The Propagation of Plastic Deformation in Solids*

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The stress wave caused by a longitudinal impact at the end of a cylindrical bar has been analyzed in the case where the impact velocity is large enough to produce plastic strain. The theory gives a method for computing the stress distribution along the bar at any instant during impact. It is shown that for a given material, there is a critical impact velocity such that when subjected to a tension impact with a velocity higher than the critical, the material should break near the impacted end with negligible plastic strain.

An experimental investigation was made concurrently with the theoretical study. Some of the most significant experimental results are presented in this paper.

I. INTRODUCTION

THE testing of impact strength of materials previous to the discovery of the laws of propagation of plastic deformation used essentially two different methods: bending of notched bars, and tension or compression of short cylindrical specimens. The first method presumes a certain standardization of the shape of the specimens and tries to establish certain relative merit numbers for the impact resistance of various materials by measuring the energy absorbed by the standardized specimens. The second method is somewhat more ambitious; its aim is the determination of the energy absorption by unit volume of the material, in the case of impact, and the comparison of this amount of energy with the work absorbed in static experiments. These tests, especially the tests of the second type, have shown that for moderate impact velocities the work absorbed in impact is, in general, somewhat larger than the work absorbed in static experiments; there were some indications that this tendency is reversed as the impact velocity increases. However, the observed drop in the amount of absorbed work was not accompanied by a decrease in elongation and no explanation was offered for this phenomenon.

As the European war started, the attention of military and other engineers concerned with the construction of bomb resisting structures was focused on the impact problem. The senior author became interested in the problem of resistance of structures to impact loads beyond the elastic limit in a conference held at the National Academy of Sciences early in 1940, under the chairmanship of Dr. V. Bush. He conceived the idea of the plastic wave in order to obtain a method for theoretical calculation of the energy absorbed by various structures, as columns, beams, plates, etc. The theory of elastic wave propagation was known. It was evident from this theory that, for any especially brittle material (i.e., materials which break at their elastic limit), a critical impact velocity exists for which fracture must occur at the point of application of the impact, so that the rest of the structure has no chance to participate in the energy absorption. It appeared that if a theory of the propagation could be developed, such a theory would reveal the existence of a critical velocity for structures loaded beyond their yield strength.

The basic theory and preliminary experiments supporting the theory, at least in its fundamental conclusions, were carried out during the year 1941. The senior author presented the fundamentals of the theory in a paper sent for publication in the Proceedings of the National Academy of Sciences in December, 1941. It was found, however, that the paper might have some bearing on problems relating to National Defense. The publication was therefore delayed, but the study was described in a classified NDRC report, A-29, "On the Propagation of Plastic Deformation in Solids," February 1942. The experimental work carried out previously by the junior author was published at the same time in the classified NDRC report, A-33, "Preliminary Experiments on Propagation of Plastic Deformation." Subsequently more complete theoretical investigations on the subject were carried out by the senior author, H. F. Bohnenblust, D. H. Hyers, and J. Charyk. The main content of these investigations will probably be published in the near future.

II. LONGITUDINAL IMPACT AT THE END OF A BAR EXTENDING TO INFINITY

The propagation of elastic strains in a cylindrical bar subjected to tension impact had already been analyzed in 1807 by Thomas Young, who established the proportionality between the elastic strain and the velocity of impact. The value of the elastic strain ϵ is in this case given by the formula

$$v_1 = c_0\epsilon,$$

in which v_1 is the velocity of impact and c_0 is the velocity of propagation of an elastic deformation. Young had pointed out in his paper that when the velocity of impact is above a certain critical value a plastic strain is initiated near the point of impact. Since Young's work, it seems that the interest of the problem has not been fully recognized, and no systematic attempt has been made to compute the stress and strain caused by an impact beyond the elastic limit of the material. In the present paper such a treatment is provided for the case of longitudinal impact.

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Consider a rod or wire extending from $x=-\infty$ to $x=0$ and assume that the endpoint at $x=0$ is suddenly put in motion with a constant velocity v_1. Let the stress-strain relation for the material be given by a function of the form $\sigma=\sigma(\epsilon)$ where σ is the stress and ϵ the strain. Stresses that depend on the time-rate of strain will be neglected; for, with the exception of the case of extremely high velocities, such stresses are small in comparison with the stresses that depend on the strain itself. To be sure, the relation $\sigma=\sigma(\epsilon)$ holds only for the first deformation of the material beyond the elastic limit; in the case of load reversal, another functional relation, which takes the hysteresis into account, has to be used. The lateral contraction of the material—that is, the contribution of the lateral contraction to the kinetic energy—is neglected in the following calculation.

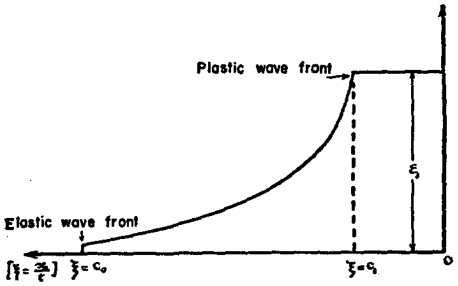

FIG. 1. Schematic representation of ϵ *versus* ξ.

With these simplifications, the equation of motion for an element of the rod or wire can be written in the form

$$\rho\frac{\partial^2 u}{\partial t^2}=\frac{d\sigma}{d\epsilon}\frac{\partial \epsilon}{\partial x}, \tag{1}$$

where u is the displacement of the element in the longitudinal direction, ρ is the density of the material, and t is the time.

It should be pointed out that in Eq. (1) the process of deformation is considered from the Lagrangean point of view. An arbitrary cross section of the bar is determined by a coordinate x, which gives its distance from the origin in the unloaded state. The stress is defined as the ratio of the internal reaction between two portions of the bar to the initial cross-sectional area of the bar. The strain is numerically equal to the change of length of a portion of the bar which, in the initial unloaded state, was of unit length. If u is the displacement of a cross section, the strain ϵ is equal to $\partial u/\partial x$. It is not assumed that ϵ is small in comparison with unity.

Since $\epsilon=\partial u/\partial x$, Eq. (1) can also be written in the form

$$\rho(\partial^2 u/\partial t^2)=T(\partial^2 u/\partial x^2), \tag{2}$$

where $T=d\sigma/d\epsilon$ is the modulus of deformation, elastic or plastic. The quantity T is considered to be a given function of the strain. The boundary conditions are $u=v_1 t$ for $x=0$, and $u=0$ for $x=-\infty$.

It is easily seen that a solution of the form

$$u=v_1[t+(x/c_1)] \tag{3}$$

with an arbitrary value of the velocity of propagation c_1, satisfies Eq. (2) and the boundary condition at $x=0$. For this solution the strain ϵ is constant and is equal to v_1/c_1.

A second solution is obtained by putting

$$T/\rho=x^2/t^2. \tag{4}$$

Since $T[=d\sigma/d\epsilon]$ is a given function of ϵ, Eq. (4) represents a solution for which ϵ is a function only of the variable $\xi=x/t$.

Assume $\epsilon=f(\xi)$; then the displacement u has the form

$$u=\int_{-\infty}^{x}\frac{\partial u}{\partial x}dx=\int_{-\infty}^{x}f(\xi)dx=t\int_{-\infty}^{\xi}f(\xi)d\xi \tag{5}$$

since $dx=td\xi$. By differentiation one readily obtains

$$\begin{aligned}\partial^2 u/\partial t^2&=(\xi^2/t)f'(\xi)\\ \partial^2 u/\partial x^2&=(1/t)f'(\xi)\end{aligned} \tag{6}$$

and substitution of Eqs. (6) in Eq. (2) shows that one of the two equations,

$$\rho\xi^2=T \tag{7}$$

or

$$f'(\xi)=0 \tag{8}$$

must hold. Equation (8) leads to the solution expressed by Eq. (3), whereas Eq. (7) gives the solution of Eq. (4).

The complete solution is obtained as follows:

(a) For $|x|<c_1 t$ the strain ϵ is constant and equal to ϵ_1;

(b) For $c_1 t<|x|<c_0 t$, where c_0 is the velocity of propagation of the elastic wave,

$$T(\epsilon)=\rho(x^2/t^2) \tag{9}$$

(c) For $|x|>c_0 t$, $\epsilon=0$.

The distribution of ϵ as a function of ξ is shown schematically in Fig. 1. The value of T for small values of ϵ—that is, within the elastic limit—is equal to E, Young's modulus of elasticity for the material. The elastic wave propagates with the velocity $c_0=(E/\rho)^{\frac{1}{2}}$. Between the plastic wave front, which is propagated with the velocity c_1, and the elastic wave front, the strain is variable, since every strain-increase from ϵ to $\epsilon+d\epsilon$ proceeds with a velocity equal to the specific value of $c=(T/\rho)^{\frac{1}{2}}$, corresponding to the strain ϵ.

The main problem is to determine the velocity c_1 of the plastic wave and the maximum strain ϵ_1 as a function of the velocity of impact v_1.

From Eq. (5) it follows:

$$v_1=\frac{u(0,t)}{t}=\int_{-\infty}^{0} f(\xi)d\xi \tag{10}$$

and by changing variables

$$v_1=-\int_0^{\epsilon_1} \xi d\epsilon$$

(See Fig. 4).

Thus, upon substituting for ξ from Eq. (7), Eq. (10) takes the form

$$v_1=\int_0^{\epsilon_1} (T/\rho)^{\frac{1}{2}}d\epsilon. \tag{11}$$

Since T is a given function of ϵ, Eq. (11) determines ϵ_1 as a function of v_1.

If the deformation remains within the elastic limit, $T=E=$ constant, and $v_1=\epsilon_1 c_0$, then $v_1=\epsilon_1(E/\rho)^{\frac{1}{2}}$. Hence, the stress σ is given by

$$\sigma_1=E\epsilon_1=\rho v_1 c_0. \tag{12}$$

Equation (12) is universally used for the calculation of the stress produced in an elastic body subjected to an impact velocity v_1. It appears that Eq. (11) replaces Eq. (12) in the case of a deformation beyond the elastic limit. If the stress remains within the elastic limit, there are two regions: for $|x|<c_0t$, $\sigma=\rho v_1 c_0$ and for $|x|>c_0t$, $\sigma=0$. In the case of plastic deformation, there are two fronts. Beyond the front of the elastic wave, $\sigma=0$; between the fronts of the elastic and plastic waves, σ increases gradually from $\sigma=0$ to a maximum value, $\sigma=\sigma_1$; and behind the front of the plastic wave, σ has the constant value σ_1, corresponding to a total strain ϵ_1—elastic plus permanent—where ϵ_1 is given by Eq. (11).

For most materials $d\sigma/d\epsilon$ approaches zero for large values of ϵ, and at some particular value of ϵ, the material breaks. Hence the integral constituting the right-hand member of Eq. (11) has a maximum value, and one obtains a critical value of the velocity v_1. It can be expected that an impact with a velocity larger than v_1 will cause an instantaneous breakdown of the material.

III. EXTENSION OF THE THEORY; BAR OF FINITE LENGTH

The characteristic parameters that define the state of strain and motion of an element are the strain $\epsilon[=\partial u/\partial x]$, the stress σ, and the velocity of the element $v=\partial u/\partial t$. The equation of motion for the element is

$$\rho(\partial v/\partial t)=\partial\sigma/\partial x$$

or, using the definition of the velocity of propagation,

$$\partial v/\partial t=c^2(\partial\epsilon/\partial x). \tag{13}$$

On the other hand, from the relations $v=\partial u/\partial t$ and $\epsilon=\partial u/\partial x$ it follows that

$$\partial v/\partial x=\partial\epsilon/\partial t. \tag{14}$$

Equations (13) and (14) are equivalent to the differential Eq. (2).

The coordinate x of the element and the time at which the magnitude of the strain of this element is equal to ϵ and its velocity is equal to v can be considered as functions of ϵ and v. Then Eqs. (13) and (14) take the form

$$\begin{aligned}\partial x/\partial\epsilon&=c^2(\partial t/\partial v)\\ \partial t/\partial\epsilon&=\partial x/\partial v.\end{aligned} \tag{15}$$

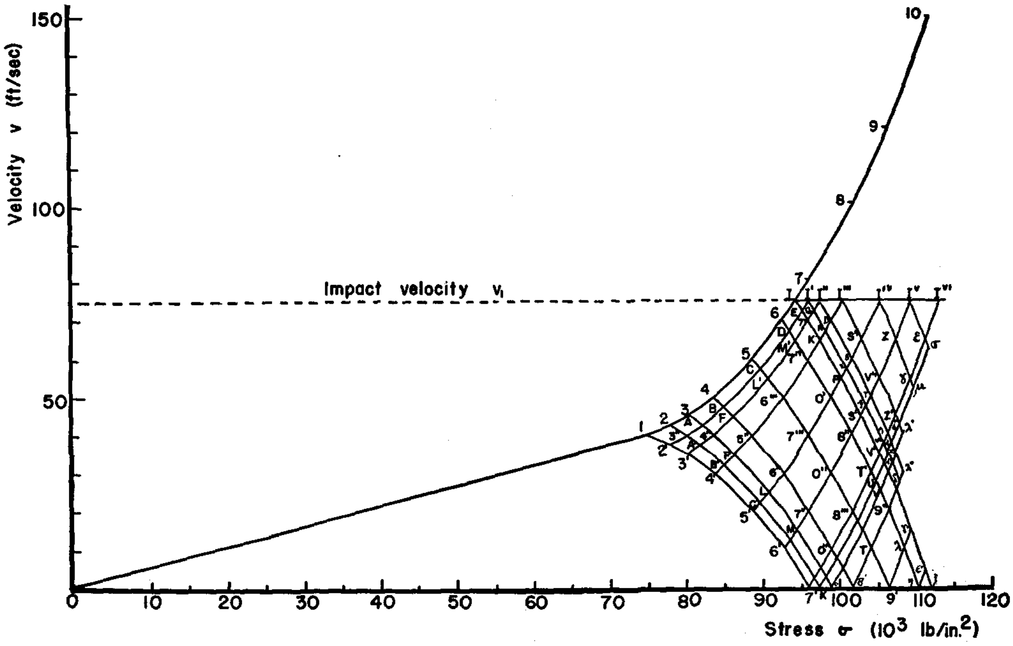

FIG. 2. Velocity *vs.* stress diagram for heat treated carbon steel.

Equations (15) become more symmetrical by the introduction of the function $\phi=\int_0^\epsilon c(\epsilon)d\epsilon$ and we obtain

$$\begin{aligned}\partial x/\partial\phi&=c(\partial t/\partial v)\\ \partial x/\partial v&=c(\partial t/\partial\phi).\end{aligned}\tag{16}$$

The process of propagation can be represented in two planes. The first, with x and t as coordinates, is the Lagrangean or physical plane; the second plane is the velocity plane, in which ϕ and v are used as coordinates. The system of Eqs. (16) has fixed characteristics in the $v\phi$ plane. Hence, a relatively simple graphical method can be developed for the solution of a broad class of impact problems in the plastic range; for example, the case of a bar of finite length with one fixed end and hit at the free end. In Figs. 2 and 3 are typical (σ, v) and (x, t) diagrams for a specimen of heat treated carbon steel subjected to an impact velocity of 75 ft./sec.

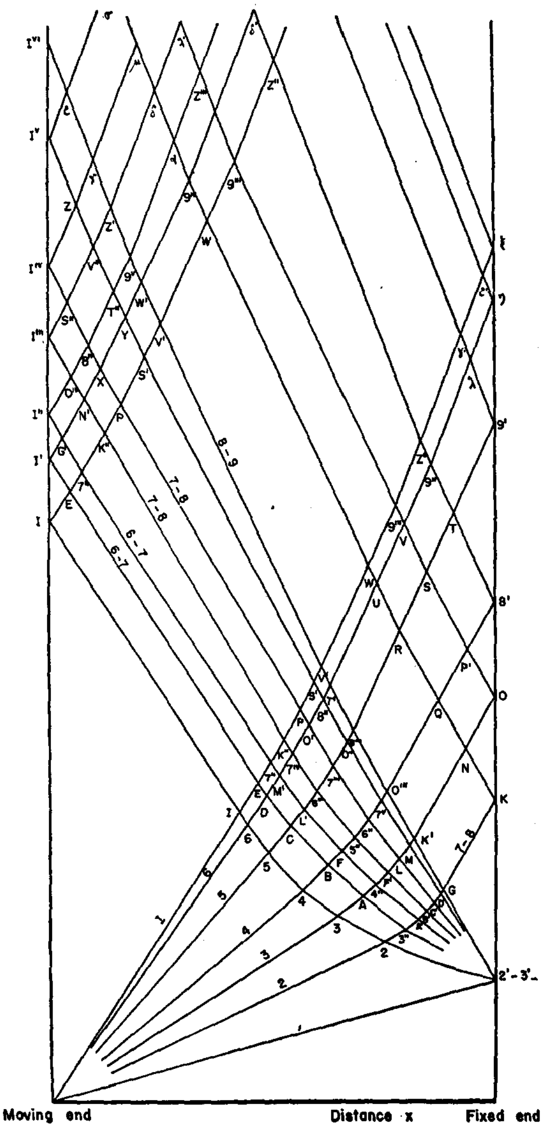

FIG. 3. Lagrange diagram for specimen of heat treated carbon steel. Impact velocity 75 ft./sec.

IV. EXPERIMENTS ON THE PROPAGATION OF PLASTIC STRAINS IN A LONG SPECIMEN SUBJECTED TO TENSION IMPACT

1. Experimental Technique

The first experiments on the propagation of plastic strain in tension were made in view of verifying the three following points resulting from the theory: (1) the existence of a plastic wave front of given amplitude; (2) the relation between the amplitude ϵ_1 of the plastic wave front and the velocity of impact v_1 (Eq. (11)); (3) the distribution of the plastic strain between the plastic and the elastic front as given by Eq. (9). The specimens used in these experiments were annealed copper wire about 100 inches long and 0.071 inch in diameter. The specimens were mounted vertically in a special testing machine built by Dr. D. S. Clark at the California Institute of Technology. The impact was produced by a hammer guided between two rails and accelerated by prestretched rubber bands. The maximum velocity attainable was approximately 200 ft./sec. The velocity was measured by a suitable electric device and a cathode-ray oscillograph. On every specimen equidistant marks were made with one inch spacings. After the test, the plastic strain was determined by measuring the displacement of each mark. Experiments were also performed in compression impact. In this case, the specimens were cylinders 12 inches long and about $\frac{3}{8}$ inch in diameter.

To control the duration of impact, the following device was employed: the bottom end of the wire is attached to a rigid piece A, as shown in Fig. 4. A vertical rod B, resting on the bottom frame of the machine, fits loosely into the tubular part of A. When a hammer H hits the piece A, the specimen elongates until A reaches the rod B. The piece A contains a circular notch N, and the rim of A breaks off at this notch after A has traveled the distance D and comes to rest on the rod B. The purpose of this arrangement is to allow the hammer to continue to move downward and also to dissipate some of its remaining energy. However, no kinetic energy is transferred to the specimen after A reaches B. The time of impact is therefore the distance D divided by the velocity of the hammer during the process of elongation.

The static stress-strain curve of one of the copper wires tested in impact was used to compute the values of $d\sigma/d\epsilon$ as a function of ϵ and the values of the velocity of propagation c, as given by the formula $c=[1/c(d\sigma/d\epsilon)]^{\frac{1}{2}}$, were then computed and a curve of c *versus* ϵ was obtained (Figs. 5 and 6). From this curve, the value of the quantity v_1 (the velocity of impact corresponding to a plastic front of amplitude ϵ_1) was calculated for each value of ϵ_1 (Fig. 7). The largest value of v_1 is obtained for the largest possible value of (16 percent in this case) and is equal to 150 ft./sec. The impact velocity of 150 ft./sec. is therefore the "critical velocity" for this material. An impact with a higher velocity

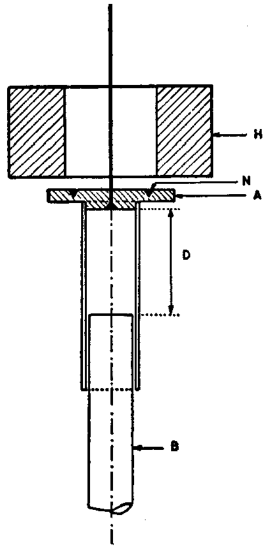

FIG. 4. Experimental device used to stop the impact after a given deformation of the specimen has been reached.

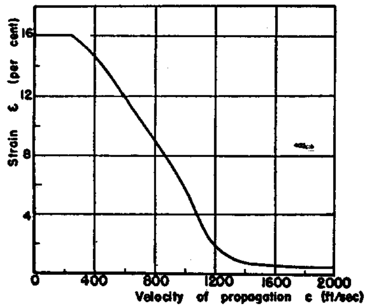

FIG. 5. Strain ϵ *versus* velocity of propagation c as calculated from the static stress-strain curve.

must produce an instantaneous breakdown of the specimen.

As mentioned above, marks were made on the specimen at intervals of one inch. The distance between the origin and each mark was measured before and after the test. If x_n denotes the distance from the origin to a mark n before the test, and if x_n' is the distance between the two points after the test, the difference $x_n'-x_n$ represents the total elongation of the specimen between these two points. The values of $x_n'-x_n$ were then plotted *versus* x. The slope at any point of the curve so obtained gave the value of the permanent strain at that point. This way of measuring the strain has been found to be the most practical one. It was not necessary to make the distinction between permanent strain and actual strain, because the elastic recovery was relatively too small to be taken into consideration.

2. Results of the High Velocity Impact Tests

The results are given in three separate subsections—(i), (ii), and (iii)—in order to show to what extent they agree with the three principal theoretical results which were expressed in Section II.

(*i*) *Existence of a Plastic Wave Front*

The first series of tests was made to establish that the amplitude of the plastic front is a function of the velocity of impact alone. This amplitude remains constant while the elastic front and the plastic front travel along the specimen. In these particular experiments, the velocity of impact was always 92.50 ft./sec., but the durations of the impact varied. To control the duration of impact the two pieces A and B, shown in Fig. 4, were placed at a proper distance apart. The curves in Fig. 8 give the distribution of the strain along the specimen. They indicate clearly that a plastic front of a given amplitude ϵ_1 is revealed by the experiments, as predicted from the theory.

(*ii*) *Relation Between the Velocity of Impact and the Amplitude of the Plastic Wave Front*

In this series of tests, the velocity of impact was varied from one test to the other. The total elongation was not necessarily the same for all the tests. The stopping device was adjusted in such a manner that during the impact the plastic front traveled a distance of between 20 and 40 inches. Figure 9 gives the distribution of the strain along the specimen in each case. The experimental values of ϵ_1 corresponding to each velocity tested are listed in Table I. For comparison with the theory, the experimental values are plotted as points in Fig. 7, whereas the solid curve represents the result of the theoretical computation. The agreement between the experimental results and the predetermined curve is fairly good. The highest velocity of impact used was 171 ft./sec. For that velocity, the specimen broke within the first inch, indicating that a velocity of 171 ft./sec. is above the critical velocity, as predicted by the theory. It must be pointed out that a considerable reduction of area was observed, which indicates that the rupture was not brittle. In addition, a plastic strain of relatively small intensity (less than

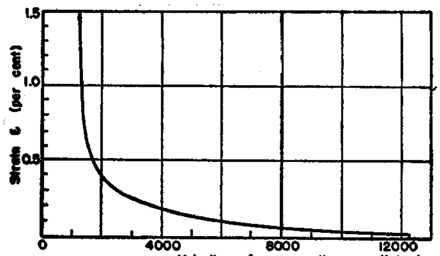

FIG. 6. Continuation of the curve of Fig. 5 up to the elastic point.

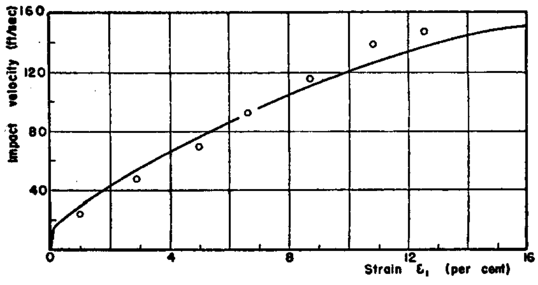

FIG. 7. Variation of the strain ϵ_1 with the impact velocity v_1.

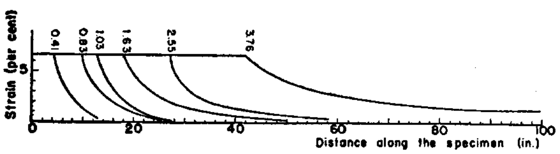

FIG. 8. Distribution of the strain ϵ along the specimen. Velocity of impact 92.5 ft./sec. The duration of impact is indicated on each curve in msec.

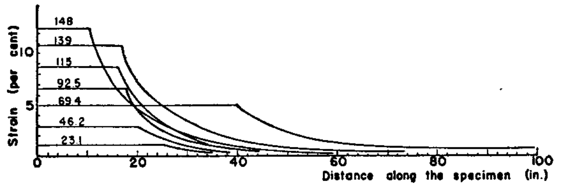

FIG. 9. Strain distribution curves for annealed copper specimens subjected to different impact velocities, indicated on each curve in ft./sec.

two or three percent) was measured along the wire over a distance of about 20 inches from the point of rupture. It is probable that a plastic wave started to propagate along the specimen during the time required to produce rupture.

(*iii*) *Shape of the Plastic Wave and Velocity of Propagation of the Plastic Front*

The distribution of the plastic strain along the wire between the plastic and elastic fronts may be calculated from Eq. (9), using the curves of Figs. 5 and 6. The dashed curves in Fig. 10 represent theoretically computed amplitude distributions of a plastic wave after a time interval equal to 0.83 msec.; the assumed velocity of impact, v_1, is 92.5 ft./sec. and ϵ_1 is 6.6 percent. The solid curve in Fig. 10 represents plots of the measured distribution of the strain. It is seen that the experimental and theoretical curves deviate considerably. This deviation is due to some inaccuracy in the value of the duration of impact, and especially to the perturbating effects brought by the sudden stopping of the impact. When the hammer which pulls the end of the wire with a constant velocity is stopped, the portion of the specimen that is in motion is not stopped instantaneously. During the deceleration period, the kinetic energy stored in the specimen is transformed into strain energy, and the total extension is larger than the distance D through which the hammer has been in contact with the specimen.

The measured shape of the plastic wave given in Fig. 10 is greatly influenced by this "stopping effect" and cannot logically be compared with the theoretical curve that represents the shape of the wave at a certain instant during impact. The stopping effect was analyzed theoretically by the authors and especially by Drs. F. Bohnenblust and J. Charyk, who collaborated in the work reported in this paper. These investigations will be published separately. They materially reduced the variance between computed and measured values.

The reflection of plastic waves at the fixed end of a specimen was also investigated theoretically and experimentally. A typical result of this investigation is shown in Fig. 11, in which a measured strain distribution curve obtained in a compression impact test is compared with a computed curve. The two curves are not in complete agreement, but the character of the observed strain distribution curve is accounted for by the theory. It may also be pointed out that the deviation between the two curves certainly reflects some influence of the strain rate, showing that the assumption of a stress-strain curve independent of the rate of strain is not entirely justified.

A very noticeable effect of the rate of strain on the elastic limit has been observed for iron specimens subjected to both tension and compression impact. The most striking result of impact tests on iron specimens

TABLE I. Results of impact tests on 80 inches long annealed copper specimens.

Impact velocity (ft./sec.)	Uniform strain (percent)	Duration of impact (msec.)	Velocity of propag. of uniform strain (ft./sec.)
23.1	1.0	1.70	1230
46.2	2.9	1.47	1130
69.4	5.0	3.37	1000
92.5	6.6	1.63	910
115.0	8.7	1.55	860
139.0	10.8	1.94	730
148.0	12.5	1.40	630
171.0		Rupture	

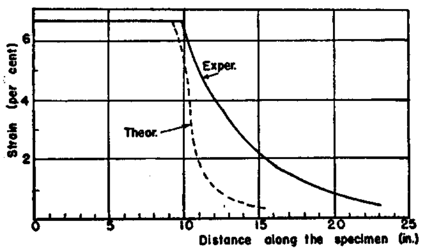

FIG. 10. Theoretical and experimental strain distribution curves for annealed copper specimen. Impact velocity 92.5 ft./sec. Duration of impact 0.83 msec.

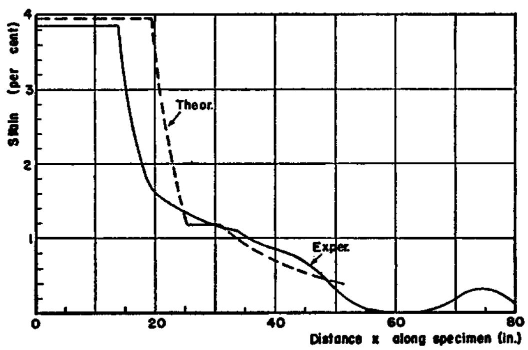

FIG. 11. Experimental and theoretical strain distribution curves for a copper wire 0.08 inch in diameter. Impact velocity 62.5 ft./sec., duration of impact 1.3 msec.

was that no plastic strain occurred near the moving end until the stress wave reached a value of the order of three times the static yield stress. The measured strain distribution curves also showed a different character, as shown in Figs. 12 and 13. It can be observed that in the case of compression the permanent set is concentrated in the neighborhood of the ends of the specimen and the center is free from plastic strain. In the tension tests there were intervals of permanent sets alternating with undisturbed regions. This is probably due to irregularities in the specimens whose influence was enhanced by the unstable character of the tension process.

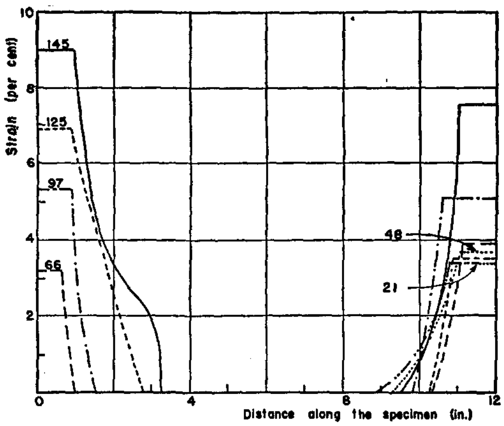

FIG. 12. Strain distribution curves for annealed mild steel specimens subjected to compression with different impact velocities, indicated on each curve in ft./sec.

3. Force-Time Relation at the Fixed End of a Specimen Subjected to Tension Impact

An important contribution of the theory of plastic wave propagation presented in this paper is a clear explanation of the dependence on time of the force measured at the fixed end of a specimen subjected to tension impact. Force *versus* time diagrams recorded at the fixed end of the specimen have been interpreted by previous investigators in the field of high velocity impact testing as dynamic stress-strain curves. The transformation of the time axis into a strain axis infers that the strain is uniform all along the specimen at any instant during impact. The theory of strain propagation shows that for impact velocities greater than approximately 10 ft./sec., this hypothesis is not justified, and a stress-strain relation cannot be obtained in such a simple way. In order to show this, the force acting at the fixed end of a specimen subjected to tension impact was computed as a function of time by the theory outlined in Section III. This force was also measured in a series of experiments made with different types of specimens. The force was recorded by means of a resistance sensitive strain gauge in conjunction with a cathode ray oscillograph. Two records obtained for copper specimens tested with an impact velocity of 100 ft./sec. are reproduced in Fig. 14, together with a theoretical stress *versus* time diagram. The agreement between theoretical and experimental curves is very satisfactory as far as their shape is concerned. The experimental curve, however, seems to be shifted to the left. This discrepancy could be explained by assuming that any strain has a velocity of propagation slightly higher than that based on the static stress-strain curve. This gives further evidence of the influence of the rate of strain on the relation between stress and strain. In any case, the interpretation of such curves as those of Fig. 14 as dynamic stress-strain curves is completely fictitious.

4. Plastic Strain Propagation and High Velocity Impact Tests

High velocity tensile impact tests have been reported by several investigators.[1-6] Some of the results[1] have shown that the energy absorbed before rupture decreased when a certain impact velocity was reached.

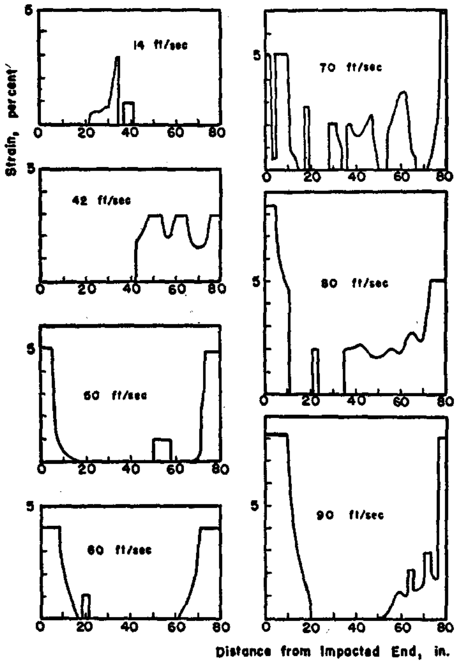

FIG. 13. Strain distribution for annealed iron wire in tension for different impact velocities.

However, the observed drop in the amount of absorbed word was not accompanied by a decrease in elongation

[1] H. C. Mann, Proc. A.S.T.M. **36**, 85 (1936).
[2] D. S. Clark and G. Datwyler, Proc. A.S.T.M. **38**, 98 (1938).
[3] D. S. Clark, *The Influence of Impact Velocity on the Tensile Characteristics of Some Aircraft Metals and Alloys,* NACA Technical Note No. 868.
[4] M. Manjoine and A. Nadai, Proc. A.S.T.M. **40**, 822 (1940). Trans. A.S.M.E., J. App. Mech. **8**, A-77 (June, 1941).
[5] DeForest, MacGregor, and Anderson, Metals Technol. **8**, 1 (December, 1941).
[6] E. R. Parker and C. Ferguson, Trans. A.S.M. **XXX**, No. 1, 68 (March, 1942).

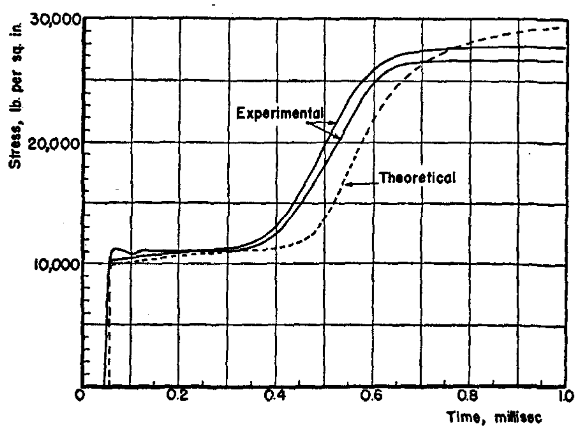

FIG. 14. Experimental and theoretical stress-time curves in tension for fixed end of annealed copper. Gage length eight inches; impact velocity 100 ft./sec.

and no explanation was offered for this phenomenon. Other investigators did not observe any drop in energy absorption, even with impact velocities as high as 200 ft./sec.

After the discovery of the laws of propagation of plastic deformation, it became evident to the authors that both the elongation and the energy should decrease more or less suddenly when the critical velocity is reached. The first tests made with long copper wires and described in Section III clearly established the existence of a critical velocity, as predicted by the theory. The rupture of the specimen, however, at velocities above the critical, was still of the ductile type, and some appreciable strain due to necking could still be measured near the end of the specimen subjected to impact. The elongation due to necking and the energy absorbed by the necking process are relatively important when the test specimen is short, or rather when the ratio between the length and the diameter of the specimen is small. Systematic experiments indicated that, in order to observe a definite drop in both energy and total elongation beyond the critical velocity, specimens should have a ratio of length to diameter at least equal to 20. These results furnished a clear explanation why a critical velocity was not observed by previous investigators, who made high velocity impact tests on short specimens.

One of the fundamental questions in high velocity impact tests is the influence of the rate of strain. The investigations of the authors clearly showed that such an influence exists. In fact, whereas the general character of the impact process was fairly well described by a theory neglecting the influence of the rate of strain, there was always a certain systematic discrepancy pointing to such an influence. However, it should be emphasized that in high speed impact tests the rate of strain varies from one point to the other and is a function of time. Computations resulting from the theory of wave propagation have shown that the average rate of strain calculated on the basis of uniform strain is far from the actual rate of strain, even when the velocity of impact is as low as 10 ft./sec. It is therefore not logical to utilize tension impact tests to study the influence of the rate of strain on the properties of metals.

Finally, it appears that for materials having a yield point like soft iron, the interpretation of high velocity impact tests is made rather difficult because of the phenomena reported in Section IV-2. The authors believe that the understanding of these phenomena requires a deeper insight into the whole question of plastic deformation and the physics of solids.

V. PRACTICAL APPLICATIONS

As previously stated, the theory of propagation of plastic deformation was developed for the understanding of the resistance of structures against impact. Hence the first application which appears to be important is the development of rules which enable the designer to compute the amount of energy which can be absorbed by a given structure. It has been shown quite clearly for the first time that the amount of energy is a function of the impact velocity, with a rather sudden decrease of the energy absorption beyond a critical velocity. The theory has already been extended to beams subjected to transverse impact. A class of ballistic problems which have not been sufficiently discussed from the point of view of plastic strain should be mentioned. Certain aspects of the problem of penetration of projectiles through metal plates are connected with plastic wave propagation. It seems, for example, that the notion of critical velocity could lead to the discrimination of the cases in which the penetrating projectile merely pushes aside the material from those in which a plug is separated from the back of the plate. Finally, the notion of plastic strain propagation might bring about a better understanding of some anomalies encountered in high speed machining.