Dynamical supersymmetry breaking in theories without Lagrangians

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We demonstrate that the supersymmetry can be dynamically broken even in theories without a Lagrangian. The example we study is the four-dimensional SU(N) gauge theory coupled to a strongly coupled superconformal theory, called T_N, which does not have a Lagrangian description at present. Such a theory without Lagrangians can be analyzed due to the supersymmetry and duality. The model is a direct generalization of the model of supersymmetry breaking on deformed moduli space in supersymmetric QCD with an SU(2) gauge group.

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I. INTRODUCTION

Before the discovery of asymptotic freedom in Yang-Mills theory, there was no ultraviolet (UV) Lagrangian which described the dynamics of hadrons such as pions and nucleons. However, the nonexistence of a Lagrangian did not stop people from studying hadron dynamics. The assumption of approximate global symmetry SU(2)$_L$ × SU(2)$_R$ × U(1) spontaneously broken to SU(2) × U(1) allows us to deduce many properties of hadron dynamics via techniques such as current algebra and effective field theory. It is also possible to gauge a subgroup of the global symmetry, U(1)$_{EM}$ ⊂ SU(2)$_L$ × SU(2)$_R$ × U(1), of such a theory which did not have a Lagrangian description in those days. This construction leads to very important dynamical consequences, such as pion decay to photons, generation of mass difference between charged and neutral pions, etc. Much of the calculation can be done without the UV Lagrangian which is QCD.

Similarly, there are theories which have no known Lagrangian description (at least for now) but nevertheless have been found to exist theoretically. One of them is the T_N theory [1] which was discovered in the study of dualities of $\mathcal{N} = 2$ supersymmetric theories. One of the most important properties is that it has the global flavor symmetry SU(N) × SU(N) × SU(N) in addition to some R symmetries. Then we can consider $\mathcal{N} = 1$ supersymmetric

systems in which a subgroup of the global symmetry is coupled to $\mathcal{N} = 1$ vector multiplets. Many dynamical properties of such theories have been understood. Instead of experimental data which was available in the case of hadron physics, we can rely on the power of supersymmetry (SUSY) and duality to study these systems.

In this paper, we demonstrate that these theories without known Lagrangian descriptions are not only theoretically interesting, but also can have applications in phenomenology. We construct a new model of dynamical supersymmetry breaking using the T_N theory. In fact, when $N = 2$, the $T_{N=2}$ theory is just a set of eight free chiral multiplets in the trifundamental representation of SU(2) × SU(2) × SU(2) ⊂ U(8), and our model is the same as the model of supersymmetry breaking on deformed moduli space of $\mathcal{N} = 1$ supersymmetric QCD proposed in [2,3]. Therefore our model can be regarded as a direct extension of their model from SU(2) to SU(N).

Since the discovery of asymptotic freedom, phenomenologists and model builders almost always start by assuming some Lagrangian and then studying its consequences. We hope our work demonstrates that it is possible to start from theories even without Lagrangians and obtain interesting results.

II. THE MODEL OF SUPERSYMMETRY BREAKING

Let us first recall some properties of the T_N theory. The T_N theory is an $\mathcal{N} = 2$ superconformal field theory which has the global symmetry SU(N)$_A$ × SU(N)$_B$ × SU(N)$_C$ × U(1)$_R$ × SU(2)$_R$, where the last two are the R symmetry of

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the $\mathcal{N} = 2$ superconformal algebra. In the language of $\mathcal{N} = 1$ supersymmetry, this theory includes, among others, the chiral operators μ_A, μ_B, and μ_C transforming in the adjoint representations of $SU(N)_A$, $SU(N)_B$, and $SU(N)_C$, respectively. They have the scaling dimension 2, and satisfy the chiral ring relation

$$\text{tr}\mu_A^k = \text{tr}\mu_B^k = \text{tr}\mu_C^k, \qquad (1)$$

where $k = 2, \ldots, N$.

When $N = 2$, the T_N theory is a set of eight free chiral multiplets $Q_{i_A i_B i_C}$ in the trifundamental representation of $SU(2)_A \times SU(2)_B \times SU(2)_C$, and the operators μ_A are given by $(\mu_A)^{j_A}_{i_A} = Q_{i_A i_B i_C} Q^{j_A i_B i_C}$ where the indices of $Q_{i_A i_B i_C}$ are raised and lowered by using the completely antisymmetric tensor $\epsilon^{i_A j_A}$, etc. The operators $\mu_{B,C}$ are given in a similar manner, and a direct computation shows that the relation (1) is satisfied in this case. However, for $N \geq 3$, we cannot interpret $\mu_{A,B,C}$ as being composites of more fundamental operators.

We consider the T_N theory coupled to the $\mathcal{N} = 1$ vector multiplet by gauging the $SU(N)_C$ flavor symmetry, and also coupled to two gauge singlet chiral multiplets M_X ($X = A$, B) transforming in the adjoint representations of $SU(N)_X$ by the following superpotential term:

$$W = -\lambda_A \text{tr}M_A\mu_A + \lambda_B \text{tr}M_B\mu_B. \qquad (2)$$

The global nonanomalous symmetry of this UV theory is $SU(N)_A \times SU(N)_B \times U(1)_R$, where $U(1)_R$ is the same as the $U(1)_R$ symmetry of the $\mathcal{N} = 2$ R symmetry of the T_N theory. The operators $\mu_{A,B}$ are neutral under this $U(1)_R$, and due to the superpotential, $M_{A,B}$ has charge 2 under this $U(1)_R$. The couplings λ_A and λ_B are dimensionless because the scaling dimensions of μ_A and μ_B are equal to 2. When $N = 2$, this superpotential is exactly the one used in [2,3].

As shown in [4], the operators of the T_N theory neutral under $SU(N)_C$ are polynomials of μ_A and μ_B. The constraints (1) are deformed by the dynamics of the $SU(N)_C$ gauge group as

$$\text{tr}\mu_A^k - \text{tr}\mu_B^k = N\Lambda^{2N}\delta^{k,N}. \qquad (3)$$

This quantum deformation in the theory without the gauge singlet fields $M_{A,B}$ was proposed in [4] and checked by 't Hooft anomaly matching and by the consistency with the results of $\mathcal{N} = 1$ SUSY QCD with $N_f = N$ flavors when an appropriate vacuum expectation value (VEV) is given to μ_A. In the case of $N = 2$ where $(\mu_A)^{j_A}_{i_A} = Q_{i_A i_B i_C} Q^{j_A i_B i_C}$ and $(\mu_B)^{j_B}_{i_B} = Q_{i_A i_B i_C} Q^{i_A j_B i_C}$, one can check that (3) is just a rewriting of the more familiar deformed moduli constraint $\det M - B\tilde{B} = \Lambda^4$ of $N_f = N = 2$ SUSY QCD [5].

The form of the constraints is not changed by the coupling to the gauge singlet fields as can be seen from the R-charge assignment. Therefore, the effective low energy superpotential is

$$W = -\lambda_A \text{tr}M_A\mu_A + \lambda_B \text{tr}M_B\mu_B$$
$$+ \sum_{k=2}^{N} \frac{X_k}{k} (\text{tr}\mu_A^k - \text{tr}\mu_B^k - N\Lambda^{2N}\delta^{k,N}), \qquad (4)$$

where we have introduced the Lagrange multipliers X_k. The F-term conditions of M_A and M_B, which require $\mu_A = \mu_B = 0$, cannot be satisfied due to the quantum deformation term. Thus the supersymmetry is spontaneously broken. When $N = 2$, this effective superpotential is exactly the one considered in [2,3], and our supersymmetry breaking mechanism is a direct generalization of the one in [2,3] to $SU(N)$.

III. ABSENCE OF THE RUNAWAY

To show that the supersymmetry breaking vacuum found in the previous section is stable, we need to check that there is no runaway behavior.

A. Pseudomoduli space

First of all, we identify the pseudomoduli direction by neglecting quantum corrections to the Kähler potential of $M_{A,B}$. By the contribution of the F components of M_A and M_B in the effective superpotential (4), the potential is bounded as

$$V \geq V_M = |\lambda_A|^2 \text{tr}(\mu_A^\dagger \mu_A) + |\lambda_B|^2 \text{tr}(\mu_B^\dagger \mu_B)$$
$$+ \left(\sum_{k=2}^{N} \frac{F_k}{k} (\text{tr}\mu_A^k - \text{tr}\mu_B^k - N\Lambda^{2N}\delta^{k,N}) + \text{c.c.} \right), \qquad (5)$$

where F_k is the F component of X_k. Let us study the minimum of V_M, obtained by solving

$$0 = |\lambda_A|^2 \mu_A^\dagger + \sum_{k=2}^{N} F_k \left(\mu_A^{k-1} - \frac{1}{N} \text{tr}\mu_A^{k-1} \right), \qquad (6)$$

$$0 = |\lambda_B|^2 \mu_B^\dagger - \sum_{k=2}^{N} F_k \left(\mu_B^{k-1} - \frac{1}{N} \text{tr}\mu_B^{k-1} \right), \qquad (7)$$

$$0 = \text{tr}\mu_A^k - \text{tr}\mu_B^k - N\Lambda^{2N}\delta^{k,N}, \qquad (8)$$

and their complex conjugates. From the above equations, it follows that $[\mu_A^\dagger, \mu_A] = [\mu_B^\dagger, \mu_B] = 0$. Therefore, $\mu_{A,B}$ can be diagonalized by using the $SU(N)_{A,B}$ transformations, so that we can parametrize the diagonal components as $\mu_A = \text{diag}(\mu_{A,1}, \ldots, \mu_{A,N})$ and $\mu_B = \text{diag}(\mu_{B,1}, \ldots, \mu_{B,N})$. Then, V_M is now

$$V_M = |\lambda_A|^2 \sum_{i=1}^{N} |\mu_{A,i}|^2 + |\lambda_B|^2 \sum_{i=1}^{N} |\mu_{B,i}|^2$$
$$\geq N(|\lambda_A|^2 |\det\mu_A|^{\frac{2}{N}} + |\lambda_B|^2 |\det\mu_B|^{\frac{2}{N}}), \qquad (9)$$

where we have used the general inequality $\frac{1}{N}\sum_{i=1}^{N} a_i \geq (\prod_{i=1}^{N} a_i)^{\frac{1}{N}}$, for $a_i \geq 0$, whose equality holds if and only if $a_1 = \cdots = a_N$.

Without loss of generality, we can assume that $|\lambda_A| \leq |\lambda_B|$. Then one can see that the minimum of the right-hand side of (9) under the constraint $\det \mu_A - \det \mu_B = (-1)^{N-1}\Lambda^{2N}$, which can be derived from $\mathrm{tr}\mu_A^k - \mathrm{tr}\mu_B^k = N\Lambda^{2N}\delta^{k,N}$, is at $\det \mu_A = (-1)^{N-1}\Lambda^{2N}$ and $\det \mu_B = 0$. When $|\lambda_A| = |\lambda_B|$, the vacuum with $A \leftrightarrow B$ is also allowed, but it does not affect the following analysis in any way. Taking into account the relation $\mathrm{tr}\mu_A^k = \mathrm{tr}\mu_B^k$ for $k \leq N - 1$, and that the equality in (9) holds when $|\mu_{A,1}| = \cdots = |\mu_{A,N}|$ and $|\mu_{B,1}| = \cdots = |\mu_{B,N}|$, we conclude that the minimum of V_M is realized at

$$\mu_A = \Lambda^2 \mathrm{diag}(1, \omega, \ldots, \omega^{N-1}), \qquad \mu_B = 0, \quad (10)$$

up to transformations by $SU(N)_A$, where $\omega = e^{2\pi i/N}$. The value of V_M at that point is

$$V_M|_{\min} = N|\lambda_A^2 \Lambda^4|. \quad (11)$$

The full potential becomes equal to V_M if the F-term conditions of $\mu_{A,B}$ are satisfied. That is, the inequality in (5) is saturated if and only if

$$0 = -\lambda_A M_A + \sum_{k=2}^{N} X_k\left(\mu_A^{k-1} - \frac{1}{N}\mathrm{tr}\mu_A^{k-1}\right), \quad (12)$$

$$0 = \lambda_B M_B - \sum_{k=2}^{N} X_k\left(\mu_B^{k-1} - \frac{1}{N}\mathrm{tr}\mu_B^{k-1}\right). \quad (13)$$

From (10) and (13), we obtain $M_B = 0$. Equation (12) says that M_A is diagonal and traceless. X_ks can then be solved, and are linear combinations of $M_{A,k}$.

In summary, the pseudomoduli space up to flavor rotations is parametrized by the diagonal entries of M_A,

$$M_A = \mathrm{diag}(M_{A,1}, \ldots, M_{A,N}), \quad (14)$$

with one constraint $\sum_{i=1}^{N} M_{A,i} = 0$, and other VEVs can be expressed in terms of $M_{A,k}$. The potential is

$$V = N|\lambda_A^2 \Lambda^4| \quad (15)$$

in this direction and in a particular constant.

B. Corrections to the pseudomoduli

So far we took the Kähler potential of $M_{A,B}$ as canonical. Let us take the quantum corrections to the Kähler potential into account. We compute the corrections at the leading order of $\lambda_A \ll 1$. We need to perform this computation in the region where $M_{A,i}$ are much larger than the dynamical scale of the $SU(N)_C$ gauge theory, to make sure that the potential shows no runaway behavior. In this region, we can neglect the effect of $SU(N)_C$ gauge interaction.

From the superpotential interaction (2), we obtain the leading order correction to the effective action of M_A as

$$-S_{\mathrm{eff}} \supset |\lambda_A|^2 \int d^4x d^4y (F_{M_A})_b^a(x)(F_{M_A}^*)_{\bar{b}}^{\bar{a}}(y)$$
$$\times \langle(\mu_A)_a^b(x)(\mu_A^*)_{\bar{a}}^{\bar{b}}(y)\rangle + \cdots, \quad (16)$$

where F_{M_A} is the F component of the chiral multiplet M_A, and the ellipsis denotes supersymmetric completion of the term explicitly written. We are using Euclidean signature here.

To evaluate the expression above, we need to know the two-point function of two μ's. When $|x - y| \ll |M_A|^{-1}$, the effect of the nonzero VEV of M_A can be neglected. Then the correlation function is simply that of the T_N theory. Since μ_A has the scaling dimension 2, we have

$$\langle(\mu_A)_a^b(x)(\mu_A^*)_{\bar{a}}^{\bar{b}}(y)\rangle \sim c|x - y|^{-4}\delta^{a\bar{a}}\delta_{b\bar{b}} \quad (17)$$

with a positive coefficient c, thanks to the unitarity of the theory. In fact, μ_A sits in the conserved current supermultiplet, and the precise value of c is known to be proportional to N [1].

When $|x - y| \gg |M_A|^{-1}$, M_A acts as a mass deformation of the T_N theory leading to a mass gap; this will lead to the behavior of the two-point function:

$$\langle(\mu_A)_a^b(x)(\mu_A^*)_{\bar{a}}^{\bar{b}}(y)\rangle \sim \text{exponentially suppressed.} \quad (18)$$

We will study this point in more detail below, and let us continue assuming its validity.

For the purpose of computing the potential, we may take F_{M_A} to be independent of the space-time coordinates. Then (16) gives

$$-S_{\mathrm{eff}} \supset |\lambda_A|^2 \int_{\epsilon_{\mathrm{UV}} < |x-y| < |M_A|^{-1}} d^4x d^4y \,\mathrm{tr}(F^\dagger F)c|x - y|^{-4}$$
$$= -2\pi^2 c|\lambda_A|^2 \int d^4x \,\mathrm{tr}(F^\dagger F) \log(\epsilon_{\mathrm{UV}}|M_A|), \quad (19)$$

where ϵ_{UV} is the UV cutoff. Therefore, the kinetic term of F_{M_A}, including the tree level term and a counterterm, is given as $[1 - 2\pi^2 c|\lambda_A|^2 \log(\epsilon_{\mathrm{UV}}|M_A|) + \delta]\mathrm{tr}(F^\dagger F)$. Here δ is a counterterm of the kinetic term, which is chosen to cancel the divergent term $\log \epsilon_{\mathrm{UV}}$. Using this, the potential (15) in the pseudomoduli space is modified to

$$V \simeq [1 + 2\pi^2 c|\lambda_A|^2 \log(|M_A|/\mu_{\mathrm{RG}})]N|\lambda_A^2 \Lambda^4|, \quad (20)$$

where μ_{RG} is a renormalization scale determined by ϵ_{UV} and δ. This prefactor containing a large logarithm can be improved by a renormalization group by taking μ_{RG} to be of order $|M_A|$ and evaluating the parameters λ_A and Λ at this renormalization scale. See [6] for details. Reflection positivity of unitary quantum field theory ensures that the constant c is positive. Then this potential is logarithmically increasing for large $|M_A|$, and hence there is no runaway behavior.

Finally, let us discuss where the actual position of the vacuum is likely to be. The most likely possibility is that the vacuum is at $M_A = 0$, where the $U(1)_R$ symmetry is unbroken. Using the low energy effective field theory of $\mu_{A,B}$ and $M_{A,B}$, we can calculate the Coleman-Weinberg potential near the point $M_A = 0$, as was done in the $N = 2$ case in [7]. We have not done this computation in our theory, but the theorem of [8] tells us that the potential minimum of this effective theory is at the point where the $U(1)_R$ is not broken, because $\mu_{A,B}$ has R charge 0 and $M_{A,B}$ has R charge 2. It is likely that this point is the global minimum in the full theory.

C. More on Eq. (18)

Let us study the long distance behavior of the two-point function of μ (18) in more detail. Let us first consider the case of $N = 2$, where $\mu_A \sim QQ$ is a composite of free chiral multiplets, as mentioned above. By the superpotential (2), the Q fields get masses of order $|M_A|$. Then the correlation function $\langle (QQ)(x)(QQ)^*(y) \rangle$ is exponentially suppressed as $\exp(-|M_A||x - y|)$ at long distances.

For $N \geq 3$, we use an $\mathcal{N} = 2$ duality of the T_N theory [1]. The $SU(N)_C$ gauge group plays no role in the discussion and we neglect it. Let us consider the T_N theory coupled to a quiver gauge group $SU(N-1) - SU(N-2) - \cdots - U(1)$, where the $SU(N-1)$ gauge fields are coupled to the subgroup of one of the three $SU(N)$'s, say $SU(N)_C$. Then this theory is dual to the Lagrangian field theory given by the quiver $SU(N)_A - SU(N)_1 - \cdots - SU(N)_{N-2} - SU(N)_B$. The $SU(N)_A$ and $SU(N)_B$ are the flavor groups of the T_N theory, and $SU(N)_i (i = 1, \ldots, N - 2)$ are gauge groups. There are bifundamental hypermultiplets between two adjacent $SU(N)$ gauge groups. In the dual side, the operator μ_A is really given as a composite of quarks $Q, \tilde{Q}$ which are bifundamentals of $SU(N)_A \times SU(N)_1$. So we have $\mu_A \sim Q\tilde{Q}$, and the VEV of M_A gives masses to these quarks. Then (18) is justified as in the $N = 2$ case, as long as the coupling constants are all finite. The weak coupling limit of $SU(N-1) - SU(N-2) - \cdots - U(1)$ corresponds roughly to an infinite coupling limit of the dual theory, but we expect no phase transition in the behavior in this limit because of the $\mathcal{N} = 2$ supersymmetry.

In the above calculation, we argued that the dual quarks $Q, \tilde{Q}$ get masses of order $|M_A|$ since they are coupled as $\mathrm{tr} M_A Q\tilde{Q}$ in the superpotential. However, in the dual quiver theory, there is the adjoint chiral field Φ_1 of the gauge group $SU(N)_1$ which is coupled to the quarks as $\mathrm{tr}\Phi_1 \tilde{Q}Q$. If the VEV of Φ_1 happens to be such that some of the quarks are almost massless, the correlation function at $|x - y| \gg |M_A|^{-1}$ behaves as $c'|x - y|^{-4}$. We consider it to be unlikely; note that in the deformed moduli space (3), the Coulomb moduli fields of the T_N theory are fixed somewhere, so the VEV of Φ_1 cannot be freely chosen. Even if it happens, the constant c' is smaller than c because at least some of the quarks are massive and the number of massless quarks are reduced in the infrared (IR). Then one can see that there is a UV contribution to the kinetic term of F_{M_A} proportional to $(c - c') \log(|M_A|)$ with $c - c' > 0$. Assuming that the IR contribution is not so bad as to spoil this UV contribution, the logarithmic increasing of the potential is not changed. For example, if the correlation function is cut off in the IR at the dynamical scale Λ, then (20) is valid just by replacing $c \to c - c'$.

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