energy, and  $\Delta \rho$  is the density difference between solid and liquid.

#### A Simple Solution

Consider the standard 'step function' problem; relevant to experiments on melt infiltration

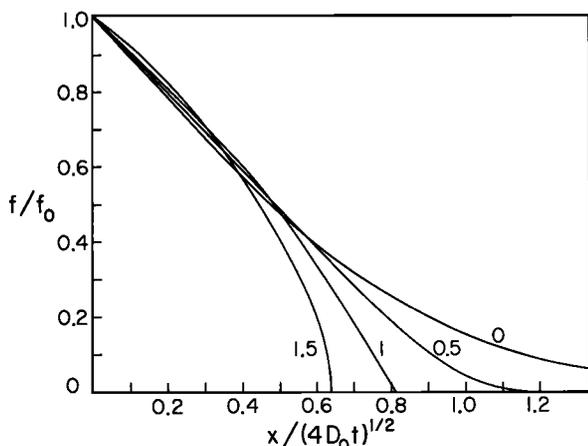


Fig. 2. Solution of the non-linear diffusion problem (equations 7, 11). The curves are labeled by the value of  $\beta$ , with  $\beta = 0$  corresponding to the usual error function solution.

(e.g., Watson, 1982):

$$\begin{aligned}
 f &= f_0, & x &= 0, & t > 0 \\
 f &= 0, & x &> 0, & t = 0
 \end{aligned}
 \tag{11}$$

For example, this represents melt in contact with dry rock and the subsequent infiltration. It is assumed that the diffusivity is positive and expressible as  $D = D_0 (f/f_0)^\beta$ . For  $\beta = 0$ , the usual error function solution applies. For  $\beta \neq 0$ , the diffusion equation (7) can be solved numerically by extension of the techniques discussed in Crank (1956), p. 162 ff. The resulting solutions are shown in Figure 2. The important point is that for all  $\beta$ , the correct long term behavior is characterized by the scaling parameter  $\xi \equiv x/(4D_0t)^{1/2}$ , the usual diffusion scaling. For  $0 < \beta < 1$ , the melt fraction and its gradient both go to zero at finite  $\xi$ . For  $\beta = 1$ , the melt fraction goes to zero at finite  $\xi$  and with finite slope. For  $\beta > 1$ , the melt fraction goes to zero at finite  $\xi$  but with infinite slope, i.e. the solution exhibits a "front." The value of  $\beta$  depends on the behavior of  $k(f)$  at small  $f$ . It is now generally agreed that  $k$  remains finite as  $f \rightarrow 0$  if  $\theta < 60^\circ$  (see discussion in McKenzie, 1984) but the functional dependence is uncertain. Many favor  $k \propto f^3$ , corresponding to  $\beta = 1.5$ , but this form is chosen to model a wide range of  $f$  and does not describe the small  $f$  limit of the numerical models (von Bargaen and Waff, 1986), where long tubules dominate (Fig. 1) and  $k \propto f^2$ . This corresponds to  $\beta = 1/2$  and the absence of a "front" in the diffusive profile.

In reality,  $D$  remains finite in the limit  $f \rightarrow 0$  because of solid state diffusion. Although this diffusion is very small, it is essential to the achievement of the long term behavior in Figure 2. In fact, all the solutions have a small error function-like tail at small  $f$ . A related problem is the formal breakdown of equation (7) when  $f$  varies significantly across a single grain diameter. However, neither of these difficulties appear to modify significantly the long time behavior, provided assumption (1) is valid.

Magnitude of the Diffusion

Assuming  $k(f) = 10^{-3} a f^2 R^2$  and  $\alpha = b \gamma_{sl} / R$ ,

$$D = ab(5 \times 10^{-7} \text{ cm}^2 \cdot \text{s}^{-1})$$

$$\left( \frac{\gamma_{sl}}{10^2 \text{ erg} \cdot \text{cm}^{-2}} \right) \left( \frac{f}{0.01} \right)^{1/2} \left( \frac{R}{0.1 \text{ cm}} \right) \left( \frac{10^3 \text{ P}}{\eta_l} \right)
 \tag{12}$$

where  $ab \sim 1$  (von Bargaen and Waff, 1986). Other parameters are referenced to plausible choices. A diffusivity of  $10^{-7} \text{ cm}^2 \cdot \text{s}^{-1}$  is much lower than heat diffusion, but much faster than most solid state diffusions and possibly comparable to solute diffusion in some magmas (Hofmann, 1980). Watson (1982) performed an experiment on melt infiltration in a simplified basalt-peridotite analog in which the initial state corresponded to the step function problem above. The infiltration of  $\sim 1.72 \text{ mm}$  in a day ( $10^5 \text{ secs}$ ) corresponds to  $D \sim 10^{-7} \text{ cm}^2 \cdot \text{s}^{-1}$ , crudely consistent with equation (12). However, it is likely that Watson was observing some chemical effects as well as simple surface tension. More experiments are desirable.

If equation (12) can be applied to fluids, then  $D$  could be spectacularly large. For example, a water-rich fluid might have  $\eta_l \sim 10^{-2} \text{ P}$  and  $D \sim 10^{-1} \text{ cm}^2 \cdot \text{s}^{-1}$ , an order of magnitude higher diffusion than heat! There are several difficulties with applying the model directly to metasomatising fluids, including the complications of chemistry (for a recent assessment, see Schneider and Egger, 1986) and the uncertain surface energies and fluid distribution (but see Murphy et al., 1984). Nevertheless, it is possible that surface tension effects could play a significant role in creating the pathways for metasomatism. More experimental work is clearly needed.

The Effect of Gravity

Including the gravitational body force, equation (5) becomes

$$u = \frac{k(f)}{\eta_l} \left[ g \Delta \rho - \frac{\alpha}{2f^{3/2}} \frac{\partial f}{\partial z} \right] \tag{13}$$

where gravity acts in the  $-z$  direction. Consider, first, the steady state solution ( $u = 0$ ) corresponding to a column, length  $D$ , of partial melt with impermeable boundaries. If the average melt fraction is  $f_0$  then the melt distribution  $f(z)$  is given by

$$\begin{aligned}
 f(z) &= \frac{f(0)}{\left[ 1 - \frac{g \Delta \rho z \sqrt{f(0)}}{\alpha} \right]^2} \\
 \int_0^D f(z) dz &= D f_0
 \end{aligned}
 \tag{14}$$

where  $z = 0$  is the bottom of the column. We can

define the critical length  $D_c$  as the one for which gravity overwhelms surface tension and  $f(D) = 1$ , corresponding to the onset of macroscopic melt segregation at the top of the column. This is given approximately by

$$D_c = \frac{\alpha}{g\Delta\rho f_0} \quad (15)$$

For  $\alpha \sim 10^3 \text{ erg.cm}^{-3}$ ,  $\Delta\rho \sim 0.2 \text{ g.cm}^{-3}$ , and  $f_0 \sim 0.02$ ,  $D_c \sim \text{few meters}$ . This is closely related to the lengthscale determined by Waff (1980) for the "balance" between surface tension and gravity. However, it does not have the interpretation Waff gave it. Although this lengthscale is small geologically, it implies that in all laboratory experiments, surface tension is more important than gravity.

If we now consider large scale melt migration as in the region beneath mid-ocean ridges, then the static solution is irrelevant and the smallness of  $D_c$  immediately implies that surface tension effects are unimportant, since the melting is distributed over geologically large lengthscales. The correction term to the usual Darcy law prediction for  $u$  will be of order  $D_c/D_t$ , where  $D_t > 10 \text{ km}$  is the lengthscale determined by mantle convection and the phase diagram for melting (cf. Fig. 1 of Turcotte, 1982).

#### Concluding Comments

The role of surface tension in melt migration is a complicated problem but some aspects are now understood. In this preliminary assessment, interesting effects have been identified with possible relevance to mantle and crustal metasomatism. The same ideas may also apply to weathering problems (formation of granitic gneiss?). In contrast, surface tension does not seem to affect greatly the existing quantitative work on large scale melt migration.

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#### References

- Bulau, J.R., H.S. Waff, and J.A. Tyburczy, Mechanical and thermodynamical constraints on fluid distribution in partial melts, *J. Geophys. Res.*, **84**, 6102-6108, 1979.
- Crank, J., *Mathematics of Diffusion*, Oxford Press, London, 347 pp, 1956
- Glicksman, M.E. and S.P. Marsh, Evolution of lengthscales in partially solidified systems. NATO Workshop on the Structure and Dynamics of

- Partially Solidified Systems Abstract Volume, ed. D. Loper, pp. 19-20, 1986.
- Hofmann, A.W., Diffusion in natural silicate melts: A critical review. In *Physics of Magmatic Processes*, ed. R.B. Hargraves, Princeton Univ. Press, Princeton, NJ, pp. 385-417, 1980.
- Jurewicz, S.R. and E.B. Watson, Distribution of partial melt in a felsic system: The importance of surface energy, *Contrib. Mineral. Petrol.*, **85**, 25-29, 1984.
- Landau, L.D. and E.M. Lifshitz, *Statistical Physics*, publ. Addison-Wesley, Reading, Mass., Ch XV, 1969.
- Marder, M., Correlations and droplet growth, *Phys. Rev. Lett.*, **55**, 2953-2956, 1985.
- McKenzie, D., The generation and compaction of partially molten rock, *J. Petrology*, **25**, 713-765, 1984.
- McKenzie, D.P., The extraction of magma from the crust and mantle, *Earth Planet. Sci. Lett.*, **74**, 81-91, 1985.
- Murphy, W.F. III, K.W. Winkler, and R.L. Kleinberg, Frame modulus reduction in sedimentary rocks: The effect of adsorption on grain contacts, *Geophys. Res. Lett.*, **11**, 805-809, 1984.
- Schneider, M.E. and D.H. Eggler, Fluids in equilibrium with peridotite minerals: Implications for mantle metasomatism, *Geochim. Cosmochim. Acta*, **50**, 711-724, 1986.
- Scott, D.R. and D.J. Stevenson, Magma solitons, *Geophys. Res. Lett.*, **11**, 1161-1164, 1984.
- Scott, D.R. and D.J. Stevenson, Magma ascent by porous flow, *J. Geophys. Res.*, **91**, 9283-9296, 1986.
- Turcotte, D.L., Magma migration, *Ann. Rev. Earth Planet. Sci.*, **10**, 397-408, 1982.
- Vaughan, P.J. and D.L. Kohlstedt, Distribution of the glass phase in hot-pressed, olivine-basalt aggregates: An electron microscopy study, *Contrib. Mineral. Petrol.*, **81**, 253-261, 1982.
- von Bargen, N. and H.S. Waff, Permeabilities, interfacial areas and curvatures of partially molten systems: Results of numerical computations of equilibrium microstructures, *J. Geophys. Res.*, **91**, 9261-9276, 1986.
- Waff, H.S., Effects of the gravitational field on liquid distribution in partial melts within the upper mantle, *J. Geophys. Res.*, **85**, 1815-1825, 1980.
- Waff, H.S. and J.R. Bulau, Equilibrium fluid distribution in an untramfied partial melt under hydrostatic stress conditions, *J. Geophys. Res.*, **84**, 6109-6114, 1979.
- Watson, E.B., Melt infiltration and magma evolution, *Geology*, **10**, 236-240, 1982.

D.J. Stevenson, Division of Geological and Planetary Sciences, Caltech, Pasadena, CA 91125.

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