Supplementary information for the letter: "Formation and Coalescence of Cosmological Supermassive Black Hole Binaries in Supermassive Star Collapse"

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PACS numbers: 04.25.D-, 04.30.Db, 97.60.Bw, 02.70.Bf

NUMERICAL CONVERGENCE

We show numerical convergence of our results by considering the Hamiltonian constraint H. $H = 0$ is one of the four elliptic constraint equations which arise from the 3+1 ADM decomposition of the Einstein equations (e.g. [1]). Any solution to the Einstein equation must satisfy these constraints. At the continuum level, if they are satisfied initially, they are automatically satisfied at all times. During a numerical evolution, however, numerical error can lead to constraint violations. As the resolution is increased, the constraints must converge to zero at the expected order of accuracy inherent to the numerical scheme.

As detailed in [2], the overall accuracy of our simulations is limited by the finite-volume hydrodynamics scheme, which is second-order accurate in regions where the fluid flow is smooth, and reduces to first order near shocks and discontinuities. Thus, we expect between first and second-order convergence. We note that the we evolve the spacetime geometry with a fourth-order finite-difference scheme [2], but the overall error is dominated by the fluid evolution.

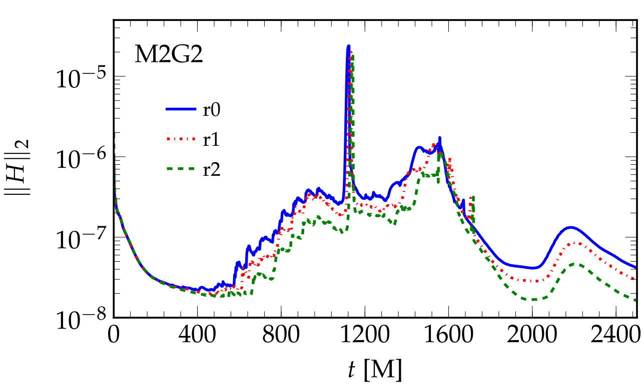

FIG. 1: Convergence of the L_2-norm of the Hamiltonian constraint $\|H\|_2$ of model M2G2 for the low $r0$, medium $r1$, and high resolution $r2$ (not rescaled). As the resolution is increased, the error decreases with a convergence rate between first and second-order as expected. The peaks occur when the black holes form, and when the black hole interior is not yet excluded from the calculation of the L_2-norm.

In Fig. 1, we show the L_2-norm of the Hamiltonian constraint $\|H\|_2$ of model $M2G2$ (which forms a pair of black holes) for three resolutions labeled by $r0$, $r1$, and $r2$. The coarse resolution, $r0$, has 25% reduced resolution compared to baseline resolution $r1$, and $r2$ has 25% increased resolution. The L_2-norm is taken over the entire domain, excluding the interior of the black hole horizons. With increasing resolution, $\|H\|_2$ decreases with a convergence rate consistent with first and second order as expected. More specifically, we find that $\|H\|_2^{r2} \leqslant 1.25\|H\|_2^{r1} \leqslant 1.25\|H\|_2^{r0}$. We note that the numerical evolution is initially non-convergent for the first $t \leqslant 250M$ due to the non-constraint preserving application of an initial density perturbation.

CONVERSION OF UNITS

TABLE I: Conversion factors for 1 solar mass $M_\odot$ in code units $c = G = M = K = 1$ to cgs units.

Quantity	code	cgs
Mass M	M	$1.9891 \times 10^{33}(M/M_\odot)$ g
Length L	M	$1.4771 \times 10^{5}(M/M_\odot)$ cm
Time T	M	$4.9271 \times 10^{-6}(M/M_\odot)$ sec
Density ρ	M^{-2}	$6.1716 \times 10^{17}(M_\odot/M)^2$ g cm^{-3}

For convenience, we provide the conversion factors from code units $c = G = M = K = 1$ to cgs units in Table I. As an example for a $M = 10^6 M_\odot$ star, a code unit time of $T = 1M$ corresponds to a physical time $t \simeq 4.93\,\mathrm{sec}$, the initial stellar equatorial radius $r_e = 80M$ corresponds to $\simeq 1.2 \times 10^8\,\mathrm{km} \simeq 0.8\,\mathrm{AU}$, and the initial central density $\rho_c = 3.38 \times 10^{-6} M^{-2}$ is $\simeq 2.1\,\mathrm{g\,cm^{-3}}$.

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