

Social Preferences under Risk: Equality of Opportunity versus Equality of Outcome[†]

By KOTA SAITO*

X and Y are brothers who like to share everything, whenever possible. They are also both huge fans of football. One day, X finds an envelope in the park that contains just one ticket to the next Super Bowl. X agonizes over whether to (i) attend the game secretly, which would make him feel guilty; or (ii) tell his brother about it and suggest they flip a coin to decide who gets to go. From an ex ante viewpoint, flipping a coin means that the brothers' chances of getting the ticket will be equal. From an ex post viewpoint, however, if X loses the coin toss, he will definitely feel envy.

This parable shows that an individual could have a preference for two distinct notions of equality: *equality of opportunity* (i.e., equality of ex ante expected pay-offs) and *equality of outcome* (i.e., equality of ex post payoffs). Moreover, the individual could face a *trade-off* between the two notions of equality.

In the literature on social preferences, many theories have been proposed on the inequality aversion of an individual, but most of them study deterministic environments. (See Camerer 2003 and Fehr and Schmidt 2006 for surveys.) However, it is not obvious how to appropriately extend such preferences to risky environments. As Fudenberg and Levine (2012, p. 606) point out, “leading theories of outcome-based social preferences fail to reflect concerns for ex ante fairness when they are extended to lotteries using expected utility theory as is implicit in some of this work.”

In the social choice literature, several papers, including Diamond (1967), Machina (1989), and Gajdos and Maurin (2004), have studied the trade-off between the two notions of equality. However, they all focus on an impartial social planner. Much less attention has been devoted to studying a *partially self-interested* individual, who

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[†]Go to <http://dx.doi.org/10.1257/aer.103.7.3084> to visit the article page for additional materials and author disclosure statement(s).

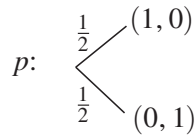


FIGURE 1. THE FIFTY-FIFTY LOTTERY (1 denotes getting the ticket and 0 denotes not getting it)

might feel envy or guilt, although this seems more germane to recent experimental evidence that shows a significance of equality of opportunity. (In this article, the terms guilt and envy refer to the utility losses from advantageous inequality and disadvantageous inequality, respectively.)

For instance, in experiments on *probabilistic dictator games*, subjects are asked to allocate probabilities of winning an indivisible good with a passive recipient. In such experiments, Karni, Salmon, and Sopher (2008); Bohnet et al. (2008); Kircher, Ludwig, and Sandroni (2013); and Bolton and Ockenfels (2010) found that a considerable number of subjects chose a probability of winning the good equally.

The purpose of this paper is to provide a step toward filling this gap. We axiomatize a model of an individual's inequality aversion under risk that captures a preference for equality of opportunity and equality of outcome. We call this model the *expected inequality-averse* (EIA) model. In the EIA model, a *unique* number captures an individual's preference for equality of opportunity *relative to* equality of outcome. This parameter determines how the individual behaves when he faces the trade-off between the two notions of equality.

The rest of this article is organized as follows. In Section I, we define the EIA model and show its uniqueness. In Section II, we provide an axiomatic foundation for the EIA model and a characterization of the parameters. In Section III, we show that the EIA model is consistent with the recent experiments on probabilistic dictator games. In Section IV, we discuss recent related literature, namely, Fudenberg and Levine (2012) and Borah (2011). Finally, in Section V, we discuss an extension of the EIA model. All proofs are in the Appendix.

I. The EIA Model

We assume that the set of payoffs is \mathbb{R} . Let $I = \{1, \dots, n\}$ be the set of individuals and let 1 denote the decision maker. A vector $\mathbf{x} = (x_1, \dots, x_n)$ in \mathbb{R}^n is called an *allocation* of payoffs among the individuals. A probability distribution p on \mathbb{R}^n with finite support is called a *lottery*. The set of all lotteries is denoted by $\Delta(\mathbb{R}^n)$. The primitive of our model is a binary relation \succsim on $\Delta(\mathbb{R}^n)$ that describes the preference of the decision maker (i.e., agent $1 \in I$). As usual, \succ and \sim denote the asymmetric and symmetric parts of \succsim , respectively. The following five axioms characterize the EIA model. The first axiom is standard.

For any $p \in \Delta(\mathbb{R}^n)$, E_p is the expectation operator with respect to p . That is, if p is a lottery that yields allocation $\mathbf{x}^s = (x_1^s, \dots, x_n^s)$ with probability λ_s for each $s \in \{1, \dots, m\}$, then $E_p(\mathbf{x}) = \sum_{s=1}^m \lambda_s \mathbf{x}^s = (\sum_{s=1}^m \lambda_s x_1^s, \dots, \sum_{s=1}^m \lambda_s x_n^s)$. In addition, for any real-valued function U on \mathbb{R}^n , $E_p(U(\mathbf{x})) = \sum_{s=1}^m \lambda_s U(\mathbf{x}^s)$.

Definition: An *expected inequality-averse (EIA)* model is a triple $(\alpha, \beta, \delta) \in \mathbb{R}_+^{n-1} \times \mathbb{R}_+^{n-1} \times [0, 1]$ such that \succsim is represented by the function V on $\Delta(\mathbb{R}^n)$ defined by

$$V(p) = \delta U(E_p(\mathbf{x})) + (1 - \delta) E_p(U(\mathbf{x})),$$

where $U(\mathbf{x}) = x_1 - \sum_{i=2}^n (\alpha_i \max\{x_i - x_1, 0\} + \beta_i \max\{x_1 - x_i, 0\})$.

The function U corresponds to the model proposed by Fehr and Schmidt (1999) and captures inequality aversion as follows: for each $i \neq 1$, the term $\alpha_i \max\{x_i - x_1, 0\}$ captures the disutility of *envy* when the decision maker’s payoff x_1 is smaller than individual i ’s payoff x_i ; the term $\beta_i \max\{x_1 - x_i, 0\}$ captures the disutility of *guilt* when the decision maker’s payoff x_1 is larger than individual i ’s payoff x_i .

With this function U , the EIA model captures preferences for the two notions of equality as follows. The term $U(E_p(\mathbf{x}))$ represents a preference for equality of *opportunity* because it captures inequality aversion in the *ex ante expected allocations* (i.e., $E_p(\mathbf{x})$). In contrast, the term $E_p(U(\mathbf{x}))$ represents a preference for equality of *outcome* because it captures expected inequality aversion in the *ex post allocations* (i.e., \mathbf{x}). Therefore, the weight δ on the first term captures a preference for equality of opportunity relative to equality of outcome.

Remark: Suppose that two EIA models (α, β, δ) and $(\alpha', \beta', \delta')$ represent \succsim . Then, (i) $(\alpha, \beta) = (\alpha', \beta')$ and (ii) if $(\alpha, \beta) \neq \mathbf{0} \neq (\alpha', \beta')$, $\delta = \delta'$.

In the remark, result (i) means that α and β are uniquely pinned down for the decision maker. Result (ii) means that δ is also uniquely pinned down when the decision maker is inequality averse (i.e., when $(\alpha, \beta) \neq \mathbf{0}$). When the decision maker is not inequality averse (i.e., $(\alpha, \beta) = \mathbf{0}$), his utility equals to his expected payoff (i.e., $V(p) = E_p(x_1)$) because $U(E_p(\mathbf{x})) = E_p(x_1) = E_p(U(\mathbf{x}))$. Hence, δ becomes irrelevant to the decision maker’s utility.

II. Axioms

Axiom (Rationality): \succsim is complete, transitive, continuous, and monotonic in equal allocations (i.e., $(x, \dots, x) \succsim (y, \dots, y)$ if and only if $x \geq y$).¹

In the next axiom, conditions (i) and (ii) capture envy and guilt, in that order. In the axiom, $(x, (y)_{-i})$ denotes allocation that gives payoff x only to individual i and gives payoff y to all the other individuals. Remember that $1 \in I$ denotes the decision maker.

Axiom (Inequality Aversion): For all $i \neq 1$, (i) $(0, (0)_{-i}) \succsim (1, (0)_{-i})$ and (ii) $(0, (0)_{-i}) \succsim (-1, (0)_{-i})$.

Since these conditions are required only for $i \neq 1$, this axiom allows the standard preference that $\mathbf{x} \succsim \mathbf{y}$ if and only if $x_1 \geq y_1$.

¹The continuity assumed here is von Neumann-Morgenstern continuity with respect to outcome mixtures and probability mixtures. Formally, (i) for all $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^n$, if $\mathbf{x} \succ \mathbf{y}$ and $\mathbf{y} \succ \mathbf{z}$, then there exist λ and μ in $(0, 1)$ such that $\lambda \mathbf{x} + (1 - \lambda)\mathbf{z} \succ \mathbf{y}$ and $\mathbf{y} \succ \mu \mathbf{x} + (1 - \mu)\mathbf{z}$; and (ii) for all $p, q, r \in \Delta(\mathbb{R}^n)$, if $p \succ q$ and $q \succ r$, then there exist λ and μ in $(0, 1)$ such that $\lambda p \oplus (1 - \lambda)r \succ q$ and $q \succ \mu p \oplus (1 - \mu)r$.

To provide a weaker version of the independence axiom, we first define an *outcome mixture* of allocations as follows:

Definition: For all $\lambda \in [0, 1]$ and $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, $\lambda\mathbf{x} + (1 - \lambda)\mathbf{y} \in \mathbb{R}^n$ is an allocation such that $\lambda\mathbf{x} + (1 - \lambda)\mathbf{y} = (\lambda x_1 + (1 - \lambda)y_1, \dots, \lambda x_n + (1 - \lambda)y_n)$.

Inequality-averse preferences may violate the independence axiom with respect to outcome mixtures. For example, in (two-agent) deterministic dictator games, a significant number of subjects prefer allocating an endowment equally rather than consuming the whole endowment by themselves (i.e., $(\frac{1}{2}, \frac{1}{2}) \succ (1, 0)$). However, the same subjects often prefer consuming the whole endowment by themselves rather than giving it away to the receiver (i.e., $(1, 0) \succ (0, 1)$). It follows that $(\frac{1}{2}, \frac{1}{2}) \succ (1, 0) \succ (0, 1)$ but $\frac{1}{2}(1, 0) + \frac{1}{2}(0, 1) \equiv (\frac{1}{2}, \frac{1}{2})$, which violates the independence axiom.²

The source of the violation here is the fact that in the mixed allocations, the rankings of the outcomes are *opposite* (i.e., the other individual is better off than the decision maker in $(0, 1)$ and worse off in $(1, 0)$). We want to assume the independence axiom only when the mixture does not involve such opposite outcomes.

Definition: Two allocations $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ are said to be *quasi-comonotonic* if there is no $i \neq 1$ such that $x_i > x_1$ and $y_i < y_1$.

Note that if \mathbf{x} and \mathbf{y} are quasi-comonotonic, then the rankings of the payoffs of any individual with respect to the decision maker are *not* opposite between \mathbf{x} and \mathbf{y} .³

Axiom (Quasi-Comonotonic Independence for Outcome Mixture): For all $\lambda \in (0, 1]$ and $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^n$ such that the pairs \mathbf{x}, \mathbf{z} , and \mathbf{y}, \mathbf{z} are each quasi-comonotonic,

$$\mathbf{x} \succsim \mathbf{y} \Leftrightarrow \lambda\mathbf{x} + (1 - \lambda)\mathbf{z} \succsim \lambda\mathbf{y} + (1 - \lambda)\mathbf{z}.$$

These three axioms characterize Fehr and Schmidt’s (1999) model on \mathbb{R}^n (i.e., the set of degenerate lotteries on allocations). (See Lemma 1 in the Appendix for the formal statement.)

The next axiom is a weaker version of the independence axiom with respect to probability mixtures. Before stating the axiom, we formally define probability mixtures by using the notation \oplus :

Definition: For all $\lambda \in [0, 1]$ and $p, q \in \Delta(\mathbb{R}^n)$, $\lambda p \oplus (1 - \lambda)q \in \Delta(\mathbb{R}^n)$ is a lottery such that $(\lambda p \oplus (1 - \lambda)q)(\mathbf{x}) = \lambda p(\mathbf{x}) + (1 - \lambda)q(\mathbf{x}) \in [0, 1]$ for each $\mathbf{x} \in \mathbb{R}^n$.⁴

²Note that mixtures here are not probability mixtures.

³The definition is a weaker version of *comonotonicity* provided by Schmeidler (1989). Schmeidler (1989) presents an interpretation of comonotonicity from the viewpoint of a *social planner’s inequality aversion*, as follows: two allocations \mathbf{x} and \mathbf{y} are comonotonic if the social rank of *any two individuals* is not reversed between \mathbf{x} and \mathbf{y} . When we focus on an *individual’s inequality-averse preferences*, what is relevant to the individual is social rank *with respect to the individual himself*, not the social rank of *any two* individuals.

⁴For example, lottery p in Figure 1 corresponds to probability mixture $\frac{1}{2}(1, 0) \oplus \frac{1}{2}(0, 1)$. For simplicity, instead of writing $\lambda\delta_{\mathbf{x}} \oplus (1 - \lambda)\delta_{\mathbf{y}}$, we write $\lambda\mathbf{x} \oplus (1 - \lambda)\mathbf{y}$, where $\delta_{\mathbf{x}}$ and $\delta_{\mathbf{y}}$ denote the degenerate lotteries that yield \mathbf{x} and \mathbf{y} with probability 1, respectively.

Just as outcome mixtures may change the rankings of ex post payoffs, probability mixtures may change the rankings of ex ante *expected* payoffs. However, probability mixtures with an *equal allocation* (x, \dots, x) should *not* change any such rankings. This suggests the following axiom:

Axiom (Constant Independence for Probability Mixture): For all $\lambda \in (0, 1]$, $p, q \in \Delta(\mathbb{R}^n)$, and $x \in \mathbb{R}$,

$$p \succsim q \Leftrightarrow \lambda p \oplus (1 - \lambda)(x, \dots, x) \succsim \lambda q \oplus (1 - \lambda)(x, \dots, x).$$

Under the above four axioms, for any allocation \mathbf{x} , it will be shown that there exists a unique real number z such that $(z, \dots, z) \sim \mathbf{x}$. Such a payoff z is denoted by $e(\mathbf{x})$ and is called an *equality equivalent*. The value $e(\mathbf{x})$ reflects the equality of an allocation \mathbf{x} .

The final key axiom, *dominance*, states that if a lottery p is superior to a lottery q in regard to both equality of opportunity and equality of outcome, then p should be preferred to q . To show the axiom, we formalize the two ways to evaluate equality of a lottery based on the two notions of equality. Consider a lottery $p = \lambda_1 \mathbf{x}^1 \oplus \dots \oplus \lambda_m \mathbf{x}^m$. Then, $E_p(\mathbf{x}) = (\sum_{s=1}^m \lambda_s x_1^s, \dots, \sum_{s=1}^m \lambda_s x_n^s)$ is the expected allocation in p . Hence, $e(E_p(\mathbf{x}))$ reflects the *evaluation of p in terms of equality of opportunity*. In contrast, since each $e(\mathbf{x}^s)$ reflects the equality of outcome \mathbf{x}^s , $E_p(e(\mathbf{x})) = \sum_{s=1}^m \lambda_s e(\mathbf{x}^s)$ is the expected *evaluation of p in terms of equality of outcome*. By using these two formalizations, we can state the axiom as follows:

Axiom (Dominance): For all $p, q \in \Delta(\mathbb{R}^n)$,

$$\left\{ \begin{array}{l} \text{(i)} \quad e(E_p(\mathbf{x})) \geq e(E_q(\mathbf{x})) \\ \text{(ii)} \quad E_p(e(\mathbf{x})) \geq E_q(e(\mathbf{x})) \end{array} \right\} \Rightarrow p \succsim q.$$

To demonstrate how to use the dominance axiom, consider two lotteries $p = \frac{1}{2}(1, 1) \oplus \frac{1}{2}(0, 0)$ and $q = \frac{1}{2}(1, 0) \oplus \frac{1}{2}(0, 1)$. Suppose that the decision maker is strictly inequality averse (i.e., $(1, 1) \succ (1, 0)$ and $(0, 0) \succ (0, 1)$). On the one hand, $E_p(\mathbf{x}) = (\frac{1}{2}, \frac{1}{2}) = E_q(\mathbf{x})$, so that $e(E_p(\mathbf{x})) = \frac{1}{2} = e(E_q(\mathbf{x}))$. Hence, p and q are equally desirable in regard to equality of opportunity.

On the other hand, by the strict inequality aversion, $1 > e(1, 0)$ and $0 > e(0, 1)$. Since $e(1, 1) = 1$ and $e(0, 0) = 0$, it follows that $E_p(e(\mathbf{x})) = \frac{1}{2}e(1, 1) + \frac{1}{2}e(0, 0) = \frac{1}{2} > \frac{1}{2}e(1, 0) + \frac{1}{2}e(0, 1) = E_q(e(\mathbf{x}))$. Hence, p is preferred to q in regard to equality of outcome. Therefore, the dominance axiom implies that p is at least as good as q .⁵

⁵We do not require a strict version of the dominance axiom (i.e., if one of the inequalities in (i) and (ii) is strict then the preference should be strict). Indeed, the strict version does not allow two special EIA models where δ is either zero or one.

The above five axioms are equivalent to an EIA model. Formally,

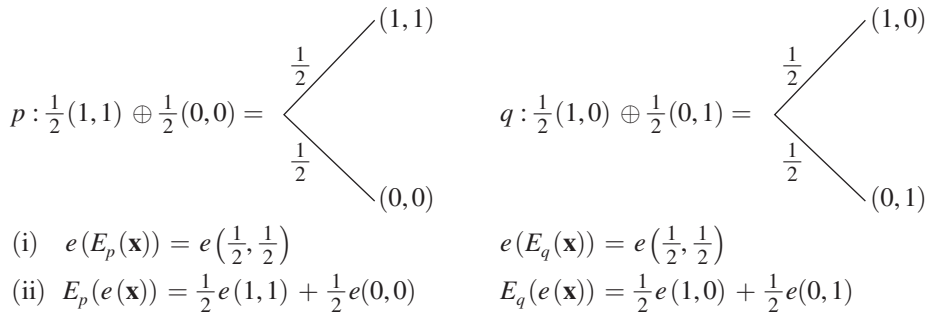


FIGURE 2. DOMINANCE

THEOREM: \succsim satisfies Rationality, Inequality Aversion, Quasi-Comonotonic Independence for Outcome Mixture, Constant Independence for Probability Mixture, and Dominance if and only if there exists an EIA model (α, β, δ) .

A. Parameter Characterization

We can identify the parameters $\alpha, \beta,$ and δ easily by experiments. We can identify α and β by eliciting real numbers α_i and β_i such that

$$(-\alpha_i, \dots, -\alpha_i) \sim (1, (0)_{-i}) \quad \text{and} \quad (-\beta_i, \dots, -\beta_i) \sim (-1, (0)_{-i}),$$

for each $i \in I \setminus \{1\}$.⁶ (Here, we use 1 and -1 just for simplicity. Instead of 1 and -1 , we could use any real numbers x and y such that $x > y$, which would be more appropriate for experiments.)⁷

After identifying α and β , we can identify δ by eliciting a real number c such that

$$(c, \dots, c) \sim \frac{1}{2} (1, (0)_{-i}) \oplus \frac{1}{2} (-1, (0)_{-i}),$$

for some $i \in I \setminus \{1\}$. Then, the EIA model shows that $c = -\frac{1}{2}(1 - \delta)(\alpha_i + \beta_i)$. Hence, $\delta = 1 + \frac{2c}{\alpha_i + \beta_i}$.⁸

In what follows, we characterize the special cases of the EIA model in which $\delta = 1$ and $\delta = 0$. The next axiom characterizes the special case of $\delta = 1$. The axiom means that the decision maker cares only about the expected payoffs.

⁶To see that α_i and β_i are nonnegative, note that by inequality aversion, $(0, (0)_{-i}) \succsim (1, (0)_{-i})$ and $(0, (0)_{-i}) \succsim (-1, (0)_{-i})$ for each $i \in I \setminus \{1\}$. Hence, by monotonicity, α_i and β_i are nonnegative.

⁷In that case, $\frac{x+y}{2}$ must be used instead of zero.

⁸To see that δ can be smaller than one, note that for inequality-averse subjects, it is natural to assume that $(0, (0)_{-i}) \succsim \frac{1}{2}(1, (0)_{-i}) \oplus \frac{1}{2}(-1, (0)_{-i})$. Hence, under monotonicity, c is negative. Thus, $\delta \leq 1$.

Axiom (Indifference to Timing of Mixture): For all $i \neq 1$,

$$(0, \dots, 0) \sim \frac{1}{2}(1, (0)_{-i}) \oplus \frac{1}{2}(-1, (0)_{-i}).$$

The other special case of $\delta = 0$ has been used in several papers such as Fehr, Klein, and Schmidt (2007) and Grund and Sliwka (2005). This special case is characterized by the axiom that states the decision maker is indifferent to probability mixtures.

Axiom (Indifference to Probability Mixture): For all $i \neq 1$ and $x, y \in \mathbb{R}$ such that $x > 0 > y$,

$$(x, (0)_{-i}) \sim (y, (0)_{-i}) \Rightarrow \frac{1}{2}(x, (0)_{-i}) \oplus \frac{1}{2}(y, (0)_{-i}) \sim (x, (0)_{-i}).$$

PROPOSITION 1: For any EIA model (α, β, δ) if $(\alpha, \beta) \neq \mathbf{0}$ then, the following holds:

- (i) \succsim exhibits indifference to timing of mixture if and only if $\delta = 1$.
- (ii) \succsim exhibits indifference to probability mixture if and only if $\delta = 0$.

PROOF:

Assume $\alpha_i > 0$ for some $i \neq 1$, without loss of generality. To prove (i), note that $V\left(\frac{1}{2}(1, (0)_{-i}) \oplus \frac{1}{2}(-1, (0)_{-i})\right) = -\frac{1}{2}(1 - \delta)(\alpha_i + \beta_i)$. Hence, $(0, (0)_{-i}) \sim \frac{1}{2}(1, (0)_{-i}) \oplus \frac{1}{2}(-1, (0)_{-i}) \Leftrightarrow \delta = 1$, so that (i) holds.

To prove (ii), choose any $x, y \in \mathbb{R}$ such that $x \succ 0 \succ y$ and $(x, (0)_{-i}) \sim (y, (0)_{-i})$. Then, $U(x, (0)_{-i}) = -\alpha_i x$ and $U(y, (0)_{-i}) = \beta_i y$. Consider the case where $\frac{1}{2}x + \frac{1}{2}y \geq 0$. In this case, $V\left(\frac{1}{2}(x, (0)_{-i}) \oplus \frac{1}{2}(y, (0)_{-i})\right) = -\delta\alpha_i\left[\frac{1}{2}x + \frac{1}{2}y\right] - (1 - \delta) \times \frac{1}{2}[\alpha_i x - \beta_i y]$. Then $V\left(\frac{1}{2}(x, (0)_{-i}) \oplus \frac{1}{2}(y, (0)_{-i})\right) - U(x, (0)_{-i}) = \delta\alpha_i\frac{1}{2}(x - y)$. Hence, $\frac{1}{2}(x, (0)_{-i}) \oplus \frac{1}{2}(y, (0)_{-i}) \sim (x, (0)_{-i}) \Leftrightarrow \delta = 0$. For the other case where $\frac{1}{2}x + \frac{1}{2}y \leq 0$, the result can be proved in the same way.

The above proposition shows that the two special cases have rather extreme implications for behavior. This would suggest that most of the subjects would exhibit the generic case where $\delta \in (0, 1)$.

III. Consistency with Experiments

In this section, we show that the EIA model can describe the experimental results in the probabilistic dictator games based on a tractable condition on the parameters (i.e., α , β , and δ). We investigate the probabilistic dictator game because this game provides a direct way to elicit preferences on lotteries over allocations. In addition, the game captures the essence of our parable of two brothers in the introduction.

In the probabilistic dictator games, a common experimental observation is that a substantial fraction of dictators share the probability of winning an indivisible good. For example, Kircher, Ludwig, and Sandroni (2013) asked 60 subjects to choose

among receiving one euro, donating one euro to the Red Cross, and randomizing between these two options with equal probability. Kircher, Ludwig, and Sandroni (2013) found that 25 percent of the subjects chose the equal randomization. See Karni, Salmon, and Sopher (2008); Bohnet et al. (2008); and Bolton and Ockenfels (2010) for similar experimental evidence.

To see that this experimental finding is consistent with the EIA model, consider a dictator whose preference is represented by the EIA model. The dictator is asked to decide a probability $\lambda \in [0, 1]$ that he will receive the indivisible good ex post. Since the recipient receives the good with the remaining probability $1 - \lambda$, the dictator obtains a lottery $p^\lambda \equiv \lambda(1, 0) \oplus (1 - \lambda)(0, 1)$.⁹ Thus, the utility obtained by choosing a probability $\lambda \in [0, 1]$ is as follows:¹⁰

$$(1) \quad V(p^\lambda) = \begin{cases} \delta(\lambda - \alpha(1 - 2\lambda)) + (1 - \delta)(\lambda(1 + \alpha - \beta) - \alpha) & \text{if } \lambda \leq \frac{1}{2}, \\ \delta(\lambda - \beta(2\lambda - 1)) + (1 - \delta)(\lambda(1 + \alpha - \beta) - \alpha) & \text{if } \lambda \geq \frac{1}{2}. \end{cases}$$

Under the commonly used assumption that the disutility from envy is larger than that from guilt (i.e., $\alpha > \beta$), the optimal λ^* (i.e., the maximizer of (1)) is characterized as follows:

PROPOSITION 2: *Suppose that $\alpha > \beta$. The optimal probability is*

$$\lambda^* = \begin{cases} \frac{1}{2} & \text{if } \delta \geq \frac{1 + \alpha - \beta}{\alpha + \beta}, \\ 1 & \text{if } \delta \leq \frac{1 + \alpha - \beta}{\alpha + \beta}. \end{cases}$$

PROOF:

Note that $\delta(\lambda - \alpha(1 - 2\lambda)) + (1 - \delta)(\lambda(1 + \alpha - \beta) - \alpha) - (\delta(\lambda - \beta(2\lambda - 1)) + (1 - \delta)(\lambda(1 + \alpha - \beta) - \alpha)) = -\delta(\alpha - \beta)(1 - 2\lambda)$. Since $\alpha > \beta$, therefore, $\lambda < \frac{1}{2}$ is not optimal.

For any $\lambda \in [\frac{1}{2}, 1]$, define $f(\lambda) \equiv V(p^\lambda) = (\alpha + \beta)\delta - \alpha + (1 + \alpha - \beta - \delta(\alpha + \beta))\lambda$. By the linearity, f attains the maximum at $\lambda = \frac{1}{2}$ or $\lambda = 1$. Moreover, $f(\frac{1}{2}) > f(1) \Leftrightarrow \frac{1}{2}[1 - (1 - \delta)(\alpha + \beta)] > 1 - \beta \Leftrightarrow \delta > \frac{1 + \alpha - \beta}{\alpha + \beta}$.

Therefore, the dictator shares the probability of winning the good if and only if $\delta \geq \frac{1 + \alpha - \beta}{\alpha + \beta}$. Note the similarity between the probabilistic dictator game and the

⁹This specification of the allocation is without loss of generality. We can obtain the same result with x and y such that $x > y$, instead of one and zero. By the linearity of V with respect to constant allocations, $V(\lambda(x, y) \oplus (1 - \lambda)(y, x)) = V(\lambda(x - y, 0) \oplus (1 - \lambda)(0, x - y)) + V(y, y) = (x - y)V(\lambda(1, 0) \oplus (1 - \lambda)(0, 1)) + V(y, y)$. This calculation shows that the maximizer λ for $V(\lambda(x, y) \oplus (1 - \lambda)(y, x))$ must be the same as the maximizer λ for $V(p^\lambda)$ for $x - y > 0$.

¹⁰By definition, $V(p^\lambda) = \delta(\lambda - \alpha \max\{1 - 2\lambda, 0\} - \beta \max\{2\lambda - 1, 0\}) + (1 - \delta)(\lambda(1 - \beta) + (1 - \lambda)(-\alpha))$. A direct calculation shows the equation for each case.

parable of the two brothers, in which X agonizes over whether to (i) attend the game secretly or (ii) flip a coin to decide who gets to go. The result implies that X should flip a coin (i.e., choose a lottery $p = \frac{1}{2}(1, 0) \oplus \frac{1}{2}(0, 1)$) if and only if $\delta > \frac{1 + \alpha - \beta}{\alpha + \beta}$. Given the values of α and β that are often assumed in the literature, this inequality condition is compatible with the restriction that $\delta \in [0, 1]$.¹¹

IV. Related Literature

In this section, we discuss the recent related studies, Fudenberg and Levine (2012); Borah (2011); and Rohde (2010). Fudenberg and Levine (2012) investigate a two-agent and two-state model, in which each state realizes equally likely. In the setup, Fudenberg and Levine (2012) show two main results. Their first result shows that the independence axiom (with respect to probability mixtures) is *incompatible* with a preference for equality of opportunity. This result is *consistent* with our theorem because the EIA model captures a preference for equality of opportunity if and only if $\delta > 0$ and $(\alpha, \beta) \neq \mathbf{0}$. Moreover, the EIA model also *violates* the independence axiom if and only if $\delta > 0$ and $(\alpha, \beta) \neq \mathbf{0}$.

The second result in Fudenberg and Levine (2012) shows that there are *no* preferences that satisfy all of the following four properties: transitivity, a preference for equality of outcome, state independence, and their Property 5 (Correlation Invariance), which states that $\frac{1}{2}(x_1, x_2) \oplus \frac{1}{2}(y_1, y_2) \sim \frac{1}{2}(x_1, y_2) \oplus \frac{1}{2}(y_1, x_2)$. Our theorem is also *consistent* with this result as well, because any EIA model satisfies transitivity and state independence. (The state independence is satisfied trivially because we assume objective probability without state space.) In addition, the EIA model captures equality of outcome if and only if $\delta < 1$ and $(\alpha, \beta) \neq \mathbf{0}$. Moreover, the EIA model *violates* Property 5 if and only if $\delta < 1$ and $(\alpha, \beta) \neq \mathbf{0}$. To see this latter equivalence, note that Property 5 implies the indifference between the two lotteries $p \equiv \frac{1}{2}(1, 1) \oplus \frac{1}{2}(0, 0)$ and $q \equiv \frac{1}{2}(1, 0) \oplus \frac{1}{2}(0, 1)$ in Figure 2, while the EIA model predicts that p is strictly preferred to q if and only if $\delta < 1$ and $(\alpha, \beta) \neq \mathbf{0}$.

Borah (2011) proposes a model of social preferences under risk, in which a function g describes a preference for equality of opportunity. The function g is defined *solely* on the *marginal distribution* p_2 of a lottery p on the other agent's allocations. Because of this feature, Borah's (2011) model implies unnatural preferences for equality of opportunity.¹² For instance, two lotteries $q \equiv \frac{1}{2}(1, 0) \oplus \frac{1}{2}(0, 1)$ and $l \equiv \frac{1}{2}(1, 0) \oplus \frac{1}{2}(1, 1)$ must be equally desirable in terms of equality of opportunity, because $q_2 = l_2$. (Both marginal distributions on the other agent's allocations are the same 50-50 lottery between zero and one.) However, it would be natural to say that the lottery q is equal in opportunity but the lottery l is unequal in opportunity,

¹¹ For $\alpha = 1$ and $\beta = 0.6$, which are assumed for 30 percent of the population in Fehr and Schmidt (1999), the condition is equivalent to $\delta \geq 0.875$. For $\alpha = 2$ and $\beta = 0.6$, which are assumed for 10 percent of the population in Fehr and Schmidt (1999), the condition is equivalent to $\delta \geq 0.93$.

¹² In a two-agent environment, Borah's (2011) model is $W(p) = (\sum_{\mathbf{x} \in \mathbb{R}^I} p(\mathbf{x})w(\mathbf{x})) + g(p_2)$, where $w(\mathbf{x})$ captures a preference for equality of outcome, and $g(p_2)$ captures a preference for equality of opportunity. He calls $\sum_{\mathbf{x} \in \mathbb{R}^I} p(\mathbf{x})w(\mathbf{x})$ *baseline expected utility evaluation* and $g(p_2)$ *adjustment*. He says "Such an adjustment may be needed because the expected utility evaluation only incorporates the decision maker's concern for end-outcomes and not opportunities."

because in q both agents have an equal chance to get one, but in l only the decision maker gets one with certainty.

In contrast, the EIA model captures a preference for equality of opportunity through the term $U(E_{p_1}(x_1), E_{p_2}(x_2))$. (Note that $E_p(x_i) = E_{p_i}(x_i)$ for each $i \in I$.) This term depends not only on p_2 but also on the *marginal distribution* p_1 of a lottery p on the decision maker's allocations. Because of this difference between the EIA model and Borah's (2011) model, *neither* model is a special case of the other. In fact, if $\delta > 0$ and $(\alpha, \beta) \neq \mathbf{0}$, then the EIA model predicts that the decision maker will feel disutilities from inequality in opportunity in the lottery $l \equiv \frac{1}{2}(1, 0) \oplus \frac{1}{2}(1, 1)$ but not in the lottery $q \equiv \frac{1}{2}(1, 0) \oplus \frac{1}{2}(0, 1)$.

In a deterministic environment, Rohde (2010) also obtains a characterization of Fehr and Schmidt's (1999) model. Rohde's (2010) axioms differ from ours.¹³ Rohde's (2010) results and ours were obtained simultaneously and independently. Moreover, since Rohde (2010) studies a deterministic environment, she does not consider the trade-off between the two notions of equality, which is the main issue in our article.

V. Concluding Remark

One important feature of the EIA model is that the model reduces into the function U proposed by Fehr and Schmidt (1999) on the set of deterministic allocations. To conclude the article, we discuss an extension to incorporate other models of social preferences.

To state the extension, first we need to define a property of a function U on \mathbb{R}^n . U is *constant linear*, if $U(\lambda \mathbf{x} + (1 - \lambda)(c, \dots, c)) = \lambda U(\mathbf{x}) + (1 - \lambda)U(c, \dots, c)$ for all $\mathbf{x} \in \mathbb{R}^n$, $c \in \mathbb{R}$, and $\lambda \in [0, 1]$. In Lemma 3 in the Appendix, we prove the following result. For any binary relation \succsim on $\Delta(\mathbb{R}^n)$, if (i) there exists a concave and constant linear function U on \mathbb{R}^n that represents \succsim on \mathbb{R}^n (i.e., $\mathbf{x} \succsim \mathbf{y} \Leftrightarrow U(\mathbf{x}) \geq U(\mathbf{y})$) and (ii) \succsim satisfies Rationality, Constant Independence for Probability Mixture, and Dominance, then there exists $\delta \in [0, 1]$ such that \succsim is represented by $V(p) \equiv \delta U(E_p(\mathbf{x})) + (1 - \delta)E_p(U(\mathbf{x}))$.

Note that Fehr and Schmidt's (1999) model satisfies the concavity and the certainty linearity. Charness and Rabin's (2002) model, $U^{CR}(\mathbf{x}) \equiv (1 - \lambda)x_1 + \lambda(\gamma \sum_{i \in I} x_i + (1 - \gamma)\min_{i \in I} x_i)$, also satisfies the two properties. Therefore, if the function U^{CR} represents \succsim on \mathbb{R}^n and \succsim satisfies the all properties in the condition (ii) above, then Lemma 3 shows that there exists $\delta \in [0, 1]$ such that \succsim is represented by $V(p) \equiv \delta U^{CR}(E_p(\mathbf{x})) + (1 - \delta)E_p(U^{CR}(\mathbf{x}))$.

APPENDIX: PROOF OF THEOREM AND REMARK

To prove the main theorem and the remark, we will prove three lemmas. Lemma 1 provides the representation theorem of the model proposed by Fehr and Schmidt (1999). Lemma 2 shows a representation theorem on a subset of \mathbb{R}^2 . By using

¹³Rohde (2010) assumes one axiom, Constant Equivalence, which we do not assume.

Lemma 2, Lemma 3 proves the result mentioned in Section V. The proofs of the theorem and the remark are in Section D.

A. Lemma 1

LEMMA 1: A preference \succsim on \mathbb{R}^n satisfies Rationality, Inequality Aversion, and Quasi-Comonotonic Independence for Outcome Mixture if and only if there exist $(\alpha, \beta) \in \mathbb{R}_+^{n-1} \times \mathbb{R}_+^{n-1}$ such that \succsim is represented by $U(\mathbf{x}) = x_1 - \sum_{i=2}^n (\alpha_i \max\{x_i - x_1, 0\} + \beta_i \max\{x_1 - x_i, 0\})$.

PROOF OF LEMMA 1:

The necessity of the axioms is trivial. In the following, we show the sufficiency. Fix \succsim that satisfies the axioms.

Step 1: There exists a utility function $U : \mathbb{R}^n \rightarrow \mathbb{R}$ such that (i) for all $\lambda \in [0, 1]$ and $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, if \mathbf{x} and \mathbf{y} are quasi-comonotonic, then $U(\lambda \mathbf{x} + (1 - \lambda)\mathbf{y}) = \lambda U(\mathbf{x}) + (1 - \lambda)U(\mathbf{y})$, (ii) for all $\mathbf{x} \in \mathbb{R}^n$ and $a \in \mathbb{R}_+$, $U(a\mathbf{x}) = aU(\mathbf{x})$, and (iii) U is unique up to positive affine transformation.

PROOF OF STEP 1:

Since an equal allocation (z, \dots, z) is quasi-comonotonic with any other allocations, Quasi-Comonotonic Independence for Outcome Mixture implies that for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, $z \in \mathbb{R}$, and $\lambda \in (0, 1)$, $\mathbf{x} \succsim \mathbf{y} \Leftrightarrow \lambda \mathbf{x} + (1 - \lambda)(z, \dots, z) \succsim \lambda \mathbf{y} + (1 - \lambda) \times (z, \dots, z)$. By the standard argument with the von Neumann-Morgenstern Theorem, the above property implies that there exists a utility function U on \mathbb{R}^n such that for all $\mathbf{x} \in \mathbb{R}^n$, $z \in \mathbb{R}$, and $\lambda \in (0, 1)$,

$$(A1) \quad U(\lambda \mathbf{x} + (1 - \lambda)(z, \dots, z)) = \lambda U(\mathbf{x}) + (1 - \lambda)U(z, \dots, z).$$

In addition, U is unique up to positive affine transformation, so that (iii) holds. Hence, we can normalize U by $U(1, \dots, 1) = 1$ and $U(-1, \dots, -1) = -1$, without loss of generality. Then, $U(0, \dots, 0) = 0$. Therefore, (A1) shows that for all $\mathbf{x} \in \mathbb{R}$ and $a \in [0, 1]$ such that $a \leq 1$, $U(a\mathbf{x}) = aU(\mathbf{x})$. For all $x \in \mathbb{R}$ and $a > 1$, $\mathbf{x} \sim \frac{1}{a}(a\mathbf{x}) + (1 - \frac{1}{a})(0, \dots, 0)$. Hence, $U(\mathbf{x}) = \frac{1}{a}U(a\mathbf{x})$, so that $U(a\mathbf{x}) = aU(\mathbf{x})$. Hence, (ii) holds.

To show (i), choose $\lambda \in [0, 1]$ and $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ such that \mathbf{x} and \mathbf{y} are quasi-comonotonic. By (ii), there exists a positive integer n such that $\frac{1}{n}U(\mathbf{x}) = U(\frac{1}{n}\mathbf{x}) \in [-1, 1]$. Since $[-1, 1] \subset \{U(z, \dots, z) \mid z \in \mathbb{R}\}$, there exists $z \in \mathbb{R}$ such that $(z, \dots, z) \sim \frac{1}{n}\mathbf{x}$. In addition, $\frac{1}{n}\mathbf{x}$ and $\frac{1}{n}\mathbf{y}$ are also quasi-comonotonic. Hence, Quasi-Comonotonic Independence for Outcome Mixture shows that $\lambda(\frac{1}{n}\mathbf{x}) + (1 - \lambda)(\frac{1}{n}\mathbf{y}) \sim \lambda(z, \dots, z) + (1 - \lambda)(\frac{1}{n}\mathbf{y})$. Therefore, by using (A1) and (ii), we obtain $\frac{1}{n}U(\lambda \mathbf{x} + (1 - \lambda)\mathbf{y}) = U(\lambda(\frac{1}{n}\mathbf{x}) + (1 - \lambda)(\frac{1}{n}\mathbf{y})) = U(\lambda(z, \dots, z) + (1 - \lambda)(\frac{1}{n}\mathbf{y})) = \lambda U(z, \dots, z) + (1 - \lambda)U(\frac{1}{n}\mathbf{y}) = \lambda(\frac{1}{n}U(\mathbf{x})) + (1 - \lambda)(\frac{1}{n}U(\mathbf{y}))$, so that (i) holds.

For all $i \neq 1$, define $\alpha_i = -U(1, (0)_{-i})$ and $\beta_i = -U(-1, (0)_{-i})$. By Inequality Aversion, $\alpha_i \geq 0$ and $\beta_i \geq 0$ for all $i \neq 1$. Fix any $\mathbf{x} \in \mathbb{R}^n$.

Step 2: Let $\bar{I} = \{i \in I \mid x_i > x_1\}$ and $\underline{I} = \{i \in I \mid x_i < x_1\}$. Then, (i) for all $i \in \bar{I}$, $U(x_i - x_1, (0)_{-i}) = -\alpha_i \max\{x_i - x_1, 0\}$, (ii) for all $i \in \underline{I}$, $U(x_i - x_1, (0)_{-i}) = -\beta_i \max\{x_1 - x_i, 0\}$, and (iii) for all $i \in I \setminus (\bar{I} \cup \underline{I})$, $U(x_i - x_1, (0)_{-i}) = 0$.

PROOF OF STEP 2:

To show (i), fix $i \in \bar{I}$. By Step 1 (ii), $U(x_i - x_1, (0)_{-i}) = (x_i - x_1)U(1, (0)_{-i}) = -\alpha_i \max\{x_i - x_1, 0\}$. (ii) and (iii) can be proved in the same way.

By the steps, the following calculations complete the proof of Lemma 1. Note that $\frac{1}{n} \mathbf{x} \sim \frac{1}{n}(x_1, \dots, x_1) + \sum_{i=2}^n \frac{1}{n}(x_i - x_1, (0)_{-i})$ and any pair among $\{(x_1, \dots, x_1), (x_2 - x_1, (0)_{-2}), \dots, (x_n - x_1, (0)_{-n})\}$ is quasi-comonotonic. Therefore, $U(\mathbf{x}) = nU(\frac{1}{n} \mathbf{x}) = nU(\frac{1}{n}(x_1, \dots, x_1) + \sum_{i=2}^n \frac{1}{n}(x_i - x_1, (0)_{-i})) = U(x_1, \dots, x_1) + \sum_{i=2}^n U(x_i - x_1, (0)_{-i}) = x_1 - \sum_{i=2}^n (\alpha_i \max\{x_i - x_1, 0\} + \beta_i \max\{x_1 - x_i, 0\})$.

B. Lemma 2

Let \mathcal{D} be a subset of \mathbb{R}^2 and let $\hat{\succsim}$ be a binary relation on \mathcal{D} . $\hat{\succ}$ and $\hat{\sim}$ denote the asymmetric and symmetric parts of $\hat{\succsim}$, respectively.

Axiom (Monotonicity): (i) $(a, b) \hat{\succ} (a', b')$ if $a \geq a'$ and $b \geq b'$. (ii) $(c, c) \hat{\succ} (c', c')$ if and only if $c \geq c'$.

Axiom (Constant Independence): For all $(a, b), (a', b'), (c, c) \in \mathcal{D}$ and $\lambda \in [0, 1]$, $(a, b) \hat{\succ} (a', b')$ if and only if $\lambda(a, b) + (1 - \lambda)(c, c) \hat{\succ} \lambda(a', b') + (1 - \lambda)(c, c)$.

Axiom (Constant Continuity): For all $(a, b), (a', b'), (c, c) \in \mathcal{D}$, if $(a, b) \hat{\succ} (a', b') \hat{\succ} (c, c)$ or $(c, c) \hat{\succ} (a', b') \hat{\succ} (a, b)$, then there exists $\lambda \in [0, 1]$ such that $(a', b') \hat{\sim} \lambda(a, b) + (1 - \lambda)(c, c)$.

LEMMA 2: Suppose that (i) for any $(a, b) \in \mathcal{D}$, $a \geq b$, (ii) for all $(a, b), (c, c) \in \mathcal{D}$ and $\lambda \in [0, 1]$, $\lambda(a, b) + (1 - \lambda)(c, c) \in \mathcal{D}$, and (iii) $(c, c) \in \mathcal{D}$ for any $c \in \mathbb{R}$. If a preference $\hat{\succsim}$ on \mathcal{D} satisfies Completeness, Transitivity, Monotonicity, Constant Independence, and Constant Continuity, then there exists $\delta \in [0, 1]$ such that $(a, b) \hat{\succ} (a', b') \Leftrightarrow \delta a + (1 - \delta)b \geq \delta a' + (1 - \delta)b'$.

PROOF OF LEMMA 2:

If $(a, a) \hat{\sim} (a, b)$ for any $(a, b) \in \mathcal{D}$, then by Monotonicity (i), $(a, b) \hat{\succ} (a', b') \Leftrightarrow (a, a) \hat{\succ} (a', a') \Leftrightarrow a \geq a'$. Hence, Lemma 2 holds for $\delta = 1$.

In the following, we assume $(a^*, a^*) \hat{\succ} (a^*, b^*)$ for some $(a^*, b^*) \in \mathcal{D}$. (Note that by assumption (iii), $(a^*, a^*) \in \mathcal{D}$.) This assumption implies that $a^* \neq b^*$. Then, by assumption (i), $a^* > b^*$. By assumption (iii), $\bar{c} > a^*$ for some \bar{c} . Monotonicity (ii)

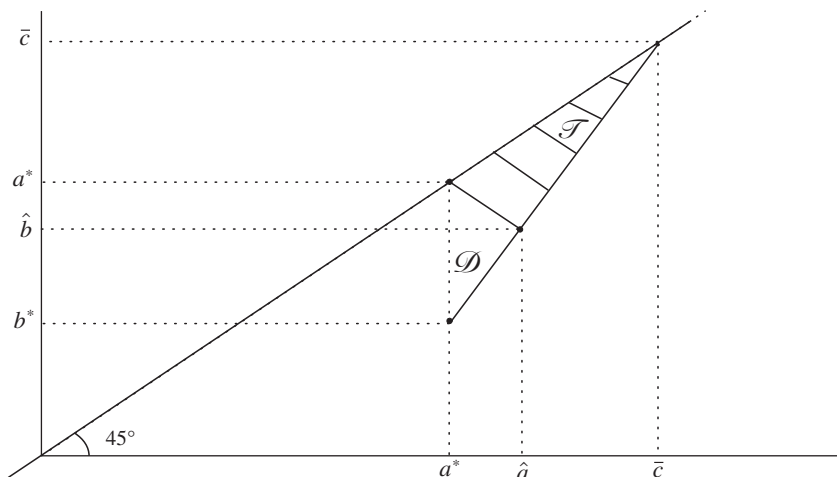


FIGURE A1. INDIFFERENCE CURVES OF $\hat{\succsim}$ ON \mathcal{F}

shows $(\bar{c}, \bar{c}) \hat{\succ} (a^*, a^*)$. Then by Constant Continuity, there exist $\bar{\lambda} \in (0, 1)$ such that $(a^*, a^*) \hat{\sim} \bar{\lambda}(a^*, b^*) + (1 - \bar{\lambda})(\bar{c}, \bar{c})$. Define $\hat{a} = \bar{\lambda}a^* + (1 - \bar{\lambda})\bar{c}$ and $\hat{b} = \bar{\lambda}b^* + (1 - \bar{\lambda})\bar{c}$. Then, $(\hat{a}, \hat{b}) \hat{\sim} (a^*, a^*)$, $\hat{a} > a^*$, and $\hat{a} > \hat{b}$.

Let \mathcal{F} be a triangle including the interior which consists of the vertices (\bar{c}, \bar{c}) , (a^*, a^*) , and (\hat{a}, \hat{b}) . (Formally, $\mathcal{F} = \{(a, b) \in \mathbb{R}^2 \mid a \geq b, \langle (a^* - \hat{a}, \hat{b} - a^*), (a, b) - (a^*, a^*) \rangle \geq 0, \langle (\bar{c} - \hat{a}, \hat{b} - \bar{c}), (a, b) - (\bar{c}, \bar{c}) \rangle \geq 0\}$, where $\langle \cdot, \cdot \rangle$ is an inner product.) In the following, we show the existence of the desired $\delta \in [0, 1]$ on \mathcal{F} and then extend the result on \mathcal{D} .

Step 1: (i) $(a^*, a^*) \neq (\hat{a}, \hat{b}) \neq (\bar{c}, \bar{c})$; (ii) if $(a, b) \in \mathcal{F}$, then $(a, b) \in \mathcal{D}$.

PROOF OF STEP 1:

Since $a^* > b^*$ and $\bar{\lambda} > 0$, $\hat{a} > \hat{b}$. Therefore, $(a^*, a^*) \neq (\hat{a}, \hat{b}) \neq (\bar{c}, \bar{c})$. Let $(a, b) \in \mathcal{F}$ to show $(a, b) \in \mathcal{D}$. Since \mathcal{F} is a triangle, there exist $\lambda, \mu \in [0, 1]$ such that $(a, b) = \lambda(\bar{c}, \bar{c}) + \mu(a^*, a^*) + (1 - \lambda - \mu)(\hat{a}, \hat{b}) = (\lambda + \mu)(c, c) + (1 - \lambda - \mu)(\hat{a}, \hat{b})$, where $c \equiv \frac{\lambda}{\lambda + \mu}\bar{c} + \frac{\mu}{\lambda + \mu}a^*$. Since $(\hat{a}, \hat{b}) \in \mathcal{D}$, it follows from assumptions (ii) and (iii) that $(a, b) \in \mathcal{D}$.

Step 2: There exists $\delta \in [0, 1]$ such that for any $(a, b), (a', b') \in \mathcal{F}$, $(a, b) \hat{\succ} (a', b') \Leftrightarrow \delta a + (1 - \delta)b \geq \delta a' + (1 - \delta)b'$.

PROOF OF STEP 2:

Substep 2.1: For all $(a, b) \in \mathcal{F}$, there exists a unique number $\lambda \in [0, 1]$ such that $(a, b) \hat{\sim} \lambda(\bar{c}, \bar{c}) + (1 - \lambda)(a^*, a^*)$.

PROOF OF SUBSTEP 2.1:

Choose any $(a, b) \in \mathcal{F}$. Since \mathcal{F} is the triangle, there exist $\lambda, \mu \in [0, 1]$ such that $(a, b) = \lambda(\bar{c}, \bar{c}) + \mu(a^*, a^*) + (1 - \lambda - \mu)(\hat{a}, \hat{b})$. Since $(\hat{a}, \hat{b}) \hat{\sim} (a^*, a^*)$, Transitivity

and Constant Independence show $(a, b) \sim \lambda(\bar{c}, \bar{c}) + (1 - \lambda)(a^*, a^*)$. Since $\bar{c} > a^*$, Monotonicity (ii) shows that λ is unique. Hence, Substep 2.1 is proved.

For all $(a, b) \in \mathcal{T}$, define $c(a, b) = \lambda\bar{c} + (1 - \lambda)a^*$, where λ is obtained in Substep 2.1.

Substep 2.2: For all $(a, b) \in \mathcal{T}$, $\frac{a - c(a, b)}{a - b} = \frac{\hat{a} - a^*}{\hat{a} - \hat{b}}$.

PROOF OF SUBSTEP 2.2:

Choose any $(a, b) \in \mathcal{T}$. Since $c(a, b) = \lambda\bar{c} + (1 - \lambda)a^*$, $\frac{a - c(a, b)}{a - b} = \frac{a - \lambda\bar{c} - (1 - \lambda)a^*}{a - b}$. Since $(a, b) \in \mathcal{T}$, $(a, b) = \lambda'(\bar{c}, \bar{c}) + \mu(a^*, a^*) + (1 - \lambda' - \mu)(\hat{a}, \hat{b})$ for some $\lambda', \mu \in [0, 1]$. Since $(a^*, a^*) \sim (\hat{a}, \hat{b})$, Transitivity and Constant Independence show $(a, b) \sim \lambda'(\bar{c}, \bar{c}) + (1 - \lambda')(a^*, a^*)$. Hence, Substep 2.1 shows $\lambda' = \lambda$, so that $(a, b) = \lambda(\bar{c}, \bar{c}) + \mu(a^*, a^*) + (1 - \lambda - \mu)(\hat{a}, \hat{b})$. Thus, $a - \lambda\bar{c} - (1 - \lambda)a^* = (\lambda\bar{c} + \mu a^* + (1 - \lambda - \mu)\hat{a}) - \lambda\bar{c} - (1 - \lambda)a^* = (1 - \lambda - \mu)(\hat{a} - a^*)$ and $a - b = (1 - \lambda - \mu)(\hat{a} - \hat{b})$. Hence, $\frac{a - \lambda\bar{c} - (1 - \lambda)a^*}{a - b} = \frac{\hat{a} - a^*}{\hat{a} - \hat{b}}$, so that Substep 2.2 is proved.

Define

$$\delta = \frac{a^* - \hat{b}}{\hat{a} - \hat{b}}.$$

Substep 2.3: $\delta \in [0, 1]$.

PROOF OF SUBSTEP 2.3:

Since $\hat{a} > \hat{b}$ and $\hat{a} \geq a^*$ by definitions, $\delta = 1 - \frac{\hat{a} - a^*}{\hat{a} - \hat{b}} \leq 1$. To show $\delta \geq 0$, it suffices to prove $a^* \geq \hat{b}$. Suppose by way of contradiction that $a^* < \hat{b}$. Remember that $a^* < \hat{a}$ and $(a^*, a^*) \sim (\hat{a}, \hat{b})$. There exists a positive ε such that $\min\{\hat{a} - a^*, \hat{b} - a^*\} > \varepsilon$. Then, $\hat{a} > a^* + \varepsilon$ and $\hat{b} > a^* + \varepsilon$. By Monotonicity (i), $(\hat{a}, \hat{b}) \hat{\succ} (a^* + \varepsilon, a^* + \varepsilon)$. However, by Monotonicity (ii), $(a^* + \varepsilon, a^* + \varepsilon) \hat{\succ} (a^*, a^*) \sim (\hat{a}, \hat{b}) \hat{\succ} (a^* + \varepsilon, a^* + \varepsilon)$, which is a contradiction. Hence, $a^* \geq \hat{b}$. Then, $\delta \in [0, 1]$.

Substep 2.4: For any $(a, b), (a', b') \in \mathcal{T}$, $(a, b) \hat{\succ} (a', b') \Leftrightarrow \delta a + (1 - \delta)b \geq \delta a' + (1 - \delta)b'$.

PROOF OF SUBSTEP 2.4:

By substituting $\frac{\hat{a} - a^*}{\hat{a} - \hat{b}} = 1 - \delta$ into the result of Substep 2.2, we conclude that for all $(a, b) \in \mathcal{T}$, $c(a, b) = \delta a + (1 - \delta)b$. Therefore, for any $(a, b), (a', b') \in \mathcal{T}$ by Substeps 2.1 and 2.2, $(a, b) \hat{\succ} (a', b') \Leftrightarrow c(a, b) \geq c(a', b') \Leftrightarrow \delta a + (1 - \delta)b \geq \delta a' + (1 - \delta)b'$.

Substeps 2.3 and 2.4 establish Step 2.

Step 3: For all $(a, b), (a', b') \in \mathcal{D}$, $(a, b) \hat{\succ} (a', b') \Leftrightarrow \delta a + (1 - \delta)b \geq \delta a' + (1 - \delta)b'$.

PROOF OF STEP 3:

Let $c^* = \frac{1}{2}\bar{c} + \frac{1}{2}a^*$. Choose any $(a, b), (a', b') \in \mathcal{D}$. By assumption (i), $a \geq b$ and $a' \geq b'$. Hence, there exists $\lambda \in (0, 1]$ such that $\lambda(a, b) + (1 - \lambda)(c^*, c^*)$ and $\lambda(a', b') + (1 - \lambda)(c^*, c^*)$ belong to \mathcal{F} . Step 2 and Constant Independence show $(a, b) \succsim (a', b') \Leftrightarrow \lambda(a, b) + (1 - \lambda)(c^*, c^*) \succsim \lambda(a', b') + (1 - \lambda)(c^*, c^*) \Leftrightarrow \delta(\lambda a + (1 - \lambda)c^*) + (1 - \delta)(\lambda b + (1 - \lambda)c^*) \geq \delta(\lambda a' + (1 - \lambda)c^*) + (1 - \delta)(\lambda b' + (1 - \lambda)c^*) \Leftrightarrow \delta a + (1 - \delta)b \geq \delta a' + (1 - \delta)b'$.

C. Lemma 3

Let \succsim be a binary relation on $\Delta(\mathbb{R}^n)$.

LEMMA 3: Suppose that (i) there exists a concave and constant linear function U on \mathbb{R}^n that represents \succsim on \mathbb{R}^n (i.e., $\mathbf{x} \succsim \mathbf{y} \Leftrightarrow U(\mathbf{x}) \geq U(\mathbf{y})$) and (ii) \succsim satisfies Rationality, Constant Independence for Probability Mixture, and Dominance. Then, there exists $\delta \in [0, 1]$ such that \succsim is represented by $\delta U(E_p(\mathbf{x})) + (1 - \delta)E_p(U(\mathbf{x}))$.

PROOF OF LEMMA 3:

Step 1: There exists a utility function V such that (i) $V(\lambda p \oplus (1 - \lambda)(x, \dots, x)) = \lambda V(p) + (1 - \lambda)V(x, \dots, x)$ for all $p \in \Delta(\mathbb{R}^n), x \in \mathbb{R}$, and $\lambda \in [0, 1]$; (ii) $V = U$ on \mathbb{R}^n .

PROOF OF STEP 1:

As in Step 1 of the proof of Lemma 1, Constant Independence for Probability Mixture implies that there exists a utility function V such that for all $p \in \Delta(\mathbb{R}^n), x \in \mathbb{R}$, and $\lambda \in [0, 1]$, $V(\lambda p \oplus (1 - \lambda)(x, \dots, x)) = \lambda V(p) + (1 - \lambda)V(x, \dots, x)$, so that (i) holds. In addition, V is unique up to positive affine transformation. Hence, we can normalize V by $V(1, \dots, 1) = 1$ and $V(-1, \dots, -1) = -1$, without loss of generality.

To prove (ii), it suffices to show that $V = U$ on $\{(x, \dots, x) \mid x \in \mathbb{R}\}$ because V is unique up to positive affine transformation. Remember $U(\lambda \mathbf{y} + (1 - \lambda)(x, \dots, x)) = \lambda U(\mathbf{y}) + (1 - \lambda)x$ for all $\mathbf{y} \in \mathbb{R}^n, x \in \mathbb{R}$, and $\lambda \in [0, 1]$. Hence, $\lambda = 0$ implies $U(x, \dots, x) = x$. Therefore, it suffices to show $V(x, \dots, x) = x$ for all $x \in \mathbb{R}$.

Note that for all $x, y \in \mathbb{R}$, $V(x, \dots, x) \geq V(y, \dots, y) \Leftrightarrow (x, \dots, x) \succsim (y, \dots, y) \Leftrightarrow x \geq y$. Hence, there exists an increasing function $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $V(x, \dots, x) = f(x)$ for all $x \in \mathbb{R}$. To show that f is the identity function, note that Dominance trivially implies $\lambda(x, \dots, x) + (1 - \lambda)(y, \dots, y) \sim \lambda(x, \dots, x) \oplus (1 - \lambda)(y, \dots, y)$ for all $x, y \in \mathbb{R}$ and $\lambda \in [0, 1]$. Therefore, by property (i) of V , for any $x, y \in \mathbb{R}$ and $\lambda \in [0, 1]$, $\lambda f(x) + (1 - \lambda)f(y) = \lambda V(x, \dots, x) + (1 - \lambda)V(y, \dots, y) = V(\lambda(x, \dots, x) \oplus (1 - \lambda)(y, \dots, y)) = V(\lambda(x, \dots, x) + (1 - \lambda)(y, \dots, y)) = f(\lambda x + (1 - \lambda)y)$. Hence, f is affine. In addition, by the normalizations, $f(1) = 1$ and $f(-1) = -1$. Therefore, f is the identity function, or $V(x, \dots, x) = x$ for all $x \in \mathbb{R}$.

To use Lemma 2, define

$$\mathcal{D} = \{(U(E_p(\mathbf{x})), E_p(U(\mathbf{x}))) \in \mathbb{R}^2 \mid p \in \Delta(\mathbb{R}^n)\}.$$

Step 2: \mathcal{D} satisfies the properties (i), (ii), and (iii) in Lemma 2.

PROOF OF STEP 2:

To show (i), it suffices to show that $U(E_p(\mathbf{x})) \geq E_p(U(\mathbf{x}))$ for all $p \in \Delta(\mathbb{R}^n)$. Since U is concave, Jensen’s Inequality shows that $U(E_p(\mathbf{x})) \geq E_p(U(\mathbf{x}))$ for all $p \in \Delta(\mathbb{R}^n)$. Property (ii) holds by the constant linearity of U . To see (iii) holds, note that for any $x \in \mathbb{R}$, if $p \equiv \delta_{(x, \dots, x)}$, then $U(E_p(\mathbf{x})) = x = E_p(U(\mathbf{x}))$.

To define a binary relation $\hat{\succsim}$ on \mathcal{D} , we define a function v on \mathcal{D} . For all $(a, b) \in \mathcal{D}$, define

$$v(a, b) = V(p),$$

where $p \in \Delta(\mathbb{R}^n)$ such that $U(E_p(\mathbf{x})) = a$ and $E_p(U(\mathbf{x})) = b$.

Step 3: v is a well-defined weakly increasing function.

PROOF OF STEP 3:

To show that v is well defined, it suffices to show that $(U(E_p(\mathbf{x})), E_p(U(\mathbf{x}))) = (U(E_q(\mathbf{x})), E_q(U(\mathbf{x})))$ implies $V(p) = V(q)$. Let $p, q \in \Delta(\mathbb{R}^n)$ such that $(U(E_p(\mathbf{x})), E_p(U(\mathbf{x}))) = (U(E_q(\mathbf{x})), E_q(U(\mathbf{x})))$. Then, both conditions (i) and (ii) in Dominance are satisfied with equalities. Therefore, Dominance shows $V(p) = V(q)$. Weak increasingness directly follows from Dominance.

Now, we define a binary relation $\hat{\succsim}$ on \mathcal{D} . For all $(a, b), (a', b') \in \mathcal{D}$ define

$$(a, b) \hat{\succsim} (a', b') \Leftrightarrow v(a, b) \geq v(a', b').$$

Step 4: $\hat{\succsim}$ satisfies the all properties in Lemma 2 (i.e., Completeness, Transitivity, Monotonicity, Constant Independence, and Constant Continuity).

PROOF OF STEP 4:

Since v is a well-defined increasing function, $\hat{\succsim}$ satisfies Completeness, Transitivity, and Monotonicity (i). Moreover, $(c, c) \hat{\succsim} (c', c') \Leftrightarrow v(c, c) \geq v(c', c') \Leftrightarrow V(c, \dots, c) \geq V(c', \dots, c') \Leftrightarrow c \geq c'$; hence, Monotonicity (ii) is also satisfied.

Constant Independence follows from Step 1 and the constant linearity of U . To see this, choose any $(a, b), (a', b'), (z, z) \in \mathcal{D}$ and $\lambda \in [0, 1]$. Then, there exist lotteries p and q such that $(a, b) = (U(E_p(\mathbf{x})), E_p(U(\mathbf{x})))$ and $(a', b') = (U(E_q(\mathbf{x})), E_q(U(\mathbf{x})))$. To simplify notation, let $p' = \lambda p \oplus (1 - \lambda)(z, \dots, z)$ and $q' = \lambda q \oplus (1 - \lambda) \times (z, \dots, z)$. By Step 1(i), then $(a, b) \hat{\succsim} (a', b') \Leftrightarrow V(p) \geq V(q) \Leftrightarrow V(p') \geq V(q') \Leftrightarrow (U(E_{p'}(\mathbf{x})), E_{p'}(U(\mathbf{x}))) \hat{\succsim} (U(E_{q'}(\mathbf{x})), E_{q'}(U(\mathbf{x})))$. Moreover, by the constant linearity of U , $(U(E_{p'}(\mathbf{x})), E_{p'}(U(\mathbf{x}))) = \lambda(a, b) + (1 - \lambda)(z, z)$ and $(U(E_{q'}(\mathbf{x})), E_{q'}(U(\mathbf{x}))) = \lambda(a', b') + (1 - \lambda)(z, z)$. This completes the proof of Constant Independence.

Finally, we show that $\hat{\succsim}$ satisfies Constant Continuity. Choose any $(a, b), (a', b'), (z, z) \in \mathcal{D}$ such that $(a, b) \hat{\succsim} (a', b') \hat{\succsim} (z, z)$. Then, there exist $p, q \in \Delta(\mathbb{R}^n)$

such that $(U(E_p(\mathbf{x})), E_p(U(\mathbf{x}))) = (a, b)$ and $(U(E_q(\mathbf{x})), E_q(U(\mathbf{x}))) = (a', b')$. Then $p \succsim q \succsim (z, \dots, z)$. Then by Continuity, there exists $\lambda \in [0, 1]$ such that $q \sim \lambda p \oplus (1 - \lambda)(z, \dots, z)$. By definition, this implies that $(a', b') \sim \lambda(a, b) + (1 - \lambda)(z, z)$. The other case where $(z, z) \succsim (a', b') \succsim (a, b)$ is proved in the same way.

Given Steps 2 and 4 and Lemma 2, it follows that $p \succsim q \Leftrightarrow V(p) \geq V(q) \Leftrightarrow (U(E_p(\mathbf{x})), E_p(U(\mathbf{x}))) \succsim (U(E_q(\mathbf{x})), E_q(U(\mathbf{x}))) \Leftrightarrow \delta U(E_p(\mathbf{x})) + (1 - \delta)E_p(U(\mathbf{x})) \geq \delta U(E_q(\mathbf{x})) + (1 - \delta)E_q(U(\mathbf{x}))$ for all $p, q \in \Delta(\mathbb{R}^n)$. This completes the proof of Lemma 3.

D. Proof of Theorem and Remark

The necessity of the axioms is trivial. To show Continuity, note that the EIA model is a weighted sum of max functions. The sufficiency follows from Lemmas 1 and 3 because $U(\mathbf{x}) = x_1 - \sum_{i=2}^n (\alpha_i \max\{x_i - x_1, 0\} + \beta_i \max\{x_1 - x_i, 0\})$ is concave and constant linear.¹⁴

In the following, we provide the proof of Remark. Suppose that (δ, α, β) and $(\delta', \alpha', \beta')$ represent \succsim . Let V and V' be the functions defined by (δ, α, β) and $(\delta', \alpha', \beta')$, respectively.

First, we show $(\alpha, \beta) = (\alpha', \beta')$. Suppose to the contrary that $(\alpha, \beta) \neq (\alpha', \beta')$. Assume $\alpha'_i > \alpha_i$ for some $i \neq 1$, without loss of generality. Therefore, $V(1, (0)_{-i}) = -\alpha_i = V(-\alpha_i, \dots, -\alpha_i) = -\alpha_i > -\alpha'_i = V(-\alpha'_i, \dots, -\alpha'_i)$ and $V'(1, (0)_{-i}) = -\alpha'_i = V'(-\alpha'_i, \dots, -\alpha'_i)$. Thus, $(1, (0)_{-i}) \sim (-\alpha_i, \dots, -\alpha_i) \succ (-\alpha'_i, \dots, -\alpha'_i) \sim (1, (0)_{-i})$, which is a contradiction.

Second, we show that if $(\alpha, \beta) \neq \mathbf{0}$, then $\delta = \delta'$. Suppose to the contrary that $(\alpha, \beta) \neq \mathbf{0}$ but $\delta \neq \delta'$. Without loss of generality, assume $\delta > \delta'$ and $\alpha_i > 0$ for some $i \neq 1$. Then, $V(\frac{1}{2}(1, (0)_{-i}) \oplus \frac{1}{2}(-1, (0)_{-i})) = -\frac{1}{2}(1 - \delta)(\alpha_i + \beta_i) = V(-\frac{1}{2}(1 - \delta)(\alpha_i + \beta_i), \dots, -\frac{1}{2}(1 - \delta)(\alpha_i + \beta_i))$. Therefore, $\frac{1}{2}(1, (0)_{-i}) \oplus \frac{1}{2}(-1, (0)_{-i}) \sim (-\frac{1}{2}(1 - \delta)(\alpha_i + \beta_i), \dots, -\frac{1}{2}(1 - \delta)(\alpha_i + \beta_i)) \succ (-\frac{1}{2}(1 - \delta') \times (\alpha_i + \beta_i), \dots, -\frac{1}{2}(1 - \delta')(\alpha_i + \beta_i)) \sim \frac{1}{2}(1, (0)_{-i}) \oplus \frac{1}{2}(-1, (0)_{-i})$, which is a contradiction.

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¹⁴To see the concavity, remember (i) for any convex function ϕ and positive number a , $a \max\{\phi(\cdot), 0\}$ is convex; (ii) a sum of convex functions is convex; (iii) for any convex function ϕ , $-\phi$ is concave. By (i), $\alpha_i \max\{x_i - x_1, 0\}$ and $\beta_i \max\{x_1 - x_i, 0\}$ are convex. Then, by (ii), $\alpha_i \max\{x_i - x_1, 0\} + \beta_i \max\{x_1 - x_i, 0\}$ is convex. Since U is a sum of concave functions, U is also concave.

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