below $\kappa_N = 0$ is included in the range

$$R_i^2 |k_i^2 - k_i^2(\kappa_N = 0)| \lesssim 1$$

throughout which effective-range theory might be expected to be good.[3]

From associating the $\Lambda\pi$ resonance with the $\bar{K}N$ s state, it follows that if we can measure the orbital angular momentum of the $\Lambda\pi$ (or the resonant $\Sigma\pi$ state) the $K\Lambda N$ (or $K\Sigma N$) parity will be measured. One possibility of determining l_Λ is from the Λ polarization. Referring to (1), we measure θ from the perpendicular to the plane of production of the resonant $\Lambda\pi$ and the spectator particle, and consider $s_{1/2}$ and $p_{1/2}$ $\Lambda\pi$ systems. If P is the degree of polarization of the resonant systems, the Λ polarization is

$$s_{1/2}\text{: } P,$$
$$p_{1/2}\text{: } P(\cos^2\theta - \sin^2\theta).$$

Of course P depends on the production mechanism, angle, and energy: One may not readily find a situation with large P.

A different possibility is to look at the energy dependence of $|\alpha|^2$ over the resonance. The $\Sigma\pi$ threshold is very favorably located close to the resonance, so that with any detailed data, s- or p-wave resonances could be distinguished. The present data already provide some indication of the $K\Sigma N$ parity. Experimentally,[7] $|\alpha|^2$ is found to be roughly 1 at the $\bar{K}N$ threshold, whereas averaged over the resonant region,[1] $|\alpha|^2 \ll 1$. Theoretically, we have from (4),[8] in first approximation,

$$|\alpha|^2 \propto k_\Sigma^{2l_\Sigma + 1} / k_\Lambda^{2l_\Lambda + 1}. \tag{8}$$

Thus, from (8), we find that $|\alpha|^2$ decreases from its value at the $\bar{K}N$ threshold to that at the peak of the resonance by a factor of 2 for $l_\Sigma = l_\Lambda = 1$ (even KYN parities). On the other hand, if we consider $l_\Sigma = 0$, $|\alpha|^2$ remains essentially constant in this energy region, which may be in disagreement with experiment.

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†On leave from Indiana University, Bloomington, Indiana.

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[2]M. Ferro-Luzzi, J. P. Berge, J. Kirz, J. J. Murray, A. H. Rosenfeld, and M. Watson, Bull. Am. Phys. Soc. 5, 509 (1960).

[3]M. Ross and G. Shaw, Ann. Phys. (to be published).

[4]M. Ross and G. Shaw, Ann. Phys. 9, 391 (1960).

[5]T. Kotani and M. Ross, Nuovo cimento 14, 1282 (1959), see Eq. (59).

[6]R. Dalitz and S. Tuan, Ann. Phys. 10, 307 (1960), see footnote 14.

[7]L. Alvarez, Ninth International Conference on High-Energy Physics, Kiev, 1959 (to be published).

[8]Alternately, one can argue that the $\bar{K}N$ quasi-bound state would be characterized by a fixed probability ratio for $\Sigma\pi/\Lambda\pi$ in the region of interaction; (8) then follows from the small values of kr involved.

UNIFIED APPROACH TO HIGH- AND LOW-ENERGY STRONG INTERACTIONS ON THE BASIS OF THE MANDELSTAM REPRESENTATION*

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We wish here to outline an approach to the theory of strong interactions, based on the Mandelstam representation, that treats high- and low-energy phenomena in a unified manner. Our approach, which will be described in detail elsewhere, extends somewhat the original program proposed by Mandelstam.[1] The most striking achievement thus far is the inference, from the observed constancy of high-energy total cross sections, that in the low-energy elastic region the P-wave interaction should be strong; at the same time, bound states or large low-energy phase shifts for $J \geqslant 2$ seem inconsistent with the theory. Furthermore, the divergence difficulty encountered by Chew and Mandelstam[2] in handling low-energy P resonances appears to be removed by the Pomeranchuk relation[3] between high-energy particle and antiparticle cross sections.

We use the pion-pion interaction to illustrate

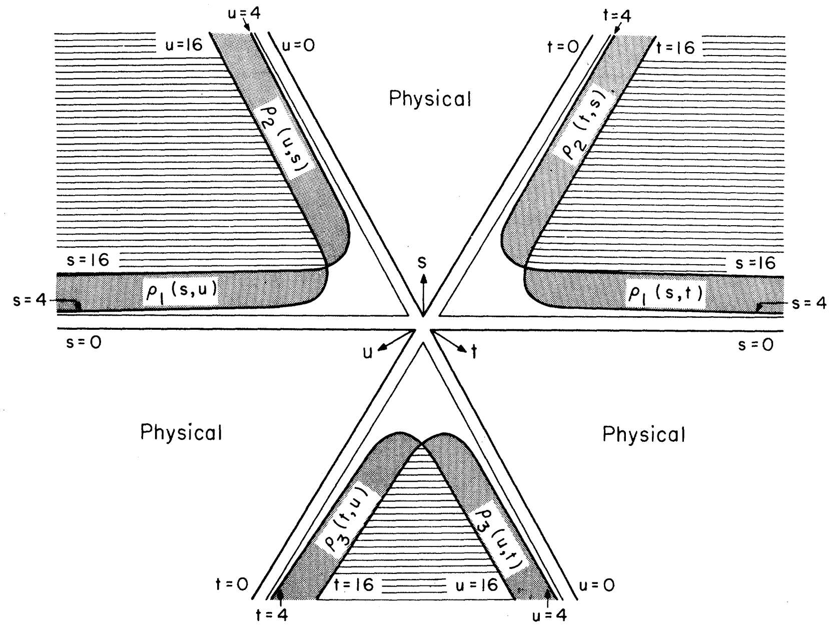

FIG. 1. The Mandelstam diagram for elastic $\pi\pi$ scattering. In a physical region the variable indicated is the square of the barycentric-system total energy—in pion mass units. The assignment of locations to ρ_1, ρ_2, and ρ_3 corresponds to the amplitude $A(s,t,u)$ of reference 4. Corresponding assignments for $B(s,t,u)$ and $C(s,t,u)$ are obtained by interchange of channel labels.[4]

our approach, although it will be clear that the essential features may be generalized. Figure 1 shows the Mandelstam diagram for $\pi\pi$ elastic scattering, where the variables s, t, and u have the usual meaning.[4] The wedge-shaped physical regions are labeled as such, and the shaded areas indicate where the double spectral functions fail to vanish. Our central assumption is that the double spectral functions in the heavily shaded areas dominate those parts of the physical regions whose distance from a boundary is of the order of magnitude of the width of the heavily shaded strips, i.e., 16 pion mass units (squared). This gives us a theory that covers not only low energies but arbitrarily high energies with low momentum transfer.

The above assumption seems a priori reasonable on the usual geometrical grounds, but in addition it is fortified by two empirical circumstances. First, total cross sections generally are largest in the low-energy elastic region. Second, diffraction peaks in elastic scattering are always observed in the forward direction at high energies. The latter means, for example, that near the right-hand boundary of the upper physical region in Fig. 1 (the s region) the quantity, $\mathrm{Im}A(s,t)$, for s large and fixed, falls off sharply as t decreases from zero (forward direction). The width of the diffraction peak in NN and πN scattering corresponds to $\Delta t \sim 20$, and it will be surprising if the same is not true for $\pi\pi$ scattering as well as for other strongly interacting particle combinations. Now, we have

$$\mathrm{Im}A(s,t) \sim \frac{1}{\pi}\int_4^\infty dt' \,\frac{\rho(t',s)}{t'-t},$$

so a t dependence of the kind required for diffraction implies a concentration of the double spectral function within a distance from the boundary of the order Δt. Such a concentration in the double spectral region labeled ρ_2 in Fig. 1, when applied to the t channel (the lower right-hand physical region), is in agreement with the first of the above-mentioned empirical circumstances, i.e., the largeness of total cross sections at low energy. We shall return to this point at the end of this Letter.

Fortunately, the double spectral functions in the heavily shaded areas are determined by relatively tractable elastic unitarity conditions, as first pointed out by Mandelstam[1] and recently re-emphasized by Cutkosky.[5] Because of space limitations we do not write these conditions here, although they will play a central role in actual dynamical calculations. Suffice it to say that in the $\pi\pi$ problem there are three distinct "strip" functions to be calculated, which we have indicated in Fig. 1 as ρ_1, ρ_2, and ρ_3. Each of these is determined exactly in the strip regions by an integral over bilinear combinations of absorptive parts of elastic amplitudes. It is only in the interior regions that inelastic amplitudes explicitly play a role.

In his original paper[1] Mandelstam stated the opinion that attempting to calculate the parts of the double spectral function on which we are now focusing attention is not worthwhile because inelastic effects are implied, in contradiction with the basic approximation, which is elastic. We believe that there is no inconsistency; the elastic unitarity condition is to be employed only where it is correct, in order to calculate double spectral functions in the strip regions. Inelastic scattering is not calculated completely, but that part occurring at low momentum transfers (implied, for example, in the s physical region in Fig. 1 by the existence of ρ_2) should be well approximated.

It is worthwhile digressing momentarily to relate these ideas to those recently expressed by Salzman and Salzman[6] and by Drell,[7] as well as by Pomeranchuk.[8] If we focus attention on the s physical region of Fig. 1, the double spectral function ρ_1 corresponds to diagrams in which only two particles are present in intermediate states but any number may be exchanged. In other words, ρ_1 is calculated from the Cutkosky diagram[5] shown in Fig. 2(a) and represents purely elastic effects in the s channel. On the other hand, ρ_2 is calculated from diagram 2(b), in which any number of particles are allowed in intermediate states but only two are exchanged (it is elastic in the t channel). Obviously, then, we are calculating here the diffraction scattering associated with inelastic transitions in which a single pion is exchanged. This is just the mechanism of Salzman and Salzman, Drell, and Pomeranchuk. We believe that our approach is more systematic, since it raises no questions it cannot answer about cross sections in unphysical regions; however, we can only discuss total cross sections and elastic scattering, not the distribution of inelastic events.

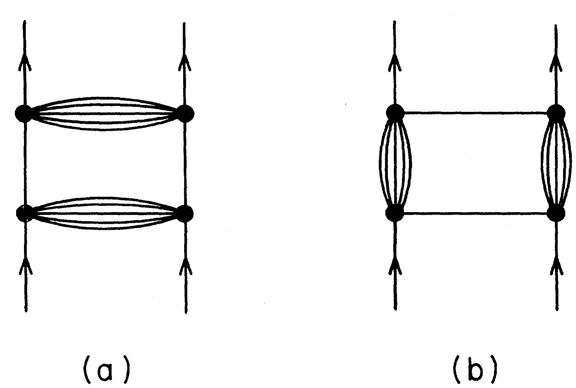

FIG. 2. The two Cutkosky diagrams needed to calculate the double spectral functions in the strip regions.

Returning to the main argument, we now consider a second postulate: that total cross sections for strongly interacting particles asymptotically approach constants at very high energies. Even within the scheme of approximation proposed here we believe that our combined requirements of unitarity and analyticity are inconsistent with total cross sections that increase indefinitely with energy. In any event, such a situation seems nonsensical and would, we believe, be impossible if unitarity were completely enforced. On the other hand, there are almost certainly solutions, such as the S-dominant solutions discussed by Chew, Mandelstam, and Noyes,[9] for which the asymptotic behavior of the double spectral functions implies a total cross section vanishing strongly at infinity. This alternative also we reject as unreasonable, feeling that a characteristic of strong interactions is a capacity to "saturate" the unitarity condition at high energies for those states that overcome the centrifugal barrier. Such a saturation should lead to a constant total cross section.

We add a final related postulate, that the first diffraction peak approaches a constant limiting shape at very high energies, and thus we arrive at the general assumption[10]:

$$\lim_{\substack{s\to\infty \\ -20\lesssim t<0}} \operatorname{Im} A(s,t) \to s f(t). \tag{1}$$

From Fig. 1, this is equivalent to

$$\lim_{\substack{s\to\infty \\ -20\lesssim t<0}} \frac{1}{\pi}\int \frac{\rho_2(t',s)}{t'-t}\,dt' \to s f(t), \tag{2}$$

which seems unnatural unless

$$\lim_{\substack{s\to\infty \\ 4<t\lesssim 20}} \rho_2(t,s) \to s\rho_2^{\,L}(t). \tag{3}$$

Although limit (3) cannot be said to follow strictly from limit (1), we adopt (3) as the basis for further argument. Similar reasoning can be employed for $\rho_1(t,s)$ and $\rho_3(t,s)$ to define $\rho_1^{\,L}(t)$ and $\rho_3^{\,L}(t)$. Then we borrow Pomeranchuk's argument[3] about consistency of the fixed t dispersion relations to conclude that

$$\rho_2^{\,L}(t) = \rho_3^{\,L}(t). \tag{4}$$

This equality corresponds to the statement that the total $(\pi^+\pi^-)$ and $(\pi^+\pi^+)$ cross sections become asymptotically the same.

It is important to remark that the behaviors (3) and (4) appear to be consistent in the strip regions with the integral unitarity conditions which are to be used to calculate the double spectral functions. Thus we believe that we shall be able to find solutions of our equations satisfying these conditions. How many free parameters are present in such solutions we shall not know until further study has been made of the consistency of our equations in the interior regions where they are not exact and where a cutoff probably will be needed.

We close by pointing out that condition (3), when the substitution law is invoked, implies that amplitudes at low energy are asymptotically proportional to t as $t\to\infty$. Thus, regardless of how many parameters are allowed, S-dominant solutions are ruled out and a strong interaction in the P wave (and no higher waves) is implied.[11] [At the same time, condition (4) appears to eliminate the inconsistency encountered by Chew and Mandelstam[2] when they attempted to ignore the double spectral functions in a consideration of P-dominant solutions.] It seems, therefore, that one has in these considerations the beginning of an understanding of why strong interactions are so uniform in their manifestations: One may conjecture that the defining characteristic of strong interactions is that they are "as strong as possible." The consequence from such a statement of a constant behavior for cross sections at high energies has always been plausible. Now we see that at low energies it appears inconsistent with general principles to have interactions so strong as to produce resonances or bound states for $J \geqslant 2$. The "strongest possible" interaction should, however, produce large low-energy P phase shifts. There remains an open question, of course, as to what extent one can determine the precise interaction strength on the basis of the "strip approximation" proposed here.

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[7]S. Drell, Phys. Rev. Letters 5, 342 (1960).

[8]I. Pomeranchuk, paper presented at the Tenth International Rochester Conference on High-Energy Nuclear Physics, 1960 (to be published).

[9]G. F. Chew, S. Mandelstam, and H. P. Noyes, Phys. Rev. 119, 478 (1960).

[10]The statements in this paragraph are uncertain with respect to logarithmic factors. Only the powers of s are to be taken seriously.

[11]In reference 1 Mandelstam discusses the relation between asymptotic behavior in t and the strength of interaction. Two papers on potential scattering by T. Regge, Nuovo cimento 14, 951 (1959), and University of Rochester Physics Department preprint, NYO-9266, 1960 (unpublished), are also very enlightening. Regge shows that the amplitude behaves like t^α, where α increases as the strength of an attractive interaction grows. For partial waves such that $l \leqslant \alpha$, there may be bound states or resonances, corresponding to the necessity for subtractions in the Mandelstam representation in these low-l states. For $l > \alpha$, subtractions are not needed and bound states or resonances are correspondingly absent. Although Regge's work is confined to potential scattering, we feel confident of the generality of this connection between asymptotic behavior in t and the maximum l that interacts strongly. When spin is present, of course, the connection will be slightly more complicated.