Hackl and Vojta Reply: Our Letter [1] proposed a Zeeman-driven narrow-band Lifshitz transition as an explanation for anomalies in the T-B phase diagram of $\mathrm{YbRh_2Si_2}$. The Comment by Friedemann *et al.* [2] argues that our ideas are problematic because (A) "unnatural fine-tuning" is involved and (B) the results contradict experimental data. Below, we rebut these arguments.

To begin, we reiterate [1] that measurements on $\mathrm{YbRh_2Si_2}$ point to the existence of small energy scales below the Kondo temperature, present even at the transition field $B_c \approx 60$ mT: distinct crossovers are seen below 0.5 K in the thermal expansion, in $\Gamma_{p,B}(T)$ [3,4], the thermopower [5], and the thermal conductivity [6]. The existence of these crossover scales limits straightforward interpretations in terms of quantum criticality.

(A) Fine-tuning: It is true that the existence of a narrow piece of band in our model [1] is an *ad hoc* assumption (justified only by the experimental observation of small crossover scales). Its existence provided, a sizeable asymmetry of the resulting peak in the density of states (DOS) is not unnatural [7]. The small effective Fermi energy $E_c \sim 50$ mK is an additional assumption, which, however, simply corresponds to placing the system near criticality—this type of assumption is required in *any* microscopic modeling of critical phenomena.

It is interesting to relate Ref. [1] to a recent theory proposal for $\mathrm{YbAlB_4}$: Ref. [8] constructed a band structure model with hybridization nodes, yielding a singularly divergent DOS. If this divergence is placed *at* the Fermi level, free-electron thermodynamics can explain many of the intriguing properties of $\mathrm{YbAlB_4}$. However, a justification for this placement is similarly lacking to date, which might point to a more general organizing principle.

The approximate concurrence of the critical fields for the Fermi-surface reconstruction and the destruction of antiferromagnetism (AFM) in $\mathrm{YbRh_2Si_2}$ is indeed nontrivial. However, the two phenomena separate for sizeable doping or pressure [9], such that it is unclear whether their connection is fundamental. In the Lifshitz scenario, AFM is assumed to be a separate (and, for some properties, secondary) phenomenon. It is plausible that the sizeable magnetization built up for $B > B_c$ is responsible for the destruction of weak AFM, furnishing a link between Lifshitz and magnetic transitions.

(B1) Observability at elevated T: Despite the thermal smearing of Fermi pockets, a distinct crossover is visible in the Hall effect at $T_{\mathrm{Hall}} \propto B$ even at temperatures $T \gg E_c$. This was shown by explicit calculation in Fig. 3 of Ref. [1], which falsifies the corresponding claim in Ref. [2].

(B2) Entropy crisis: First, the small energy scale E_c does not replace the Kondo scale but is an additional scale within the heavy-fermion band structure. Therefore, the entropy involves only a fraction of $R\ln 2$, which, moreover, will only be partially released at $T = 50$ mK due to band asymmetries. Second, the experimental $T \to 0$ γ value is not known, as γ keeps increasing at the lowest T measured, such that strict constraints on the height of the DOS peak cannot be inferred. Taken together, a peak weight corresponding to a few percent of $R\ln 2$ would be compatible with thermodynamic data. However, we feel that a detailed quantitative comparison to experiment is not appropriate at this stage because correlation effects (e.g., the tendency towards ferromagnetism) will significantly influence the results.

(B3) Low-temperature Hall effect: References [2,10] interpret the evolution of R_H in terms of a zero-temperature jump. This interpretation invokes an extrapolation to $T = 0$ which may be problematic: At 20 mK, the crossover width is roughly 20 mT, which is not small compared to B_c; i.e., even at the lowest investigated T, the crossover is broad. In fact, Fig. 3(d) of Ref. [1] provided a proof of principle that a Lifshitz scenario can be consistent with the apparent linear-in-T crossover width down to 20 mK (without jump at $T = 0$).

(B4) Enhancement of specific heat: In our scenario, the large specific heat does *not* arise from the Lifshitz transition but from the narrow piece of band causing the DOS peak: γ increases upon cooling until T becomes smaller than the energy scale on which the DOS varies—there is no inconsistency with experimental data for $B \geq B_c$. In the narrow range $B < B_c$, the shape of the DOS peak becomes relevant, and, more importantly, the interplay with antiferromagnetism needs to be considered, which is beyond the scope of Ref. [1]. We note that the weak singularities of a Lifshitz transition, alluded to in Ref. [2], will only be relevant at ultralow T (below 10 mK).

In summary, we feel that the scenario of Ref. [1] continues to be a viable candidate to explain salient features of $\mathrm{YbRh_2Si_2}$. A more quantitative modeling needs to treat correlation effects beyond effective quasiparticle band structures; initial work in this direction is in progress.

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