A VELOCITY FILTER FOR ELECTRONS AND IONS

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Abstract

If a charged particle, moving along the x-axis with a velocity v, encounters successively two identical alternating electric fields of frequency ν, which are everywhere perpendicular to the x-axis, it will emerge from the last field undisplaced and traveling in the original direction under the following conditions: (1) Each field has two similar halves whose distance between centers is a; (2) The distance between the fields, center to center, is $D=sa/n$, where s and n are odd integers; (3) The velocity of the particle is $v_0=2a\nu/n$. The distribution about the velocity v_0 is computed when particles enter and leave the system through slits of width y_0 on the x-axis. The results show that the emergent beam can be confined to a very narrow velocity range.

Applications of the velocity filter.—Possible applications of the velocity filter, chiefly to positive ray work, are mentioned. It has already proved a successful and convenient substitute for pure magnetic analysis. It can be used to analyze a beam which is inhomogeneous both in mass and velocity when combined with either electric or magnetic deflection methods. The latter combination can be arranged to give a rigorously linear mass scale.

Most methods hitherto used for isolating from a stream of charged particles, moving in a high vacuum, those having a given velocity require the use of a magnetic field. The measurement, maintenance and reproduction of a constant and homogeneous magnetic field is, in many cases, difficult or inconvenient. On the other hand the recent development of radio technique has made it a comparatively simple matter to generate and regulate high frequency oscillating electric fields, so that a method using such fields should prove a valuable addition to those now available for positive ray analysis and similar work.

The first arrangement which suggests itself is an alternating electric field at right angles to the beam. If it were possible to make such a field uniform and sharp edged then those particles whose velocity is such that they pass through the field in a whole number of cycles will evidently have received as much acceleration in one direction as in the other and will emerge parallel to their original direction of motion. Simple considerations show however that they will be displaced from their path by an amount depending on the phase of the field when they entered it. If now a second identical field be placed at such a distance from the first that these particles enter it in opposite phase to that in which they entered the first, then the displacement will be reversed in the second field and the particles will emerge from it undeviated and undisplaced.

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It is impossible in practice to make such a uniform sharp edged field but in the theory which follows it is shown that fields which are neither uniform nor sharp edged and whose actual form is unknown can be used equally well, provided they satisfy certain conditions of symmetry.

I. Theory

CONSIDER a charged particle moving originally along the x-axis with a velocity v which encounters successively two identical alternating fields that are everywhere perpendicular to the x-axis. The origin is chosen at the center of the first field which lies entirely between $-a$ and $+a$. For a given value of x the field strength at any instant is $f(x)\psi(t)$. The wave form of the alternating field may be written

$$\psi(t)=\sum_{r=1}^{r=\infty}C_r\sin(2r\pi\nu t+\phi_r)$$

where ϕ_r is the phase of the rth harmonic as the particle crosses the y-axis and ν is the fundamental frequency. Between $-a$ and $+a$ we may express $f(x)$ as a Fouriers series

$$f(x)=B_0/2+\sum_{q=1}^{q=\infty}B_q\cos(q\pi x/a)+\sum_{p=1}^{p=\infty}A_p\sin(p\pi x/a)$$

The acceleration and y velocity component of the particle of mass m and charge e are respectively

$$\frac{d^2y}{dt^2}=\frac{e}{m}f(x)\psi(t) \text{ and } \frac{dy}{dt}=\frac{e}{m}\int f(x)\psi(t)dt \tag{1}$$

Since the particle has a uniform x velocity component we may write $x=vt$, $dx=vdt$, so that on emerging from the first field

$$\frac{dy}{dt}=\frac{e}{mv}\int_{-a}^{+a}f(x)\psi\left(\frac{x}{v}\right)dx$$

Let $k=2\pi\nu/v$ and substitute values given above for $f(x)$ and $\psi(x/v)$. This gives

$$\frac{dy}{dt}=\frac{e}{mv}\sum_{r=1}^{r=\infty}C_r\int_{-a}^{+a}\left(B_0/2+\sum_{q=1}^{q=\infty}B_q\cos(q\pi x/a)+\sum_{p=1}^{p=\infty}A_p\sin(p\pi x/a)\right)\sin(rkx+\phi_r)dx$$

Integrating this equation gives

$$\frac{dy}{dt}=\frac{e}{mv}\sum_{r=1}^{r=\infty}C_r\left(\left\{\frac{B_0}{rk}+\sum_{q=1}^{q=\infty}(-1)^qB_q\frac{2rka^2}{r^2k^2a^2-q^2\pi^2}\right\}\sin rka\sin\phi_r\right.$$

$$\left.+\sum_{p=1}^{p=\infty}(-1)^pA_p\frac{2p\pi a}{r^2k^2a^2p^2\pi^2}\sin rka\cos\phi_r\right) \tag{2}$$

When $ka=n\pi$, n being an integer, all terms are zero except those for which $rn=q=p$. Let $rn=u$ and the above expression becomes

$$\frac{dy}{dt}=\frac{ae}{mv}\sum_{r=1}^{r=\infty}C_r(B_pu\sin\phi_r+A_pp\cos\phi_r)\frac{2(-1)^p}{\pi}\frac{\sin u\pi}{u^2-p^2}$$

Let $u\rightarrow p$ and substitute $ae/mv=n\pi e/kmv=ne/2mv$ and we have

$$\frac{dy}{dt}=\frac{ne}{2mv}\sum_{r=1}^{r=\infty}C_r(B_{rn}\sin\phi_r+A_{rn}\cos\phi_r) \tag{3}$$

The Fouriers coefficients B_{rn} and A_{rn} are

$$B_{rn}=\frac{1}{a}\int_{-a}^{+a}f(x)\cos\frac{rn\pi x}{a}dx \text{ and } A_{rn}=\frac{1}{a}\int_{-a}^{+a}f(x)\sin\frac{rn\pi x}{a}dx.$$

If we make $f(x-a)=f(x)$ and if rn is an odd integer then both B_{rn} and A_{rn} are zero because

$$f(x-a)\cos\frac{rn\pi(x-a)}{a}=-f(x)\cos\frac{rn\pi x}{a}$$

and

$$f(x-a)\sin\frac{rn\pi(x-a)}{a}=-f(x)\sin\frac{rn\pi x}{a}$$

Thus particles emerging from the first field will have no y velocity component if there are only odd harmonics in the alternating field, if $f(x-a)=f(x)$, and if the velocity of the particle is

$$v_0=2a\nu/n \tag{4}$$

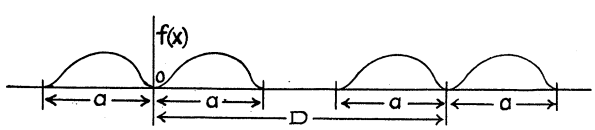

Fig. 1.

where ν is the frequency of the alternating field, $2a$ is the length of the field, and n is an odd integer. A form of $f(x)$ which meets the above requirements is shown in Fig. 1.

Displacement in Fields

The y velocity component at a point x in the first field is, from Eq. (1),

$$\frac{dy}{dt}=\frac{e}{mv}\sum_{r=1}^{r=\infty}C_r\int_{-a}^{x}\left(B_0/2+\sum_{q=1}^{q=\infty}B_q\cos(q\pi x/a)\right.$$
$$\left.+\sum_{p=1}^{p=\infty}A_p\sin(p\pi x/a)\right)\sin(rkx+\phi_r)dx$$

The displacement on emerging from the first field will be

$$Y=\frac{1}{v}\int_{-a}^{+a}\frac{dy}{dt}\,dx$$

Integrating this twice we obtain finally

$$Y=\frac{a^2e}{mv^2}\sum_{r=1}^{r=\infty}C_r\left\{\begin{array}{l}\dfrac{B_0}{r^2k^2a^2}(-\sin rka\cos\phi_r+rka\cos(rka-\phi_r))\\ +\displaystyle\sum_{q=1}^{q=\infty}(-1)^q2B_q\left[\frac{1}{r^2k^2a^2-q^2\pi^2}\left(\frac{-(r^2k^2a^2+q^2\pi^2)}{r^2k^2a^2-q^2\pi^2}\sin rka\cos\phi_r\right.\right.\\ \qquad\left.\left.+rka\cos(rka-\phi_r)\right)\right]\\ +\displaystyle\sum_{p=1}^{p=\infty}(-1)^p2A_p\left[\frac{1}{r^2k^2a^2-p^2\pi^2}\left(\frac{2rka\,p\pi}{r^2k^2a^2-p^2\pi^2}\sin rka\sin\phi_r\right.\right.\\ \qquad\left.\left.+p\pi\sin(rka-\phi_r)\right)\right]\end{array}\right\}\quad(5)$$

When $ka=n\pi$ all the first terms in the above square brackets drop out except those for which $rn=q=p$. In this case let $rn=u$ and let $u\rightarrow p$. Evaluating the indeterminate form gives eventually the following expression in which $p\neq rn$, $q\neq rn$, $ka=n\pi$, and rn is odd.

$$Y=\frac{a^2e}{mv^2}\sum_{r=1}^{r=\infty}C_r\left\{\begin{array}{l}-\left(\dfrac{B_0}{r^2n^2\pi^2}+\displaystyle\sum_{q=1}^{q=\infty}(-1)^q\frac{2B_q}{(r^2n^2-q^2)\pi^2}\right)rn\pi\cos\phi_r\\ \qquad+\displaystyle\sum_{p=1}^{p=\infty}(-1)^p\frac{2A_p}{(r^2n^2-p^2)\pi^2}\,p\pi\sin\phi_r\\ +\dfrac{B_{rn}}{2rn\pi}(\cos\phi_r+2rn\pi\sin\phi_r)+\dfrac{A_{rn}}{2rn\pi}(\sin\phi_r+2rn\pi\cos\phi_r)\end{array}\right\}$$

But we have chosen rn odd and made $f(x-a)=f(x)$ so that B_{rn} and A_{rn} are zero and odd values of q and p drop out, since for them B_q and A_p are also zero. Substituting the value $a^2e/\pi mv^2=(nv/2\nu)^2\ e/\pi mv^2=n^2e/4\pi m\nu^2$ gives the equation

$$Y=\frac{n^2e}{4\pi mv^2}\sum_{r=1}^{r=\infty}C_r\left\{-\left(\frac{B_0}{rn}+\sum_{q=2}^{q=\infty}\frac{2rnB_q}{r^2n^2-q^2}\right)\cos\phi_r+\sum_{p=2}^{p=\infty}\frac{2pA_p}{r^2n^2-p^2}\sin\phi_r\right\} \quad (6)$$

Where there appear only even values of q and p and odd values of r and n.

Final Displacement

Suppose the charged particles pass through a second field identical with the first but with its center at a distance D from the origin. Consider the particle having the velocity $v_0=2a\nu/n$ which passed the origin when the phase of the rth harmonic was ϕ_r. It passes D when the phase of this harmonic is ϕ_r'. Since it traveled parallel to the x-axis between fields its final displacement is

$$Y_f=\frac{n^2e}{4\pi mv^2}\sum_{r=1}^{r=\infty}C_r\left\{-\left(\frac{B_0}{rn}+\sum_{q=2}^{q=\infty}\frac{2rnB_q}{r^2n^2-q^2}\right)(\cos\phi_r+\cos\phi_r')\right.$$
$$\left.+\sum_{p=2}^{p=\infty}\frac{2pA_p}{r^2n^2-p^2}(\sin\phi_r+\sin\phi_r')\right\}$$

For the rth harmonic this will be zero if $\phi_r'=\pi s_r+\phi_r$ where s_r is any odd integer. This condition is satisfied if $D=s_ra/rn$. We can choose $s_r=rs$ where s is an odd integer and it will then be satisfied for all odd harmonics at once since

$$D=rsa/rn=sa/n \quad (7)$$

Other Velocities Transmitted

Certain other velocities will also be transmitted for, with a given setting, $D/a=s/n=k$ and if any other odd integers, m and p, can be found such that $k=m/p$ then the corresponding velocity, $v=2a\nu/p$ will pass. If s and n are incommensurable then p must be an odd multiple of s so that the other velocities which pass the filter will be $v'=v/h$ where h is an odd integer. These velocities are all less than v. If the alternating field is sufficiently strong so that the spread of the velocity between the fields is the maximum permitted by diaphrams in the apparatus then any lower velocity v' will generally have a wider spread and be cut out partially by the diaphrams.

Velocities Differing Slightly from v_0

We will now find the displacement of a particle with the velocity $v=v_0+\Delta v$ at a distance d beyond the end of the second field. Since the easiest experimental arrangement makes $f(x)=f(-x)$ we will consider this case, leaving only cosine terms in our Fouriers Series. If the time of passage from $x=a$ to $x=D+a$ is τ_1 and the time from $x=D+a$ to

$x=D+a+d$ is τ_2; if the y velocity component on leaving the first field is $(dy/dt)_1$ and on leaving the second field it is $(dy/dt)_2$ and if the displacement in the first field is y_1 and in the second y_2; then the displacement when $x=D+a+d$ is

$$y'=\tau_1(dy/dt)_1+y_1+y_2+\tau_2(dy/dt)_2=y_0+y_1+y_2+y_3$$

Starting with Eq. (2), expanding trigonometric functions in terms of the small angles $\Delta v/v_0$, neglecting powers of $\Delta v/v_0$ higher than the second and neglecting $\Delta v/v_0$ compared with one in the coefficient of q we obtain y_0. By a somewhat similar procedure, starting with Eq. (5) we obtain y_1 and y_2. We get y_4 in a manner similar to y_0. Adding the results gives

$$y'=\frac{nes}{4mv^2}\sum_{r=1}^{r=\infty}C_r\left\{\begin{aligned}&B_0\left[\left(\frac{r\pi(2n-s+2nd/a)}{2}\cos\phi_r-3\sin\phi_r\right)\left(\frac{\Delta v}{v_0}\right)^2\right.\\&\left.+2\sin\phi_r\left(\frac{\Delta v}{v_0}\right)\right]\\&+\sum_{q=2}^{q=\infty}\frac{2r^2n^2B_q}{r^2n^2-q^2}\left[\left(\frac{-3r^2n^2+5q^2}{r^2n^2-q^2}\cdot\sin\phi_r\right.\right.\\&\left.\left.+\frac{r\pi(2n-s+2nd/a)}{2}\cos\phi_r\right)\left(\frac{\Delta v}{v_0}\right)^2+2\sin\phi_r\left(\frac{\Delta v}{v_0}\right)\right]\end{aligned}\right\}$$

Since $(\Delta v/v_0)^2$ is very small the $(\Delta v/v_0)^2$ term is negligible except when $\sin\phi_r$ is of the order of $(\Delta v/v_0)$. In this case the $\sin\phi_r$ term in the coefficient of $(\Delta v/v_0)^2$ may be neglected since it is of the order of $(\Delta v/v_0)^3$, so we have finally

$$y'=\frac{nes}{4mv^2}\sum_{r=1}^{r=\infty}C_r\left[\left(B_0+\sum_{q=2}^{q=\infty}\frac{2r^2n^2B_q}{r^2n^2-q^2}\right)\left(\frac{r\pi(2n-s+2nd/a)}{2}\cos\phi_r\left(\frac{\Delta v}{v_0}\right)^2\right.\right.$$
$$\left.\left.+2\sin\phi_r\left(\frac{\Delta v}{v_0}\right)\right)\right] \tag{8}$$

In our further considerations we will assume that only the fundamental or one of the harmonics is present. In this case Eq. (8) takes the form

$$y'=\frac{Anes}{4mv^2}\left(B_0+\sum_{q=2}^{q=\infty}\frac{2r^2n^2B_q}{r^2n^2-q^2}\right)\left(\frac{r\pi(2n-s+2nd/a)}{2}\cos\phi\left(\frac{\Delta v}{v_0}\right)^2\right.$$
$$\left.+2\sin\phi\left(\frac{\Delta v}{v_0}\right)\right) \tag{8'}$$

where A is the maximum value of the field. From Eq. (6) we know that the maximum width of the beam between the fields is

$$W=2Y_{\max}=\frac{Anes}{4mv^2}\frac{2}{\pi rs}\left(B_0+\sum_{q=2}^{q=\infty}\frac{2r^2n^2B_q}{r^2n^2-q^2}\right) \tag{9}$$

Substituting this value in (8′) gives

$$y'=\frac{Wsr\pi}{2}\left(\frac{r\pi(2n-s+2nd/a)}{2}\cos\phi\left(\frac{\Delta v}{v_0}\right)^2+2\sin\phi\left(\frac{\Delta v}{v_0}\right)\right) \tag{10}$$

Intensity Considerations

In practice the beam will not be confined to the infinitely narrow X-axis but will have a finite width y_0. Hence there will be in the emergent beam, which we will also confine by a slit to a width y_0 along the x-axis, a certain distribution about the velocity v_0. For an apparatus of given dimensions we may compute the relative number which will emerge with any velocity $v=v_0+\Delta v$. Let y_0 be the width of the slit, i.e. the maximum value which y' can have and pass. Make the substitutions

$$\frac{\Delta v}{v_0}=\frac{y_0}{Wsr\pi}\sqrt{\frac{-1+\sqrt{1+4R^2\psi^2}}{2R^2}} \text{ where } R=\frac{(2n-s+2nd/a)y_0}{4Ws}$$

then Eq. (10) becomes

$$y'/y_0=R\left(\frac{-1+\sqrt{1+4R^2\psi^2}}{2R^2}\right)\cos\phi+\sqrt{\frac{-1+\sqrt{1+4R^2\psi^2}}{2R^2}}\sin\phi \tag{11}$$

Let $\tan\theta=(1/\sqrt{2})\sqrt{-1+\sqrt{1+4R^2\psi^2}}$ and this takes the form

$$y'/y_0=\psi\sin(\phi+\theta) \tag{12}$$

The displacement y' is always a maximum when $\sin(\phi+\theta)=1$. When this maximum is less than y_0 all values of ϕ contribute to the emergent intensity, but when it exceeds y_0 only certain values of ϕ contribute. The point where this change occurs is

$$\psi_m=\pm 1 \text{ or } \left(\frac{\Delta v}{v_0}\right)_m=\pm\frac{y_0}{Wsr\pi}\sqrt{\frac{-1+\sqrt{1+4R^2}}{2R^2}} \tag{13}$$

If I_0 is the intensity of the transmitted beam of particles with the velocity $v_0=2a\nu/n$, then the intensity of any other value of $\Delta v/v_0$ and ϕ will be

$$I_{(\Delta v/v_0,\phi)}=\frac{y_0-|y'|}{y_0}I_0=\left(1-\frac{|y'|}{y_0}\right)I_0$$

Where $|y'|$ is the absolute value of the displacement for a given $\Delta v/v_0$ and ϕ. It is assumed that the entering beam is equally intense in all velocities and phases but enters parallel to the x-axis. $I_{(\Delta v/v_0,\,\phi)}$ is zero when $|y'|/y_0>1$. To get the value of $I_{(\Delta v/v_0)}$ we must sum up for all values of ϕ which permit that value of $\Delta v/v_0$ to pass the filter. From Eq. (13) all values of ϕ contribute when $\psi\leqq 1$. The expression for $I_{(\Delta v/v_0)}$ in this region is then

$$I_{(\Delta v/v_0)}=\frac{I_0}{\pi}\int_{-\theta}^{\pi-\theta}(1-|\psi\sin(\phi+\theta)|)d\phi=\left(1-\frac{2|\psi|}{\pi}\right)I_0 \tag{14}$$

When $\psi\geqq 1$ only values of ϕ from $-\theta$ to ϕ_1 and from $\pi-2\theta-\phi_1$ to $\pi-\theta$ contribute. Here ϕ_1 has the value $\phi=\sin^{-1}(1/\psi)-\theta$ so that

$$I_{(\Delta v/v_0)}=\frac{2I_0}{\pi}\int_{-\theta}^{\phi_1}(1-|\psi\sin(\phi+\theta)|)d\phi$$

$$=\frac{2}{\pi}\left(\sin^{-1}\frac{1}{\psi}-\psi\left(1-\sqrt{1-\frac{1}{\psi^2}}\right)\right)I_0 \tag{15}$$

This expression can be expanded, giving

$$I=\frac{1}{\pi\psi}\left(1+\frac{1}{3\cdot 4}\frac{1}{\psi^2}+\frac{1\cdot 3}{4}\frac{1}{5\cdot 6}\frac{1}{\psi^4}+\frac{1\cdot 3\cdot 5}{4\cdot 6}\frac{1}{7\cdot 8}\frac{1}{\psi^6}+\cdots\right)I_0$$

$$=(\pi\psi)^{-1}(1+.0833\psi^{-2}+.0250\psi^{-4}+.0112\psi^{-6}+.0061\psi^{-8}+\cdots)I_0$$

From this a table of I/I_0 in terms of ψ can be computed.

ψ	0	.5	1.0	1.5	2.0	3.0	5.0	8.0	12.0
I/I_0	1.000	.682	.363	.221	.163	.107	.064	.040	.027

The value of $\Delta v/v_0$ corresponding to any of the above values of I/I_0 can now readily be found from

$$\frac{\Delta v}{v_0}=\frac{\pm y_0}{Wsr\pi}\sqrt{\frac{-1+\sqrt{1+4R^2\psi^2}}{2R^2}}=\frac{\pm y_0\psi}{Wsr\pi}\sqrt{1-R^2\psi^2+\cdots}$$

In most cases R can be neglected. At least it can be considered constant for a given apparatus in which the ratio a/D does not vary greatly and the second slit is at the end of the second field ($d=0$) for then

$$R=\frac{(2n-s+2nd/a)y_0}{4Ws}=\frac{(2a/D-1)y_0}{4W}$$

D, a, y_0 and W are all dimensions in the apparatus. If R is neglected $\Delta v/v_0$ can be computed from the formula

$$\frac{\Delta v}{v_0}=\frac{y_0\psi}{Wsr\pi} \tag{16}$$

The distribution of velocities emerging, computed from Eqs. (14), (15) and (16) are shown in Fig. 2. The dimensions of the apparatus now set up are used. These are $a=4.9$ cms, $D=24.5$ cms to 27.5 cms, $y_0=.02$ cms and $W=.3$ cms. The ordinates represent the relative intensity and each abscissa scale gives the corresponding value of $\Delta v/v_0$ when the n is chosen as indicated in the vertical column of figures.

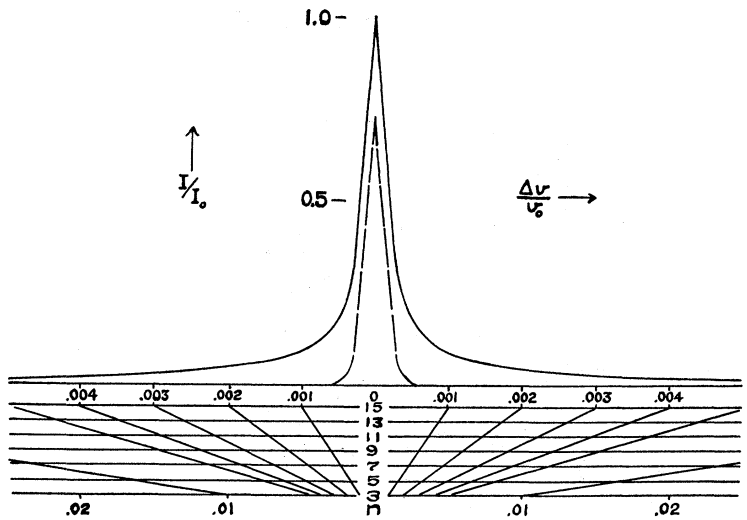

Fig. 2.

We have assumed heretofore that the entire beam of width W is permitted to pass between the two fields. An increase in resolving power with only a small loss in intensity can be obtained by cutting off the outer edges of this beam with a diaphram. If the width passed is W_0 we can see from Eqs. (6) and (9) that those particles which enter in the phase ϕ will be eliminated if $W_0/W<\cos\phi<1$. The effect of this on the intensity distribution curve can be found by modifying Eqs. (14) and (15). We neglect R (and hence θ) and let $\cos\phi_0=W_0/W$ so that when $\psi\leqq 1$ we have instead of Eq. (14):

$$I=\frac{I_0}{\pi}\int_{\phi_0}^{\pi-\phi_0}(1-|\psi\sin\phi|)d\phi=\left(1-\frac{2}{\pi}\left(\cos^{-1}\frac{W_0}{W}+\frac{|\psi|W_0}{W}\right)\right)I_0 \tag{17}$$

When $1 \leqq \psi \leqq 1/\sqrt{1-(W_0/W)^2}$ we have instead of Eq. (15):

$$I = \frac{2I_0}{\pi}\int_{\phi_0}^{\phi_1} (1-|\psi \sin\phi|)d\phi$$

$$= \frac{2}{\pi}\left(\sin^{-1}\frac{1}{\psi} - \cos^{-1}\frac{W_0}{W} + \psi\left(\sqrt{1-\frac{1}{\psi^2}} - \frac{W_0}{W}\right)\right) I_0 \qquad (18)$$

When $1/\sqrt{1-(W_0/W)^2} \leqq \psi$, $I=0$.

The distribution curve obtained by letting $W_0/W = .9$ is shown by the broken curve in Fig. 2.

If the above dimensions are used and the frequency applied is $\nu = 3\times 10^{-6}$ 1/sec, corresponding to a radio wave-length of 100 meters, we can cover the following voltage ranges of positive rays by choosing proper values of n as shown.

n	Positive ray with velocity v_0					
1	437 volt	H^+	to	1748 volt	He^+
3	193 "	He^+	to	770 "	0^+
5	208 "	C^+	to	781 "	Sc^+
7	204 "	Na^+	to	753 "	Rb^+
9	208 "	K^+	to	737 "	Ba^+
11	209 "	Ni^+	to	750 "	Bi^+
13	203 "	Br^+	to	612 "	U^+
15	206 "	Ag^+	to	458 "	U^+

For a given choice of n the alternating potential used to obtain the theoretical distribution will be proportional to the accelerating potential required to give the charged particles the velocity v_0. The factor of proportionality can only be found experimentally since the form of $f(x)$ is unknown.

The chief source of error in the above theory lies in the assumption that "the field is everywhere perpendicular to the x-axis." Clearly this can only be strictly true along the axis itself. At other points there will be a longitudinal component retarding or accelerating the particle along the x-axis. The actual computation of this effect involves assumptions as to the nature of the field which are uncertain and depend on the individual apparatus. If the curvature of the field is sufficiently great the velocity with which a particle leaves the first field parallel to the x-axis will vary appreciably from v_0 for those values of the phase which correspond to large displacements. Only those particles which have the velocity v_0 will reach the second field in the correct phase to reverse the displacement and thus pass the slit. Also, in general, those particles which do not travel between fields parallel to the x-axis will be lost. Therefore, the chief effect of the curvature of the field, will be to weaken

the intensity of the emergent beam rather than to destroy its purity. Similar considerations hold for most of the accidental errors such as irregularities in the dimensions or positions of condensers, errors in the setting of D and so forth.

II. Experimental Arrangement

The experimental set up required to meet the conditions of the theory is obvious. We must apply an alternating potential of frequency ν to four condensers in parallel. These four will be grouped in two pairs. In each pair the distance between the centers of the condensers is a, and the distance between the pairs, center to center, is D. We must put the condensers plates far enough apart so that the curvature of the lines of force in the region used is negligible. All condensers must be of identical construction and must be placed in identical chambers in which are suitable apertures to permit the beam to pass through. Since it is desirable to be able to use different values of n for different regions the distance D should be adjustable.

III. Applications of the Velocity Filter

The more obvious applications of the velocity filter are:

1. It can be used instead of a magnetic field to analyze the velocities in a beam of similarly charged particles of equal mass.
2. It can be used to determine the relative masses present in a stream of similarly charged atoms or molecules which have fallen through the same potential.
3. Combined with electrostatic or magnetic deflection methods it can be used to find the masses of the constituents of a stream of positive rays containing all masses and all velocities. For precision work this possesses certain decided advantages to be discussed later.

The second application mentioned above has already proved successful. Dr. Klein, in this laboratory, completed last fall a velocity filter which he uses to identify the positive ions from the various thermionic emitters with which he works in his study of the secondary electron emission from positive ion bombardment. He varies the frequency applied to the filter until a maximum current passes through. A wavemeter and a voltmeter reading give the frequency and the accelerating potential and e/m is easily computed. The method seems quite convenient and accurate, the apparatus working successfully at the first trial.

Dr. Mattauch and the author have just set up a filter of higher resolving power which may be used as the one just mentioned or as in application three above. In the latter case it sorts out one velocity from a

heterogeneous beam of positive rays. The frequency determining the set of possible velocities can be kept constant with a piezoelectric oscillator. We may attach to the filter a magnetic or electrostatic analyzing device. Magnetic analysis will be used where a photographic record is desirable as in exploring work. We deflect our beam in a semicircle by means of a magnet with semidisk polepieces 25 cm in diameter, receiving them on a photographic plate. If positive ions are to make much impression on a photographic plate they must have a velocity of 5000 volts or greater. This would require an inconveniently high frequency on the filter and also, for heavy ions, a prohibitively powerful magnet. The difficulty is solved by applying 15,000 or 20,000 volts accelerating field immediately in front of the plate. At this point all particles, having completed a semicircle, are traveling parallel to each other and at right angles to the photographic plate, so a symmetrical accelerating field produces no distortion. It is unnecessary that this field be constant. The displacements on the plate are proportional to the radii of curvature in the magnetic field and these are directly proportional to the masses, since the velocity is constant. Thus we have a rigorously linear mass scale. All factors affect the image symmetrically so there is nothing to produce a relative shift between mass lines except inhomogeneities in the polepieces. The latter are of soft annealed steel run far below saturation and irregularities should be very small if they exist at all. They can be determined by comparing with the electrostatic method to be described next.

The electrostatic method may prove more satisfactory than the above for accurate work. The positive rays after passing the filter are deflected between two curved plates two mm apart and caught in a Faraday cylinder connected with an electrometer. Only a small range of masses passes between the plates with a given field but by altering the field successive masses are brought on the slit. The masses are directly proportional to the potential difference applied to the plates and can be compared as accurately as the potentials can be measured. All masses can be checked directly against oxygen, we hope with great precision, and our computations indicate that we should be able to determine the masses of most atoms within one hundredth of a unit of atomic weight. Such determinations would be of considerable interest in atomic theory.

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