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Citation: Physics of Fluids (1994-present) **25**, 105114 (2013); doi: 10.1063/1.4826315
View online: http://dx.doi.org/10.1063/1.4826315
View Table of Contents: http://scitation.aip.org/content/aip/journal/pof2/25/10?ver=pdfcov
Published by the AIP Publishing

PHYSICS OF FLUIDS **25**, 105114 (2013)

A proposed modification to Lundgren's physical space velocity forcing method for isotropic turbulence

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(Received 23 July 2013; accepted 20 September 2013; published online 28 October 2013)

As an alternative to spectral space velocity field forcing techniques commonly used in simulation studies of isotropic turbulence, Lundgren [Linearly forced isotropic turbulence," in *Annual Research Briefs* (Center for Turbulence Research, Stanford, 2003), pp. 461–473] proposed and Rosales and Meneveau ["Linear forcing in numerical simulations of isotropic turbulence: Physical space implementations and convergence properties," Phys. Fluids **17**, 095106 (2005)] validated a physical space forcing method termed "linear forcing." Linear forcing has the advantages of being less memory intensive, less computationally expensive, and more easily extended to variable density simulations. However, this forcing method generates turbulent statistics that are highly oscillatory, requiring extended simulation run times to attain time-invariant properties. A slight modification of the forcing term is proposed, and it is shown to reduce this oscillatory nature without altering the turbulent physics. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4826315]

I. INTRODUCTION

In simulation studies of isotropic, triply periodic box turbulence, the velocity fields are forced artificially via various methods to prevent the decay of the turbulent fluctuations. This has the effect of perpetuating in time the turbulent flow field such that meaningful data can be collected for analysis. Velocity forcing methods simply entail the addition of a source term to the governing momentum equations. However, it has been well documented that some forcing methods produce turbulent quantities (i.e., turbulent kinetic energy, dissipation rate) that can be subject to significant statistical variation.[1] The literature provides several examples of forcing methods that have been designed to reduce such temporal fluctuations. These efforts are varied and include artificially freezing the energy content in the largest flow scales,[2] fixing the ratio of energy content between subsequent waveshells,[3,4] and imposing a model energy spectrum to which forcing is done in proportion.[1]

Until recently, all forcing methods and their corresponding approaches to reduce fluctuations relied on a source term implemented in spectral space. These methods[1–8] prevent the decay of turbulent fluctuations by injecting energy into a contrived region of wavespace, generally within a narrow band of waveshells. Two limitations of such methods are their dependence on periodic boundary conditions and the difficulties associated with extending them to variable density simulations. In response to this, Lundgren[9] proposed a physical space forcing method, where the source term assumes the form of a pseudo-shear term, Au_i. This method has been shown to capture experimentally observed structure function curving,[9] although its convergence towards Kolmogorov's 4/5-law occurs more slowly than with spectral forcing techniques.[10] Its shear-like term injects energy into the velocity field in direct proportion to the magnitude of the velocity fluctuations themselves. The effect of this approach is to inject energy over all scales of the flow, not over only a narrow band of waveshells. As the largest flow scales are subject to the largest fluctuations, these scales are correspondingly the

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most impacted by the source term; the smallest scales are virtually unaffected. Although this method has been successful in producing and perpetuating isotropic turbulent conditions, it is not a perfect technique for turbulent energy production, as is, also, the case with any spectral forcing method.

II. PROPOSED MODIFICATION TO LUNDGREN'S LINEAR FORCING METHOD

When implemented as proposed by Lundgren,[9] the linearly forced momentum equations take the form

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}\right) + A u_i. \tag{1}$$

The forcing parameter, A, controls the magnitude of the energy added to the velocity field. This parameter is determined by the user, and it is sufficient (with the viscosity, ν, and a defined length-scale, l) to completely prescribe all pertinent physical parameters,[11] including the Reynolds number, Re_λ, the turbulent kinetic energy, k, the dissipation rate, ϵ, and the eddy turn-over time, τ. To understand how this method is able to control the resulting turbulent field, consider the turbulent kinetic energy equation derived from Eq. (1),

$$\frac{dk}{dt} = -\epsilon + 2Ak, \tag{2}$$

where, during the spatial (volume) averaging step, denoted as $\langle \cdot \rangle$, incompressibility ($\partial u_i/\partial x_i = 0$) and homogeneity ($\langle \nabla \cdot (\,) \rangle = 0$) have been assumed, and the definitions $k = \langle \frac{1}{2} u_i u_i \rangle$ and $\epsilon = \langle \nu \frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j} \rangle = 2\nu \langle s_{ij} s_{ij} \rangle$ were used for turbulent kinetic energy and dissipation rate, respectively. Applying the condition of statistical stationarity, Eq. (2) reduces to simply a balance between the dissipation rate and a scalar multiple of the turbulent kinetic energy,

$$0 = -\epsilon + 2Ak. \tag{3}$$

From Eq. (3), the physical significance of the forcing parameter, A, becomes clear; A is simply the inverse of twice the eddy turn-over time, τ, or $A = (2\tau)^{-1}$, with $\tau = k/\epsilon$. Thus, the forcing parameter provided by the user imposes the time-scale over which energy is injected into the turbulent velocity field.

Rosales and Meneveau[11] found that this linear forcing technique generates a turbulent velocity field that asymptotically approaches a unique solution. This asymptotic state is characterized by an integral length-scale, which is approximately 20% of the computational domain. The integral length-scale, l, can be expressed in terms of physical parameters as $l = (u'^2)^{3/2}/\epsilon$, where u'^2 is the variance of the velocity field (i.e., $k = \frac{3}{2}u'^2$). If such an asymptotic state exists, as defined by Eq. (3), then, together with the definitions of the integral length-scale and the turbulent kinetic energy provided, the asymptotic values for key turbulent metrics can be evaluated. For example, the turbulent Reynolds number, Re, and its Taylor-microscale counterpart, Re_λ, can be expressed as

$$Re = \frac{l\, u'}{\nu} = \frac{3A\, l^2}{\nu}, \qquad Re_\lambda = \frac{\lambda_g u'}{\nu} = \left(\frac{45\, A\, l^2}{\nu}\right)^{1/2}, \tag{4}$$

and the characteristic velocity, u', mean turbulent kinetic energy, k_0, and mean dissipation rate, ϵ_0, as

$$u' = 3Al, \qquad k_0 = \frac{27}{2}A^2 l^2, \qquad \epsilon_0 = 27\, l^2 A^3. \tag{5}$$

Note that to obtain the Taylor-microscale, λ_g, the relation for the dissipation rate under isotropic conditions[12] was used, $\epsilon = 15\, \nu u'^2/\lambda_g^2$. Note further that there are two degrees of freedom available to the user, namely, the forcing parameter, A, and the viscosity, ν.

However, it was noted by Rosales and Meneveau,[11] as well as by Lundgren[9] in the original work, that the turbulent statistics generated under this method were sometimes subject to large oscillations around the above average values. Additionally, these oscillations were found to increase with increasing Re_λ. To reduce the amplitude of these oscillations, this work proposes a slight modification to Lundgren's[9] original momentum source term. This modification changes the original source term from Au_i to $A\left(\frac{k_0}{k}\right)u_i$, resulting in forced momentum equations of the form

$$\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}\right) + A\left(\frac{k_0}{k}\right)u_i, \tag{6}$$

where k is the instantaneously calculated turbulent kinetic energy and k_0 is the desired steady-state turbulent kinetic energy (Eq. (5)). Changing the source term in this manner is conceptually consistent with implementing a relaxation term or a damping coefficient as implemented by Overholt and Pope.[1] The velocity field is driven towards the desired turbulent kinetic energy value in a more constrained fashion, thereby reducing the amplitude of its oscillations. Note that in the (long-time) limit of $k = k_0$, this term is equivalent to the original source term. Also, the turbulent parameters under this modification are controlled in the same fashion. After specifying Re_λ, the value for A required for a given ν can be calculated straightforwardly from Eq. (4) and the long-time kinetic energy and dissipation rate can be determined from Eq. (5). The modification proposed does mitigate the "localness" of Lundgren's[9] original method, as a globally averaged quantity, k, is added to the source term. However, the stability resulting from this modification, which is discussed later, justifies this mitigation.

It is found that this modified source term does not significantly or detrimentally impact the generated turbulent fields; its sole effect is to reduce the oscillatory behavior of the turbulent statistics. This can be verified both analytically and graphically via a comparison between the turbulent fields produced under the action of the original and modified source terms. The analytical justification for this claim is first addressed.

The turbulent kinetic energy equation corresponding to Eq. (6) is

$$\frac{dk}{dt} = -\epsilon + A\frac{k_0}{k}\langle u_i^2\rangle = -\epsilon + 2Ak_0, \tag{7}$$

where incompressibility and homogeneity are assumed. At stationarity, it is obtained

$$0 = -\epsilon + 2Ak_0. \tag{8}$$

Note that the only difference between this equation and that of the original source term (Eqs. (2) and (3)) is that now, instead of the instantaneous turbulent kinetic energy being of importance, only the long-time asymptotic (stationary) turbulent kinetic energy is important. This has the effect of reducing the variation in the resulting dissipation rate (ϵ). Further, the physical meaning of the forcing parameter A is preserved under this proposed modification. It is still related to the eddy turn-over time via $A = (2\tau_0)^{-1}$, where $\tau_0 = k_0/\epsilon$. This eddy turn-over time is equivalent to the τ from the original source term once stationarity sets in, as $k = k_0$ and $\epsilon = \epsilon_0$. It is of note, also, that using this modified source term is more consistent with spectrally-based forcing schemes. Spectral schemes generally inject a fixed, constant amount of energy into the computational domain during each timestep. As the modified source term results in a term in the turbulent kinetic energy equation which depends only on k_0 and A, both of which have constant, temporally unchanging values, it is conceptually similar to the more widely used spectral forcing schemes.

III. SIMULATION STUDY

In addition to analytical support for the claim that the modified source term has only the intended effects of reducing unwanted oscillations in the calculated turbulent statistics, simulation-based (practical/empirical) verification is now provided. A comparison between turbulent physics produced by the modified and original source terms is performed for two Re_λ cases: $Re_\lambda = 110$ and $Re_\lambda = 140$ on a $N^3 = 384^3$ grid and a $N^3 = 512^3$ grid, respectively. For the $Re_\lambda = 110$ cases, the forcing parameters are $A = 0.96$ and $\nu = 0.005$. For the $Re_\lambda = 140$ cases, the forcing parameters are $A = 1.40$ and $\nu = 0.005$. In all cases, the grid resolution is kept at $\kappa_{max}\eta \geq 1.5$.

The initial velocity fields were Gaussianly distributed following the initialization procedure in Eswaran and Pope.[7] In the plots to be referenced, the legend entries "Original" and "Modified" denote the results obtained when implementing the original and modified source terms, respectively.

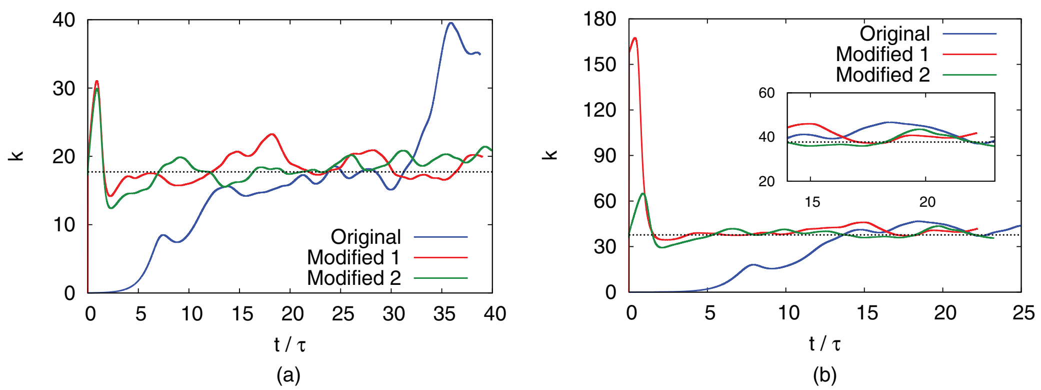

FIG. 1. Time evolution of turbulent kinetic energy. The (black) dashed line denotes the expected stationary value, k_0, calculated from Eq. (5). (a) $Re_\lambda = 110$. (b) $Re_\lambda = 140$.

The "Original" and "Modified 1" data were subject to initial conditions of $k(t = 0) = 0.014$ and $\epsilon(t = 0) = 7.3 \times 10^{-4}$ for both Re_λ; "Modified 2" data had initial conditions of $k(t = 0) = k_0 = 17$ and $\epsilon(t = 0) = 0.87$ for $Re_\lambda = 110$ and $k(t = 0) = k_0 = 36$ and $\epsilon(t = 0) = 1.83$ for $Re_\lambda = 140$. As will be shown in Figs. 1–6, the results appear to be independent of the initial conditions implemented. The code package used to perform these simulations is NGA.[13] The code is physical (non-spectral), suitable for low Mach number flows, and uses a standard staggered grid. The velocity field is solved implicitly via a second-order accurate finite-difference scheme, and this scheme is discretely energy conserving. The time advancement is accomplished by a semi-implicit Crank-Nicolson method.

The first two statistics of interest are the time evolution of the turbulent kinetic energy and the dissipation rate, which are depicted in Figs. 1 and 2. As is apparent from the statistics for the original source term, there is considerable variation in turbulent kinetic energy and dissipation rate even after stationary conditions have set in (approximately $t/\tau \geq 15$ for $Re_\lambda = 110$ and $Re_\lambda = 140$). As shown in Figs. 1(a) and 2(a), large jumps in calculated turbulent statistics are possible when the original source term is used (e.g., $t/\tau \geq 30$), and these cannot be modulated. The modified source term, as evidenced by both the $Re_\lambda = 110$ and $Re_\lambda = 140$ cases, produces markedly smoother statistics, free from significant deviations from the asymptotic stationary values. It is important to note, also, that statistical stationarity is obtained much more rapidly with the modified source term ($t/\tau \geq 4$ for both Re_λ) than with the original source term ($t/\tau \geq 15$ for both Re_λ). Regardless, however, both the original and the modified source terms produce equivalent eddy turn-over times, as depicted in Fig. 3. This is significant, as it supports the earlier claim that only the variations are being damped by the modified source term; the underlying physics are largely unchanged.

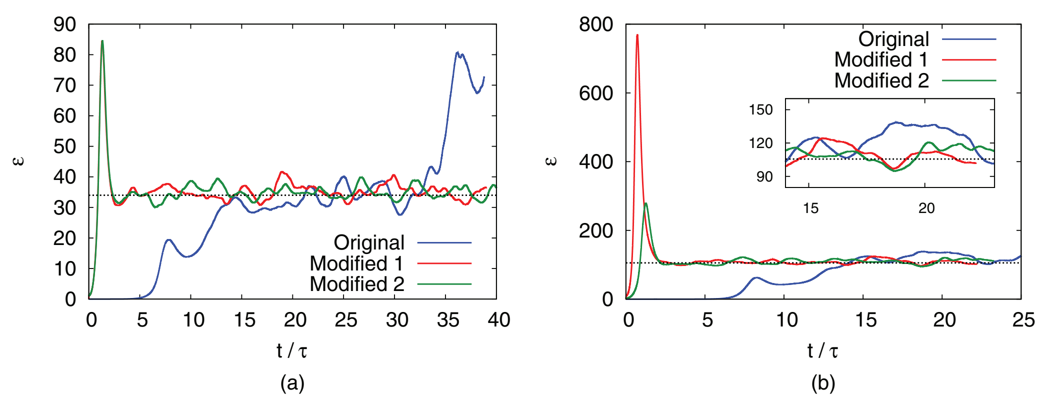

FIG. 2. Time evolution of dissipation rate. The (black) dashed line denotes the expected stationary value, ϵ_0, calculated from Eq. (5). (a) $Re_\lambda = 110$. (b) $Re_\lambda = 140$.

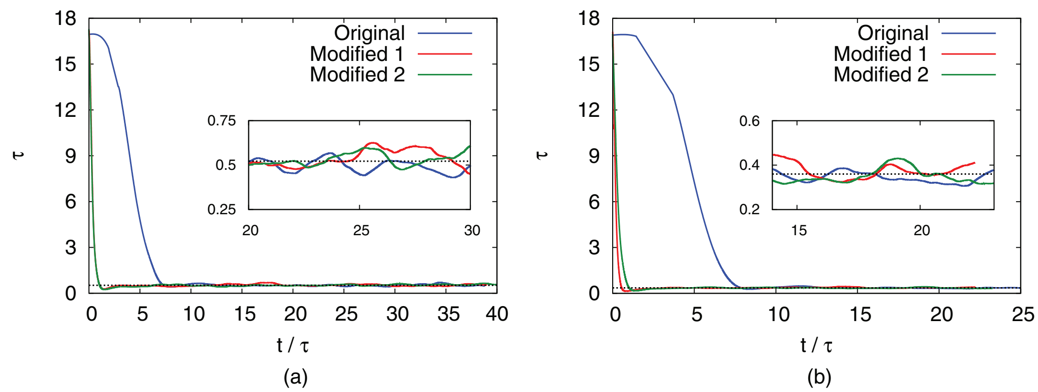

FIG. 3. Time evolution of eddy turn-over time. The (black) dashed line denotes the expected stationary value, τ_0, calculated from $\tau_0 = (2A)^{-1} = k_0/\epsilon_0$. (a) $Re_\lambda = 110$. (b) $Re_\lambda = 140$.

Since all relevant turbulent fields (e.g., energy spectrum, $E(\kappa)$, dissipation spectrum, $D(\kappa)$, transfer spectrum, $T(\kappa)$) are related directly to the dissipation rate and turbulent kinetic energy, the variation in these metrics correspondingly decreases. The practical ramifications of this is quite significant, as fewer datasets are now required to obtain statistically stationary (time-independent) statistics. This translates into shorter simulations and a reduced computational burden.

As the key turbulent statistics indicate that the modified source term is having the intended effect of reducing large amplitude oscillations without significantly altering any asymptotic behavior, the spectra generated are presented now to verify that the spectral distribution of energy has not been affected. The energy, dissipation, and transfer spectra for the six cases are provided in Figs. 4–6. In these three sets of spectra, the distribution in wavespace is unchanged; the magnitudes of the curves, however, do vary slightly (as expected) between the turbulent fields obtained with the original and modified source terms. This slight variation is most pronounced in the dissipation spectra (Fig. 5), and these differences in magnitude can be attributed to the oscillatory behavior of the turbulent fields obtained with the original source term. The critical feature of Figs. 4–6 is that the respective spectrum shapes are preserved when implementing the modified source term.

IV. LINEAR PERTURBATION (STABILITY) ANALYSIS

The objective of applying a velocity field forcing method is to prevent the decay of the turbulent fluctuations. While it is difficult (if not impossible) to prove convergence towards a unique statistically stationary state irrespective of initial conditions, all numerical tests performed tend to suggest that

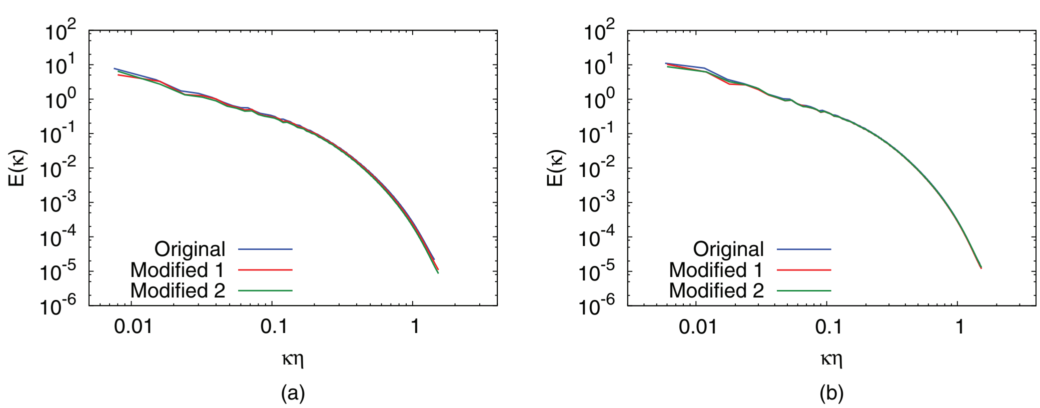

FIG. 4. Energy spectra at statistical stationarity (averaged over a minimum of 10 τ). Here, η is the Kolmogorov length-scale, defined as $\eta = (\nu^3/\epsilon)^{1/4}$. (a) $Re_\lambda = 110$. (b) $Re_\lambda = 140$.

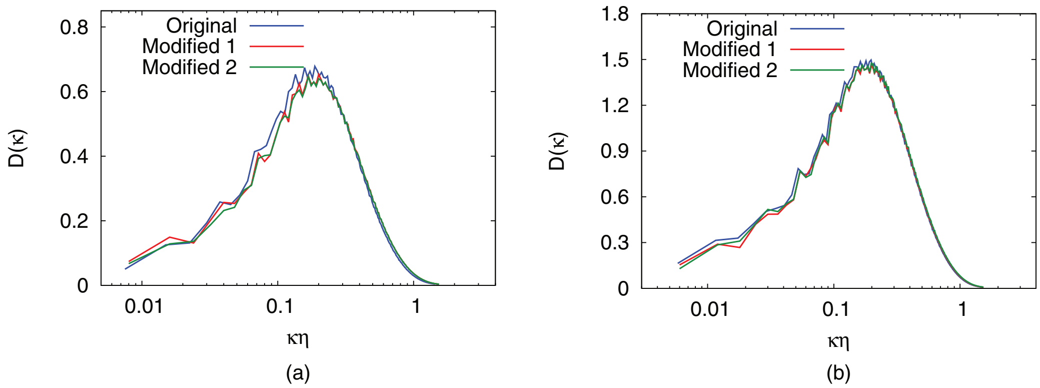

FIG. 5. Dissipation spectra at statistical stationarity (averaged over a minimum of 10 τ). The dissipation spectrum is defined as $D(\kappa) = 2\nu\kappa^2 E(\kappa)$. (a) $Re_\lambda = 110$. (b) $Re_\lambda = 140$.

this is the case. However, it has been shown that the original form of the source term induces significant oscillation in the long-time behavior of its produced turbulent statistics, while the modified source term does not. To better understand the reasons behind these oscillations, a straightforward, perturbation-based analysis of the two relevant governing equations (turbulent kinetic energy and dissipation rate) around the asymptotic values of k_0 and ϵ_0 is conducted. The pertinent turbulent kinetic energy equations are Eq. (2) for the original source term and Eq. (7) for the modified source term. These expressions involve the dissipation rate directly, necessitating an evolution equation for this parameter also. Although an analytical transport equation for the dissipation rate is attainable by manipulation of the momentum equations (Eqs. (1) and (6)), the resulting expressions are not closed. As an approximation, a $k - \epsilon$ model evolution equation[14] is assumed, which can be written in a general form as

$$\frac{\partial \epsilon}{\partial t} + U_j \frac{\partial \epsilon}{\partial x_j} = C_{\epsilon 1} \frac{\epsilon}{k} \tau_{ij} \frac{\partial U_i}{\partial x_j} - C_{\epsilon 2} \frac{\epsilon^2}{k} + \frac{\partial}{\partial x_j} \left((\nu + \nu_T/\sigma_\epsilon) \frac{\partial \epsilon}{\partial x_j} \right) + f, \tag{9}$$

where U_j and ν_T denote mean velocity and turbulent eddy-viscosity, σ_ϵ, $C_{\epsilon 1}$, and $C_{\epsilon 2}$ are positive constants resulting from closure approximations, and f is a source term resulting from the velocity field forcing method implemented. Under the present configuration (isotropic, triply periodic box

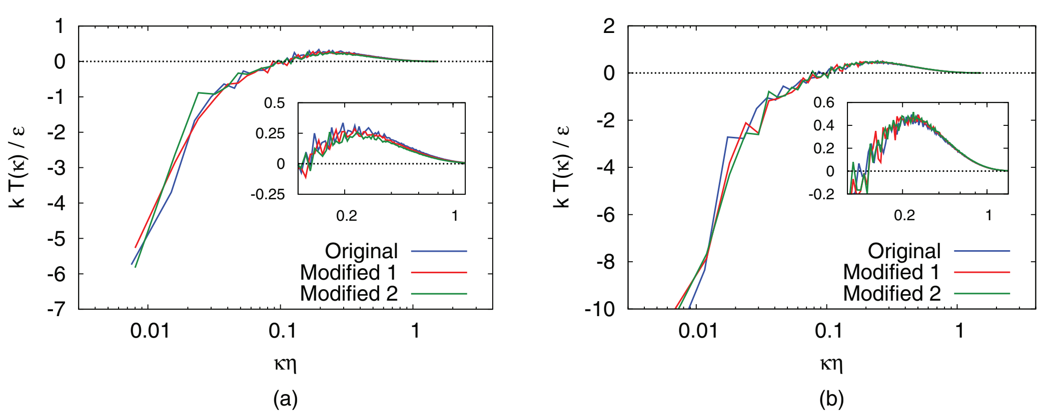

FIG. 6. Transfer spectra at statistical stationarity (averaged over a minimum of 10 τ). The transfer spectrum is defined as $T(\kappa) = \langle -\hat{u}_i \mathscr{F}\left(u_j \frac{\partial u_i}{\partial x_j}\right)\rangle$, a scalar function of the wavenumber. Here, $\mathscr{F}(\cdot)$ denotes the Fourier transform and $\hat{u}$ denotes the Fourier-transformed velocity field. (a) $Re_\lambda = 110$. (b) $Re_\lambda = 140$.

turbulence) and using the conditions of homogeneity and a zero mean velocity, this reduces to

$$\frac{\partial \epsilon}{\partial t} = -C_{\epsilon 2}\frac{\epsilon^2}{k} + f, \tag{10}$$

where $f = 2A\epsilon$ under the action of the original momentum source term and $f = 2A\epsilon(k_0/k)$ under the action of the proposed modified source term. It is important to note that the above expression is only a model and may not describe adequately the evolution of ϵ under all conditions.

The turbulent kinetic energy and dissipation rate are perturbed about their asymptotic (time-invariant) mean values, k_0 and ϵ_0, according to $k = k_0 + k'$ and $\epsilon = \epsilon_0 + \epsilon'$. These perturbed expressions are inserted into Eqs. (2), (7), and (10). For the original source term, the results are

$$0 = -\epsilon_0 + 2Ak_0, \tag{11a}$$

$$\frac{dk'}{dt} = -\epsilon' + 2Ak', \tag{11b}$$

$$0 = -C_{\epsilon 2}\frac{\epsilon_0}{\tau_0} + 2A\epsilon_0, \tag{11c}$$

$$\frac{d\epsilon'}{dt} = \frac{C_{\epsilon 2}}{\tau_0^2}k' + \epsilon'\left(2A - 2\frac{C_{\epsilon 2}}{\tau_0}\right), \tag{11d}$$

where only terms that are at most first-order (linear) in the perturbed quantity have been kept. For the modified source term, the results are

$$0 = -\epsilon_0 + 2Ak_0, \tag{12a}$$

$$\frac{dk'}{dt} = -\epsilon', \tag{12b}$$

$$0 = -C_{\epsilon 2}\frac{\epsilon_0}{\tau_0} + 2A\epsilon_0, \tag{12c}$$

$$\frac{d\epsilon'}{dt} = \left(\frac{C_{\epsilon 2}}{\tau_0^2} - \frac{2A}{\tau_0}\right)k' + \left(2A - 2\frac{C_{\epsilon 2}}{\tau_0}\right)\epsilon'. \tag{12d}$$

To obtain these linearized perturbation equations, the denominators of the dissipation rate equations (Eqs. (11) and (12)) were Taylor-expanded for small k'. Under statistically stationary conditions and, irrespective of the source term used (original or modified), it is recovered $A = \epsilon_0/(2\,k_0) = 1/(2\,\tau_0)$. Additionally, it is found that a necessary (but not sufficient) condition for the existence of an asymptotic state is that $C_{\epsilon 2} = 1$. (This result is independent of the form of the source term.) This value for $C_{\epsilon 2}$ differs from that of a standard $k - \epsilon$ model,[15,16] as it now corresponds to a stationary, forced turbulent field, not a decaying one. As such, Eq. (10) with $C_{\epsilon 2} = 1$ may not be used to describe the initial stages of the forced velocity field (prior to reaching statistical stationarity) and may not be used to prove convergence independent of the initial conditions (i.e., k_0 and ϵ_0).

Using Eqs. (11) and (12), the needed coupled turbulent kinetic energy-dissipation rate system can be specified. For the original source term, this system takes the form in Eq. (13a). For the modified source term, this system takes the form in Eq. (13b). For the modified forcing method proposed to be stable, a necessary condition is that perturbations about the asymptotic values of k_0 and ϵ_0 should temporally decrease; such behavior is indicated by the eigenvalues of the coupled equation system

$$\frac{d}{dt}\begin{bmatrix} k' \\ \epsilon' \end{bmatrix} = \frac{1}{\tau_0}\begin{bmatrix} 1 & -\tau_0 \\ \frac{1}{\tau_0} & -1 \end{bmatrix}\begin{bmatrix} k' \\ \epsilon' \end{bmatrix}, \tag{13a}$$

$$\frac{d}{dt}\begin{bmatrix} k' \\ \epsilon' \end{bmatrix} = \frac{1}{\tau_0}\begin{bmatrix} 0 & -\tau_0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} k' \\ \epsilon' \end{bmatrix}. \tag{13b}$$

For the original momentum source term, the eigenvalues are found to be zero, $\lambda_1 = \lambda_2 = 0$. Eigenvalues of zero are associated with marginal stability, implying that oscillations will be neither compelled to grow nor to decay in time. There is no mechanism to dampen or reduce the amplitudes of the fluctuating turbulent quantities. It is believed that this is the cause for the sensitivity of the turbulent kinetic energy and dissipation rate statistics depicted in Figs. 1 and 2.

Alternatively, when the eigenvalues corresponding to the system for the modified source term are calculated, one eigenvalue is found to be negative, $\lambda_1 = -1/\tau_0$, and the other is found to be zero, $\lambda_2 = 0$. The negative eigenvalue suggests that variations in calculated turbulent quantities will be driven towards progressively smaller amplitudes. This negative eigenvalue is responsible for the improved long-time behavior of the pertinent turbulent field statistics, and justifies the proposed modification to Lundgren's[9] original source term.

V. EXTENSION TO NON-HOMOGENEOUS FLOWS

Although the simulations used in this study are homogeneous and isotropic, Lundgren's[9] physical space method can be implemented in non-homogenous configurations. In fact, its application to such geometries is one of its key capabilities. Under such conditions, the instantaneous, domain-averaged, total turbulent kinetic, k, which is needed in the source term, may not be calculated readily. In instances where this is the case, a reasonable local averaging approach can be applied. For example, the needed k could be approximated via planar averages in any two homogeneous directions. If no such homogeneous directions exist or if they are deemed inappropriate, then, alternatively, k could be approximated by computing the turbulent kinetic energy lying within a region encapsulating the data point of concern. In such an instance, the volume-average should be of length at least the integral length-scale of the velocity field to ensure that a sufficiently large percentage of the total kinetic energy is being captured in the averaging process. These approaches would also partially preserve the "local" nature of Lundgren's[9] original forcing method.

VI. SUMMARY

In summary, although Lundgren's[9] original velocity field forcing technique can successfully drive a turbulent field to and sustain it at the desired Re_λ, the turbulent statistics are subject to considerable and large oscillations in their long-time behavior. A practical implication of these large amplitude fluctuations is that simulations must be conducted for a significantly longer period of time in order to obtain time-invariant quantities. Through a linear perturbation analysis, the cause for this undulating statistical behavior has been connected to the form of the momentum source term appended to the Navier-Stokes equations and to the resulting stability characteristics of the forced-turbulent kinetic energy-dissipation rate equation system. A modification to Lundgren's[9] momentum source term has been proposed, which is more consistent with existing spectral forcing methods. Upon application of this modified source term, the temporal behavior of the turbulent statistics was found to be improved, while the spectral characteristics of the velocity field were preserved. Moreover, statistical stationarity was reached much earlier in the simulation when the proposed modification was implemented. As direct numerical simulation studies are computationally intensive from the outset, this reduction in the time necessary to attain temporally invariant turbulent physics when using the proposed modified source term is of practical significance.

[1] M. R. Overholt and S. B. Pope, "A deterministic forcing scheme for direct numerical simulations of turbulence," Comput. Fluids **27**, 11 (1998).
[2] E. D. Siggia and G. S. Patterson, "Intermittency effects in a numerical simulation of stationary three-dimensional turbulence," J. Fluid Mech. **86**, 567 (1978).
[3] S. Chen, G. D. Doolen, R. H. Kraichnan, and Z. She, "On statistical correlations between velocity increments and locally averaged dissipation in homogeneous turbulence," Phys. Fluids A **5**, 458 (1993).
[4] Y. Zhou, "Degrees of locality of energy transfer in the inertial range," Phys. Fluids A **5**, 1092 (1993).
[5] R. M. Kerr, "Theoretical investigation of a passive scalar such as temperature in isotropic turbulence," Ph.D. thesis, Cooperative Thesis No. 64, Cornell University and NCAR, 1981.
[6] K. Alvelius, "Random forcing of three-dimensional homogeneous turbulence," Phys. Fluids **11**, 1880 (1999).

[7] V. Eswaran and S. B. Pope, "An examination of forcing in direct numerical simulations of turbulence," Comput. Fluids **16**, 257 (1988).
[8] N. P. Sullivan, S. Mahalingam, and R. M. Kerr, "Deterministic forcing of homogeneous, isotropic turbulence," Phys. Fluids **6**, 1612 (1994).
[9] T. S. Lundgren, "Linearly forced isotropic turbulence," in *Annual Research Briefs* (Center for Turbulence Research, Stanford, 2003), pp. 461–473.
[10] T. S. Lundgren, "Kolmogorov two-thirds law by matched asymptotic expansion," Phys. Fluids **14**, 638 (2002).
[11] C. Rosales and C. Meneveau, "Linear forcing in numerical simulations of isotropic turbulence: Physical space implementations and convergence properties," Phys. Fluids **17**, 095106 (2005).
[12] S. B. Pope, *Turbulent Flows* (Cambridge University Press, New York, 2000).
[13] O. Desjardins, G. Blanquart, G. Balarac, and H. Pitsch, "High order conservative finite difference scheme for variable density low Mach number turbulent flows," J. Comput. Phys. **227**, 7125 (2008).
[14] D. C. Wilcox, *Turbulence Modeling for CFD* (DCW Industries Inc., La Canada, 2000).
[15] B. E. Launder and B. I. Sharma, "Application of the energy-dissipation model of turbulence to the calculation of flow near a spinning disk," Lett. Heat Mass Transfer **1**, 131 (1974).
[16] V. Yakhot and S. A. Orszag, "Renormalization group analysis of turbulence: 1. Basic theory," J. Sci. Comput. **1**, 3 (1986).