The divergences in a are certainly no larger than the different errors of measurement. So far as I see, therefore, it does not seem possible to reach the relatively low value ($a = .81$) usually quoted for the diffusion of hydrogen into air by the present method.

* Advance note from a Report to the Carnegie Inst. of Washington, D. C.

** These PROCEEDINGS, 10, 1924, p. 153.

WHITE LIGHT INTERFERENCE FRINGES WITH A THICK GLASS PLATE IN ONE PATH*

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The object of this paper is to describe and account for certain interference phenomena which are observed with white light, when a thick plate of glass, or other dispersive medium, is placed in the path of one of the beams in the Michelson interferometer. The mirrors are adjusted in sodium light for circular fringes, and the paths made equal. The glass plate is then inserted in one of the paths, and that path is shortened, or the other path lengthened, by moving the mirror, till the sharpest sodium maximum is reached. The path containing the glass plate will be shorter than the other by about one-half the thickness of the plate. If white light is now substituted, the fringes should be in the field. It is better before throwing on the white light, to re-adjust the mirrors for perfect parallelism of the two beams, which may be known by the almost startling distinctness of the sodium fringes in this position. It will be easier if the first trials are made with a plate not more than 5 mm. thick.

With a thick glass plate in one path, the fringes are not all in the field at one time. It is necessary to vary the adjustable path by a millimeter or more, depending on the thickness and dispersion of the glass, in order to run through the range. Toward one end of the range the fringes are alternate circular bands of dark green and bright red. Toward the other end, the bands are bright green and dark red. In the middle of the range the bands are almost colorless, the bright circles being yellowish, and the dark ones a bluish gray. There is no determinate central fringe; the changes in coloration are so gradual that no difference can be seen in passing over even as many as a hundred fringes if the glass is fairly thick. The following table gives the results of a few trials in counting the fringes with different thicknesses of glass. The figures are not strictly comparable among themselves, as the plates were not all of the same kind of glass.

Thickness of glass plate in mm.	0	3.18	7.44	15.04	30.08
Number of fringes counted	20	255	555	1490	2500

The bands are so uniform as to suggest monochromatic light, though alternate bands are almost complementary in color. With 7.44 mm. of glass in one path, the distances passed over in counting 100 fringes at each extreme were measured, and the equivalent wave-lengths calculated. These were 6360Å and 5180Å for the bright red and the bright green ends, respectively. These are merely rough measurements, made to ascertain the general nature of the fringes. In counting fringes in these experiments, the adjustable path was slowly lengthened (or shortened) and the number of fringes vanishing (or forming) at the center counted.

The fact that interference effects are still visible, in spite of the great dispersive action of a thick glass plate, is most unexpected, and makes an explanation a matter of particular interest. The theory is given for plane wave-fronts, and is divided into two parts: I. Wave-front parallel to the mirrors. II. Wave-front oblique to the mirrors. The present paper deals with the first part. A paper covering the second part is in preparation. It should be noticed that the first part does not deal with the whole field of view in the interferometer, but only with the central spot of the fringe system.

Physical Explanation of the Fringes.—To analyze the phenomena, let us suppose at first a source of monochromatic light. The phase-difference of

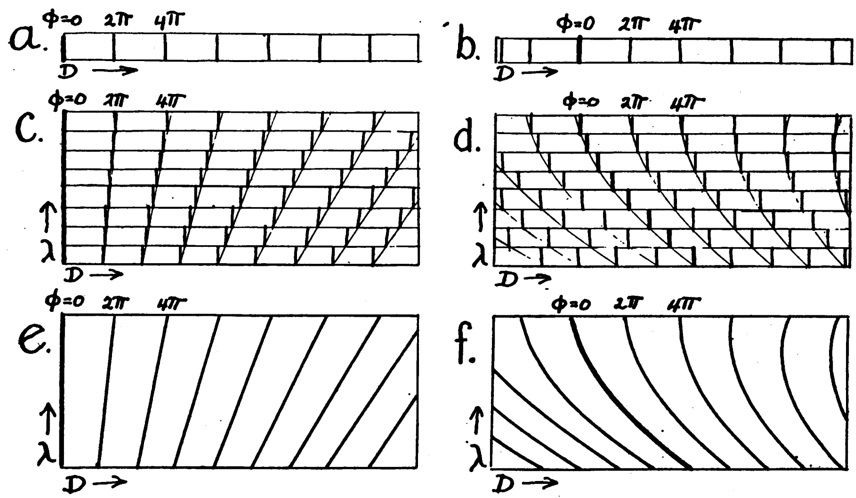

FIGURE 1

the two re-uniting beams is proportional to their path-difference, so that, if we have a field over which the difference of path increases steadily from left to right, the points where the phase-difference is an even multiple of π

(greatest light intensity) will be a series of equidistant vertical straight lines, the distance between them being proportional to the wave-length of the light employed (fig. 1A). If, now, we place a retarding medium in part of the shorter of the two paths, it will be necessary, if we wish to restore the original phase-difference, to lengthen the longer path. That is to say, any particular phase-line in the diagram will be moved to the right. The effect will be to move the system of lines as a whole to the right by an amount depending on the retarding effect of the medium (fig. 1B).

Now, if we suppose a source of light of a definite number of discrete wave-lengths, we shall have a diagram like the above for each wave-length. We can graph these one under the other as in figure 1C. Introducing the retardng medium will move each system to the right by a different amount, giving an effect shown in figure 1D, where the heavy lines are the positions of zero phase-difference in each case. Passing to the case of a light-source containing all wave-lengths, the systems of short lines merge into continu-

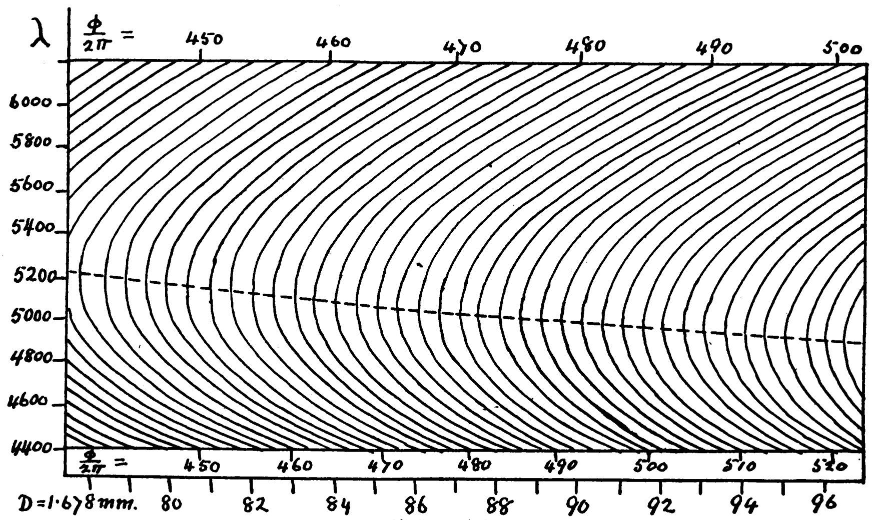

FIGURE 2

ous curves, and the phase-difference diagrams appear as in figures 1E and 1F, the former having no retarding medium. The effect of putting in a greater thickness of retarding medium is to warp the phase-lines more and more, so that we get the effect shown in figure 2, in which the point of zero path-difference is far to the left of the part drawn. The last mentioned diagram is drawn to scale, to represent the actual conditions when a glass plate 3 mm. thick is interposed in the path of one of the beams.

To explain the white-light interference effects observed when the spectrum fringes are in this condition, divide the diagram into horizontal strips of finite but narrow width $d\lambda$. This is equivalent to considering the source of white light as being composed of a number of sources, each emitting light of all wave-lengths over a narrow range $d\lambda$. If we observe the light from one of these sources, excluding all the others, there will be visible interference if, for a given path-difference, the phase-difference does not vary much over the range. If the phase-difference at one edge of the strip differs by as much as π from that at the other edge, varying continuously and gradually from one edge to the other, the variations in intensity over the strip will compensate each other, and the field will appear uniform to the eye, since there are no decided color-differences in the small range of λ over the strip. It will be seen that this last condition holds, for a constant path-difference and a small range of λ, over the whole field in figure 2, except for those strips in the neighborhood of the bend of the phase-curves. In this region each strip will show clear interference, since the phase-differences on its upper and lower edges are nearly the same. Moreover, the neighboring strips will reinforce the interference, their phase-differences being in step with it. Over the rest of the field the action of neighboring strips will produce no interference effect, giving uniform illumination. We shall have, then, interference in one narrow range of the spectrum, superimposed on uniform illumination from the rest of the spectrum. The color effects observed can be explained if we consider the field to be composed of a set of interference bands of light of a certain color (small color-range), seen against a background of uniform illumination of all other colors. The background will be complementary to the interference bands. The intensity of the interfering color ranges from twice the intensity of the other colors to zero, since the rest of the spectrum is covered by very narrow spectrum fringes, cutting down its intensity one-half. When the interfering light is in the red, for instance, we have alternately a surplus of red and no red, similarly for other positions. The part of the phase-curves drawn in figure 2 represents less than one-third of the range of D over which fringes are visible. If the entire range were drawn it would be noticed that the position of the bend in the phase-curves passes in succession through all values of λ, from about 6000 Å. at the left, to about 5000 Å. at the right. A study of the equations given in the next section brings out this fact, which is also in accord with experiment. The dotted line shows the change in wave-length of the fringe-producing light as D varies.

It will be noticed that, according to the above analysis, the interference is caused by light of such a wave-length that its phase-difference is greater than that of any other wave-length, for the given position of the mirrors. This is in accordance with the principle enunciated by Cornu,** and enlarged upon and exemplified by Lord Rayleigh,*** that the condition for white-light interference is that the phase-difference shall be a maximum or a minimum with respect to the wave-length. In symbols, $d\phi/d\lambda = 0$, where ϕ is the phase-difference of the two beams.

Lord Rayleigh gives several methods of increasing the number of white-light fringes, by means of optical systems which tend to equalize more or less the widths of the monochromatic fringes of all wave-lengths. In the case considered in this paper no such explanation of the increased number of fringes is possible, since the "width of a fringe" is simply the distance through which the mirror must be moved to change the phase-difference by 2π, and is equal to half a wave-length in each case. Hence all the monochromatic components have fringe-widths proportional to their wave-lengths.

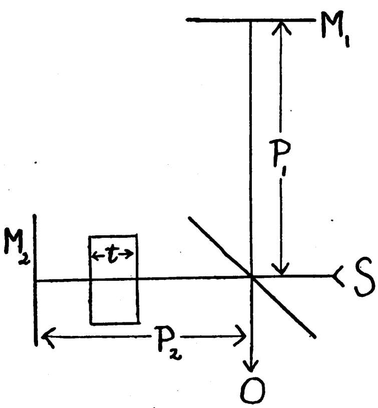

FIGURE 3

Theory of the Fringes.—In this section we are going to investigate by what distance the mirror can be displaced without destroying the interference fringes, and we shall find an expression for the number of fringes visible with plates of different thickness and dispersive power.

Let N_1 be the number of waves of length λ (in air) in P_1, (fig. 3), and N_2 the number in P_2, P_1 and P_2 being the distances from a point on the half-silvered surface to the two mirrors. Suppose a plate of glass, or other dispersive substance, of thickness t to be placed in P_2. Then, if μ is the index of refraction from air into the plate for wave-length λ, we have the relations,

$$N_1 = P_1/\lambda; \; N_2 = (P_2 - t)/\lambda + \mu t/\lambda \qquad (1)$$

Let D be the difference of the distances to the two mirrors, $P_1 - P_2$, and let ϕ be the phase-difference of the beams on re-uniting. Then $\phi = 4\pi\ (N_1 - N_2)$, since the beams pass twice over their respective paths. Directly from these relations we obtain the fundamental equation

$$\phi/4\pi\lambda = D - (\mu - 1)t \qquad (2)$$

To apply Cornu's principle, differentiate (2) with respect to λ, and put $d\phi/d\lambda = 0$ in the result. We get

$$\phi/4\pi + t\, d\mu/d\lambda = 0 \qquad (3).$$

as the condition for producing fringes in the neighborhood of λ. Eliminating ϕ between this equation and (2), we get the condition in terms of D:

$$D = (\mu - 1 - \lambda\, d\mu/d\lambda)t \qquad (4)$$

If λ_1 and λ_2 are the shortest and the longest wave-lengths of the range under consideration, by placing the corresponding values in the above equation and subtracting, we have

$$D_2 - D_1 = R = [\mu_{\lambda_2} - \lambda_2(d\mu/d\lambda)_{\lambda_2} - \mu_{\lambda_1} + \lambda_1(d\mu/d\lambda)_{\lambda_1}]t \qquad (5)$$

If μ is a continuous function of λ this can be written

$$R = t\int_{\lambda_2}^{\lambda_1} \lambda \frac{d^2\mu}{d\lambda^2}\, d\lambda \tag{6}$$

The number of fringes in this range will be $(\phi_1 - \phi_2)/2\pi$, ϕ_1 and ϕ_2 being the values of ϕ in (3) corresponding to λ_1 and λ_2 respectively. We shall have from (3),

$$(\phi_1 - \phi_2)/2\pi = n = 2t[(d\mu/d\lambda)_{\lambda_2} - (d\mu/d\lambda)_{\lambda_1}] \tag{7}$$

whence

$$n = 2t\int_{\lambda_2}^{\lambda_1} \frac{d^2\mu}{d\lambda^2}\, d\lambda \tag{8}$$

By means of Cauchy's dispersion formula, $\mu = A + B/\lambda^2$, (2) gives the following relation between D, ϕ, and λ:

$$D = \phi/4\pi\,\lambda + (A - 1)t + Bt/\lambda^2$$

plotting D against λ, using ϕ as parameter, we get a series of curves of equal phase-difference. A vertical cross-section of this set of curves at any point will give the composition of the light for that particular value of D, or in other words, for the corresponding position of the movable mirror. Figure 2 was drawn from this equation, using the values: $A = 1.5$; $B = 57 \times 10^{-10}$; $t = 3$ mm.

Putting Cauchy's value for μ in (6) and (8), and integrating, gives

$$R = 3Bt(1/\lambda_2^2 - 1/\lambda_1^2) \tag{10}$$

and

$$n = 4Bt(1/\lambda_2^3 - 1/\lambda_1^3) \tag{11}$$

These results show that the range, and the number of fringes visible, are proportional to the thickness of the glass plate, and also to the dispersive power of the glass, as measured by B. By experiment it is found that the interference is lost in the glare of the background for wave-lengths much less than 5100A, or greater than 6300A. Using these limits, and taking $B = 57 \times 10^{-10}$, we get, from (10) and (11), for a plate 3 mm. thick, $R = .063$ mm.; $n = 227$. This agrees well with actual count, as can be seen from the table at the beginning of this paper.

The writer wishes to acknowledge the kindness and patience of Dr. Paul S. Epstein in reading the manuscript critically and making many valuable suggestions.

* A brief abstract of this paper was presented to the American Physical Society, September 1923. In the meantime Sethi (*Phys. Rev.*, Jan. 1924, pp. 69–74) observed the same phenomenon independently, though his theory is different from mine.

** *J. Phys.*, Ser. 2, **1**, p. 293 (1882).

*** *Phil. Mag.*, Ser. 5, **28**, pp. 77 and 189 (1889).