A SECONDARY TRACER APPROACH TO THE DERIVATION OF GALACTIC COSMIC-RAY SOURCE ISOTOPIC ABUNDANCES

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ABSTRACT

A formalism has been developed for deriving cosmic-ray source isotopic abundances from observed local abundances using a purely secondary nuclide as a tracer of spallation production during propagation. Although the formalism is based on the leaky-box model of cosmic-ray propagation, it is shown that source abundances derived by the tracer technique are reasonably independent of detailed propagation models. The tracer formalism also permits a quantitative evaluation of the effects of observational uncertainties on deduced source abundances. It is shown that statistical errors in the observed abundances and uncertainties in the spallation cross sections are at present the dominant sources of uncertainty. The latter error can be reduced with increased detector size or exposure time, while the former can be minimized by measurements of the relative production cross sections. As a specific example, the tracer technique is applied to the isotopes of sulfur and calcium, and the level of uncertainties which must be achieved to distinguish evolutionary differences between solar-system material and cosmic ray-source material are established.

Subject headings: cosmic rays: abundances — nuclear reactions

1. INTRODUCTION

In the study of the galactic cosmic rays ($E \gtrsim 10$ MeV per amu), extensive use has been made of observations of the composition of the nuclear component for deducing parameters describing cosmic-ray propagation and for understanding the processes by which these particles are synthesized in their sources. Early in this study it was learned from observations of the L/M ratio ($\text{Li} + \text{Be} + \text{B/C} + \text{N} + \text{O}$) that cosmic rays typically traverse $\sim 5 \text{ g cm}^{-2}$ of interstellar matter between the time they are produced and the time they are observed in the vicinity of Earth. In addition, it was deduced that the relative source abundances of groups of nuclei between carbon and iron were approximately consistent with the corresponding relative abundances in solar-system material. More recently, measurements of the abundances of individual elements have tended to confirm this picture—with a few notable exceptions (see, e.g., Cassé and Meyer 1977).

Recent progress in the development of high-resolution detection systems capable of isotopic separation should soon make possible the measurement of relative abundances of all nuclides with $Z \leq 28$ in the galactic cosmic rays. The importance of such isotopic composition measurements has been reviewed by a number of authors (Silberberg, Tsao, and Shapiro 1976; Waddington 1977). These first-generation isotopic composition experiments will be characterized by exposure factors $\sim 10^8\text{--}10^9 \text{ cm}^2 \text{ sr s}$ and energy intervals of several hundred MeV per amu. These experiments will perform an exploratory in-

vestigation of isotopic composition up to the iron region of the periodic table. In so doing, they should readily identify any major differences between the isotopic composition of the cosmic-ray sources and the isotopic composition of solar-system material (as summarized by Cameron 1973). The observation of such major differences, indicative of substantial differences in either the environment or in the physical processes involved in the synthesis of these samples of material, would immediately provide information which would contribute to our understanding of the differences in the astrophysical conditions under which these two samples of material were synthesized.

However, it is possible that the composition of the cosmic-ray sources is not radically different from the composition of solar material. Such might be the case if, for example, the cosmic rays were simply a small fraction of the interstellar gas which was accelerated to relativistic energies long after its synthesis in stars. In cases such as this, one would expect differences in composition between the cosmic-ray sources and the solar-system material to be subtle, due possibly to the continual evolution of the composition of the matter processed by stars. In this context it should be noted that solar-system material was synthesized $\sim 4.5 \times 10^9$ years ago, whereas the cosmic-ray propagation time (i.e., the time since acceleration to relativistic energies) may be only $\sim 10^7$ yr (Hagen, Fisher, and Ormes 1977; Garcia-Munoz, Mason, and Simpson 1977a; Müller and Prince 1977; Webber *et al.* 1977).

If there are only subtle evolutionary differences between the cosmic-ray source material and solar-system material, it will be necessary to obtain a

significantly larger statistical sample than will be obtainable from first-generation isotope experiments in order to distinguish these differences. The present investigation was undertaken, in part, to delineate the extent to which the statistical accuracy of cosmic-ray composition experiments must be increased in order to distinguish such evolutionary differences. In addition, we have investigated the extent to which it is necessary to reduce the uncertainties in our knowledge of nuclear spallation cross sections so that errors in the calculation of corrections for secondary production during propagation will not dominate the uncertainty in the deduced source abundances. Although the details of the approach are illustrated for source abundances similar to the solar system, the approach is generally applicable to the derivation of source-abundance ratios from cosmic-ray isotope measurements.

The observation of cosmic-ray isotopic abundances will result in an approximately threefold increase in the number of individual species available for constraining models of nucleosynthesis and propagation. Separate analysis of certain groups of nuclides will make it possible to remove various systematic problems which are present when one attempts to analyze all of the abundances from Li through Ni together. First, by comparing source abundances of various isotopes of the same element, it is possible to obtain information concerning the nucleosynthesis process without introducing uncertainties due to the possible presence of charge-dependent acceleration mechanisms, such as have been suggested by various authors (Cassé, Goret, and Cesarsky 1975, Cassé and Goret 1978). Second, by obtaining an estimate of the amount of interstellar material traversed during propagation by considering only those purely secondary isotopes¹ which have essentially the same parentage as the primary species of interest, it is possible to avoid uncertainties due to possible differences in the propagation of different species.

In the course of this investigation, we have developed a formalism suitable for separately analyzing abundance information obtained by considering small groups of nuclides.

II. ANALYSIS OF COMPOSITION DATA

a) Conventional Approach

A convenient approach to the analysis of cosmic-ray composition information has been developed by Silberberg, Tsao, and Shapiro (1976). From the abundances of purely secondary elements (in particular, Li, Be, B, and the group $21 \leq Z \leq 25$) relative to the abundant parents from which they are produced by spallation, one obtains an estimate of the interstellar grammage traversed of $\sim 5\text{--}6 \text{ g cm}^{-2}$ (assuming a ratio $\text{He}/\text{H} = 0.1$ by number of atoms in the interstellar gas). One can then assume a source composition for the remaining elements, distribute the source

¹ We shall use the term "secondary isotope" to refer to those isotopes which are produced by spallation reactions during propagation.

abundance of each element among its stable isotopes in proportion to the isotopic abundances in the solar system, and propagate the resulting abundances through the distribution of interstellar path lengths previously derived. The local elemental abundances resulting from this calculation can be compared with cosmic-ray observations. By readjusting the assumed source abundances and repeating the propagation calculation, it is possible to iterate to a set of source abundances which are consistent with the observed local abundances and the derived interstellar path length.

There are several limitations to this traditional approach to the derivation of source abundances. First, the iterative, top-down approach does not provide a convenient means for determining the uncertainty in the derived source abundances and assessing the relative contribution of various experimental errors to this uncertainty.

Second, at low energies (below several hundred MeV per amu), ionization-energy loss by cosmic-ray particles in the interstellar gas causes relative abundances observed at a single energy in local interstellar space to be an average over a range of higher energies at the source. In order to take account of this effect it is conventional to assume a common spectral shape for all cosmic-ray species at the source, to follow the evolution of the shapes of the various spectra during propagation, and finally to compare the resulting local spectra with observations. The statistical accuracy obtained with present-day experiments is not sufficient to permit detailed spectral comparisons, so it is difficult both to assess the correctness of assumptions about the shape of source spectra and to determine the uncertainty which such assumptions introduce into the deduced source abundances.

b) Tracer Approach

In this section we present a new formalism which can be employed in the analysis of cosmic-ray composition observations. We assume that the propagation of cosmic rays can be adequately described by a leaky-box model (Cowsik *et al.* 1967), so that the source and local abundances can be related by a continuity equation (Gloeckler and Jokipii 1969; Meneguzzi, Audouze, and Reeves 1971):

$$0 = q_i + \sum_l \varphi_l \left[\frac{\sigma_{pli} + (n_{\text{He}}/n_{\text{H}})\sigma_{ali}}{M_{\text{H}} + (n_{\text{He}}/n_{\text{H}})M_{\text{He}}} \right] + \frac{Z_i^2}{M_i} \frac{d}{dE} (s(E)\varphi_i) - \left[\frac{\sigma_{pi} + (n_{\text{He}}/n_{\text{H}})\sigma_{ai}}{M_{\text{H}} + (n_{\text{He}}/n_{\text{H}})M_{\text{He}}} \right] \varphi_i - \frac{\varphi_i}{\Lambda} \quad (1)$$

In this equation, φ_i is the flux [$\text{cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1} (\text{MeV per amu})^{-1}$] of cosmic-ray species i ; Z_i and M_i are its charge and mass, respectively, in units of the proton charge and mass; $\sigma_{pi}(\sigma_{ai})$ is the total inelastic cross section for species i in collision with a hydrogen (helium) target; $\sigma_{pli}(\sigma_{ali})$ is the partial cross section for production of species i in a collision between species

I and a hydrogen (helium) target; $M_H(M_{He})$ is the mass (in grams) of a hydrogen (helium) atom; n_{He}/n_H is the ratio of number densities of helium to hydrogen atoms in the interstellar gas (assumed to be 0.1 here); $s(E)$ is the specific ionization (in $\text{MeV cm}^2\text{g}^{-1}$) of a proton of energy E ; Λ is the mean interstellar path length traversed by cosmic rays in the absence of nuclear collisions and ionization-energy loss; and q_i is proportional to the rate of production of species i by the cosmic-ray sources. We have assumed that the heavy fragments produced in spallation reactions have approximately the same velocity as the incident heavy nucleus, in accord with recent observations (Greiner *et al.* 1975).

It should be noted that in equation (1) the source abundances q_i are expressed in terms of the single phenomenological parameter Λ , which describes the cosmic-ray propagation, and of a collection of observable quantities.

As usual, it is possible to obtain Λ by considering species which, based on nuclear systematics, can be assumed to be absent at the sources ($q_i = 0$) (Reames 1974). As mentioned above, by considering only those secondary species which are produced by predominantly the same parents as are the isotopes for which we wish to derive a source composition, it is possible to minimize errors which might arise from differences in the propagation of different species. In order to obtain an estimate of Λ which has reasonable statistical accuracy, it is at present useful to employ an average (with appropriate weighting) of values of Λ obtained by considering a number of purely secondary species.

However, with improvements in the statistical accuracy of cosmic-ray observations, it should eventually become possible to obtain Λ from a single secondary species. For the purpose of illustration, we shall specialize to this case. There is a unique relationship between the path length Λ and the local flux of purely secondary species (given by eq. [1] when $q_i = 0$). Therefore, it is possible to use the observable flux φ_i in place of Λ as a parameter describing the interstellar propagation. In this spirit, we can solve equation (1) for Λ and then substitute this result into equation (1) rewritten for two other species, j and k , for which there are primary contributions to the observed fluxes. In this way we obtain an expression for the source ratio q_j/q_k in terms only of observable quantities:

$$\begin{aligned} \frac{q_j}{q_k} = \frac{\varphi_j}{\varphi_k} & \left[1 - \frac{\varphi_i S_j}{\varphi_j S_i} + \left\{ \frac{1}{\Lambda_j} - \frac{1}{\Lambda_i} + \frac{s(E)}{E} \right. \right. \\ & \times \left. \left. \left[\frac{Z_j^2}{M_j} (\gamma_j + \alpha) - \frac{Z_i^2}{M_i} (\gamma_i + \alpha) \right] \right\} \frac{\varphi_i}{S_i} \right] / \\ & \left[1 - \frac{\varphi_i S_k}{\varphi_k S_i} + \left\{ \frac{1}{\Lambda_k} - \frac{1}{\Lambda_i} + \frac{s(E)}{E} \right. \right. \\ & \times \left. \left. \left[\frac{Z_k^2}{M_k} (\gamma_k + \alpha) - \frac{Z_i^2}{M_i} (\gamma_i + \alpha) \right] \right\} \frac{\varphi_i}{S_i} \right] \end{aligned} \quad (2)$$

Here

$$S_i = \sum_l \varphi_l \left[\frac{\sigma_{pli} + (n_{He}/n_H) \sigma_{all}}{M_H + (n_{He}/n_H) M_{He}} \right],$$

$$\frac{1}{\Lambda_i} \equiv \frac{\sigma_{pi} + (n_{He}/n_H) \sigma_{ai}}{M_H + (n_{He}/n_H) M_{He}},$$

$$\alpha \equiv - \left[\frac{\partial \ln s}{\partial \ln \epsilon} \right]_{\epsilon=E},$$

$$\gamma_i \equiv - \left[\frac{\partial \ln \varphi_i}{\partial \ln \epsilon} \right]_{\epsilon=E}.$$

Note that the abundance ratios and spectral indices needed to evaluate equation (2) are those in local interstellar space. In order to employ values of these quantities observed near the orbit of Earth, it is necessary to apply corrections for solar modulation effects to obtain the corresponding interstellar values. In a later section of this paper, we shall present estimates of the error introduced in the calculated source ratios due to uncertainties in this demodulation procedure.

In equation (2) the source-abundance ratio q_j/q_k is expressed as the product of the corresponding local ratio, φ_j/φ_k , and a number of correction terms. The term $(S_j/S_i) \cdot \varphi_i/\varphi_j$ corrects for secondary production of species j during propagation. The term $(\Lambda_j^{-1} - \Lambda_i^{-1}) \cdot \varphi_i/S_i$ corrects for differences in the nuclear-interaction mean free path between the tracer i and the species of interest j . Also, the term multiplied by $s(E)/E$ corrects for ionization-energy loss effects on the fluxes of species i and j . Similar corrections to the observed flux of species k appear in the denominator.

It should be noted that in order to include the effects of ionization-energy loss in the present treatment (see eq. [2]), it is only necessary to know the power-law index γ at the energy of interest for the spectra of each of the three species being considered. Similarly, fragmentation cross sections need be known only for the observed energies. This is in contrast to the conventional analysis where in principle it is necessary to assume the shape of the source spectra of the species of interest as well as of all parent species at all energies above the energy of interest and the energy dependence of the relevant cross sections. This simplification is due to the fact that in the leaky-box model the slope of the local spectrum at any energy has attained a value that reflects the net contribution of energy-changing processes to the transport of cosmic rays and therefore contains all the information needed to correct the observed abundances for such effects.

At energies above several hundred MeV per amu, the correction for ionization-energy loss effects is relatively small. Also, the differences in nuclear destruction length is small if the mass of the tracer isotope is approximately the same as the masses of the

other species being considered. Under these conditions, equation (2) reduces to

$$\frac{q_j}{q_k} = \frac{\varphi_j - \varphi_i(S_j/S_i)}{\varphi_k - \varphi_i(S_k/S_i)}, \quad (3)$$

Here the primary abundance q_j of the species j is expressed (to within arbitrary scale factors) as the difference between the observed flux φ_j and the secondary contribution to the flux, $\varphi_i S_j/S_i$. The secondary production term appears as the rate of production of species j relative to species i times the observed flux (assumed to be totally secondary) of species i .

If the cosmic-ray source abundances are similar to solar-system abundances, then in many cases of interest (e.g., the isotopes of Ne, Mg, S, Ar, Ca, and Fe) a single isotope composes a large fraction of the elemental abundance, and the observed flux of this isotope will be composed predominantly of surviving primary particles. If we assume that species k is such an isotope, then equation (3) reduces to

$$\frac{q_j}{q_k} \approx \frac{\varphi_j}{\varphi_k} - \frac{\varphi_i S_j}{\varphi_k S_i}.$$

From this equation it can readily be seen that modest observational uncertainties can seriously degrade the information which can be obtained about source composition. The quantity $\varphi_j - \varphi_i S_j/S_i$ is the difference between the observed flux of species j and the secondary contribution to that flux. In a number of cases of interest, the difference in these two terms is small compared to either of the terms individually. If, for example, the difference is $\sim 10\%$ of the two terms being subtracted, then a 10% uncertainty in either of these terms will result in a 100% uncertainty in the difference. If, as in the cases being considered here, the flux φ_k is largely of primary origin, then one finds that a given fractional uncertainty in φ_j , φ_i , or S_j/S_i will be magnified in the fractional uncertainty in q_j/q_k by a factor of approximately $(\varphi_j/\varphi_k)/(q_j/q_k)$.

Table 1 shows, for a number of isotopes which are expected to be present in the cosmic-ray source, the fraction of the local flux which is composed of surviving primary particles if the source composition is solar-like and the cosmic rays propagate through an exponential distribution of path lengths with a mean value of 5.5 g cm^{-2} . It can be seen from this table that the magnification of uncertainties will be most significant in the case of the isotopes of Ca, Ar, and S. In the cases of Ne, Mg, Si, and Fe, even the less-abundant isotopes have substantial primary components at Earth, and the uncertainties in the quantities required for deriving source-abundance ratios should be enhanced by no more than factor of 2 in the resulting ratios.

Uncertainties arise from a number of sources. The local interstellar ratios φ_j/φ_k and φ_i/φ_k are uncertain because of (1) the finite statistical accuracy in the measurement of the fluxes near Earth and (2) uncertainties in the demodulation of local fluxes to obtain the corresponding interstellar values as a result

TABLE 1
PRIMARY FRACTIONS OF LOCAL FLUXES

Isotope	Primary Fraction
²⁰ Ne.....	0.87
²² Ne.....	0.43
²⁴ Mg.....	0.92
²⁵ Mg.....	0.58
²⁶ Mg.....	0.75
²⁸ Si.....	0.97
²⁹ Si.....	0.59
³⁰ Si.....	0.45
³² S.....	0.82
³⁴ S.....	0.13
³⁶ Ar.....	0.49
³⁸ Ar.....	0.11
⁴⁰ Ca.....	0.98
⁴² Ca.....	0.02
⁴⁴ Ca.....	0.07
⁵⁴ Fe.....	0.70
⁵⁶ Fe.....	1.00
⁵⁷ Fe.....	0.95
⁵⁸ Fe.....	0.88

of uncertainty in the level of solar modulation and of the inexact nature of solar modulation models. The ratio S_j/S_i of relative secondary production rates of species j and i is uncertain because of uncertainties in the nuclear fragmentation cross sections for producing these species and uncertainties in the fluxes of the parents from which the secondaries are produced. In addition, if the propagation of the cosmic rays is characterized by a nonexponential distribution of path lengths, the ratio of the secondary yields of species j and i will differ somewhat from S_j/S_i . Consequently, uncertainty concerning the exact form of the path length distribution will produce an uncertainty in S_j/S_i and ultimately in q_j/q_k . Also, if the source abundance of the tracer being used is not strictly zero, the contribution of surviving primary material to the local flux will cause an overestimate of the secondary components of the other species being considered and, consequently, an error in the value of q_j/q_k which is derived.

c) Tracer Analysis of Sulfur and Calcium Composition

In order to illustrate the application of the above formalism to the analysis of cosmic-ray observations, we will consider two triplets of isotopes: ³²S, ³⁴S, ³³S and ⁴⁰Ca, ⁴²Ca, ⁴³Ca. These cases are of considerable interest since the ratios ³⁴S/³²S and ⁴²Ca/⁴⁰Ca can serve as probes of the neutron excess in the environment in which these species were synthesized (Woosley, Arnett, and Clayton 1973). In addition, since these cases are expected to be more sensitive to uncertainties than some of the other cases of interest (see Table 1), our investigation will indicate some of the limitations on our ability to obtain source information from cosmic-ray observations and will show in which areas improved measurements can enhance this ability. In each of these cases, we will assume that the fluxes of

TABLE 2
SULFUR AND CALCIUM ABUNDANCES FROM
SOLAR-LIKE SOURCE

Isotope Ratio	Source*	Local†
$^{33}\text{S}/^{32}\text{S}$	0.0080	0.160
$^{34}\text{S}/^{32}\text{S}$	0.0444	0.238
$^{42}\text{Ca}/^{40}\text{Ca}$	0.0066	0.413
$^{43}\text{Ca}/^{40}\text{Ca}$	0.0015	0.353

* From Cameron 1973.

† Source abundances propagated through an exponential path-length distribution with $\Lambda = 5.5 \text{ g cm}^{-2}$.

the first two isotopes listed have a significant primary contribution, and that the flux of the third isotope is entirely of secondary origin. Table 2 lists the isotopic composition of these elements in solar material as well as the abundances expected in the local cosmic rays, assuming that (1) the isotopic composition of the cosmic-ray source is solar-like, and (2) the cosmic rays propagate through an exponential distribution of path-lengths with a mean path length given by $\Lambda = 5.5 \text{ g cm}^{-2}$.

In Figures 1 and 2, we show the relationship between the source ratio q_j/q_k and the local ratios φ_j/φ_k and

φ_i/φ_k as given by equation (2). The following values of the parameters in equation (2) have been adopted: $E = 700 \text{ MeV}$ per amu and hence $s(E) = 4.2 \text{ MeV cm}^2 \text{ g}^{-1}$ and $\alpha = 0.33$; spallation cross sections are calculated using the semiempirical formulas of Silberberg and Tsao (1973) with modifications summarized by Silberberg, Tsao, and Shapiro (1976); total-destruction cross sections are obtained from Cheshire *et al.* (1974) and Tsao and Silberberg (1975); local elemental abundances of parent nuclides were obtained from Silberberg, Tsao, and Shapiro (1976), while the isotopic compositions of the various parent elements were randomized (the resulting value of q_j/q_k being an average over many such randomized parentages); and the spectral index at 700 MeV per amu for each of the species was assumed to be 1.33, as obtained by Garrard (1973) (see also Garrard, Stone, and Vogt 1973) from investigation of the solar modulation of the cosmic-ray hydrogen and helium spectra. The extent to which uncertainties in these parameters affect the deduced value of q_j/q_k will be considered in the next section.

The qualitative features of Figures 1 and 2 are easily understood. The secondary isotope i serves as a tracer of secondary production during propagation. Thus, increasing φ_i/φ_k corresponds to increasing Λ , the mean interstellar path length. In the limit that $\varphi_i/\varphi_k = 0$

FIG. 1.—Calculation of source-abundance ratios from local-abundance ratios for sulfur isotopes at 700 MeV per amu. Points indicate compositions resulting from a source of solar composition ("Cameron" point) and from sources consisting of material synthesized by explosive oxygen burning in environments having various levels of neutron excess as indicated. Local abundances result from propagating the source material through an exponential distribution of path lengths with $\Lambda = 5.5 \text{ g cm}^{-2}$.

FIG. 2.—Calculations as in Figure 1 of the abundance ratios of calcium

(i.e., no secondaries have been produced), all local abundances are equal to the corresponding source abundances: $\varphi_j/\varphi_k = q_j/q_k$. Also, for small values of q_j/q_k , the curves of constant φ_i/φ_k are quite steep, corresponding to the fact that the observed flux of j is predominantly of secondary origin. Clearly, for small values of q_j/q_k , small uncertainties in φ_j/φ_k , in φ_i/φ_k , or in the precise position of the curves of constant φ_i/φ_k can propagate into much larger relative uncertainties in q_j/q_k .

Also shown in Figures 1 and 2 are the points corresponding to cosmic-ray production in sources with various levels of neutron excess as well as in a source of solar-system composition followed by propagation with a mean path length of $\Lambda = 5.5 \text{ g cm}^{-2}$ (see Table 2).

III. ANALYSIS OF UNCERTAINTIES

The dominant sources of uncertainty in q_j/q_k are (1) uncertainties in the observed local flux ratios and (2) uncertainties in the nuclear spallation cross sections used to calculate the secondary contribution to the observed fluxes. In addition, errors in the other parameters required in evaluating equation (2) make

smaller contributions to the uncertainty in q_j/q_k . We will individually examine the contributions of each of these sources to the overall uncertainty in q_j/q_k .

a) Flux Measurement Errors

In the isotopic-composition results published to date (Hagen, Fisher, and Ormes 1977; Garcia-Munoz, Mason, and Simpson 1977b; Simpson *et al.* 1977a, b; Webber *et al.* 1977), abundance uncertainties have been present due to both statistical errors and systematic errors arising from the imperfect separation between adjacent mass peaks. In cases where one is attempting to obtain a precise measurement of the relative abundance of adjacent isotopes and this abundance ratio is small ($\lesssim 0.1$), such systematic errors can be particularly troublesome. The response of a cosmic-ray spectrometer is seldom understood in sufficient detail to predict the shape of the tail (the outer-most few percent) of a mass distribution. However, it is this information which is essential when trying to separate mass distributions which are not completely resolved. Even a calibration of this response is difficult, since an accelerator calibration will not simulate the isotropic flux observed in space.

Consequently, it is important to achieve a high degree of mass resolution in order to investigate small differences between the cosmic-ray source composition and the solar-system composition. In the following discussion of statistical uncertainties, we shall assume that sufficient mass resolution has been achieved so that systematic errors contribute negligibly to the overall measurement error.

If one makes observations over a time interval during which N events are expected, distributed among the three species i, j , and k , the statistical uncertainties in the distribution of these events lead to an uncertainty in the deduced source ratio which is given by

$$\sigma_{q_j/q_k} = \left[\left[\frac{\partial(q_j/q_k)}{\partial(\varphi_j/\varphi_k)} \right]^2 (\varphi_j/\varphi_k) + \left[\frac{\partial(q_j/q_k)}{\partial(\varphi_i/\varphi_k)} \right]^2 (\varphi_i/\varphi_k) + \left\{ \left[\frac{\partial(q_j/q_k)}{\partial(\varphi_j/\varphi_k)} \right] (\varphi_j/\varphi_k) + \left[\frac{\partial(q_j/q_k)}{\partial(\varphi_i/\varphi_k)} \right] (\varphi_i/\varphi_k) \right\}^2 \right]^{1/2} \times \left[\frac{1 + (\varphi_j/\varphi_k) + (\varphi_i/\varphi_k)}{N} \right]^{1/2}$$

Notice that σ_{q_j/q_k} is proportional to $N^{-1/2}$ and that the coefficient of proportionality can be obtained either from equation (2) or from families of curves such as those shown in Figures 1 and 2. For each point on these plots it is possible to calculate a value of

σ_{q_j/q_k} , and thereby to generate a new family of curves consisting of lines of constant fractional statistical uncertainty in q_j/q_k . These curves are shown (dotted) in Figures 3 and 4, assuming that a total of 1000 events is observed, distributed among the three isotopes being considered. For each of the two elements that we are considering (S and Ca), one expects to observe between several hundred and a thousand events with present-day isotope spectrometers. It should be noted in the case of sulfur (see Fig. 3) that even with 1000 events it is only possible to distinguish a solar-like source composition from a source consisting of a single isotope (^{32}S) by $\sim 1\frac{1}{2}$ standard deviations, even if only statistical errors are considered. In the case of the calcium isotopes (Fig. 4), the situation is even more severe. Note, however, that the use of multiple tracers will improve the statistical uncertainties. For example, the use of both ^{33}S and ^{37}Ar as tracers for the S isotopes would reduce the statistical uncertainties by $\sim 2^{1/2}$.

b) Spallation Cross-Section Errors

In evaluating the secondary contribution to the observed fluxes, it is necessary to use nuclear spallation cross sections for production of the species of interest in collisions between heavier cosmic rays and interstellar hydrogen and helium. Although several

FIG. 3.—Curves of constant relative uncertainty in the calculated source abundances due to spallation cross-section errors (*dashed*) and statistical errors (*dotted*) for the sulfur isotopes.

FIG. 4.—Curves of constant relative uncertainty in the calculated source abundances due to spallation cross-section errors (dashed) and statistical errors (dotted) for the calcium isotopes.

groups (Raisbeck and Yiou 1976; Lindstrom *et al.* 1975) have recently measured many of these cross sections, the majority of the spallation cross sections required in studying species up through the iron group have not been measured. In these cases it is necessary to rely on calculated cross sections. The semiempirical formulae of Silberberg and Tsao (1973), based on studies of the systematics of the measured cross sections, are particularly useful. From comparisons by Silberberg and Tsao (1973) of their calculated cross sections with the measurements on which the formulae are based, and from comparisons of the predictions of these formulas with subsequent measurements (Lindstrom *et al.* 1975), it is estimated that the uncertainties in these calculated cross sections are typically $\sim 35\%$. Uncertainties in measured cross sections tend to be somewhat smaller, $\sim 10\text{--}20\%$ in many cases (see, e.g., Raisbeck and Yiou 1976).

In deriving cosmic-ray source abundance ratios, it is in many cases not essential to know precisely the absolute spallation cross sections. This can be seen by considering equation (3). Here the spallation cross

sections appear in terms of the form:

$$\frac{S_j}{S_i} = \frac{\sum_l \varphi_l [\sigma_{plj} + (n_{\text{He}}/n_{\text{H}})\sigma_{alj}]}{\sum_l \varphi_l [\sigma_{pli} + (n_{\text{He}}/n_{\text{H}})\sigma_{ali}]}$$

By noting that the cross sections σ_{pli} and σ_{alj} are approximately proportional (Silberberg, Tsao, and Shapiro 1976), this becomes

$$\frac{S_j}{S_i} \approx \frac{\sum_l \varphi_l \sigma_{plj}}{\sum_l \varphi_l \sigma_{pli}} \quad (4)$$

In the extreme case, where only a single parent species b dominates the production of the secondaries which we are considering, this becomes $S_j/S_i \approx \sigma_{pbj}/\sigma_{pbi}$. That is, it is only necessary to know the ratio of the cross sections for producing species i and j from this single parent. In the more realistic case, where there are a number of parents which contribute to the secondary production, a significant reduction in the uncertainty in S_j/S_i and S_k/S_i (and hence in the calculated source-abundance ratio) can still be

FIG. 5.—Reduction of uncertainty in the secondary production ratio S_j/S_i due to reduction of uncertainties in cross-section ratios from individual parents. (a) $j \equiv {}^{34}\text{S}$, $i \equiv {}^{33}\text{S}$; (b) $j \equiv {}^{32}\text{S}$, $i \equiv {}^{33}\text{S}$; (c) $j \equiv {}^{42}\text{Ca}$, $i \equiv {}^{43}\text{Ca}$; (d) $j \equiv {}^{40}\text{Ca}$, $i \equiv {}^{43}\text{Ca}$.

realized if measurements are available of the ratios for producing the two species from individual parents. The joint distribution of the cross sections σ_{pi} and σ_{pj} can be described by a bivariate normal distribution with correlation coefficient

$$\rho = 1 - \frac{1}{2}(r_{ij}^2/r^2), \quad (5)$$

where r is the relative rms uncertainty in the individual cross sections for producing species j and species i , and r_{ij} is the relative rms uncertainty in the ratio $(\sigma_{pj}/\sigma_{pi})$ of these cross sections. No correlation has been assumed between cross sections for production from different parents.

In Figure 5 we show the relative uncertainties in the ratio S_j/S_i for the pairs of isotopes ${}^{34}\text{S}$, ${}^{33}\text{S}$; ${}^{32}\text{S}$, ${}^{33}\text{S}$; ${}^{42}\text{Ca}$, ${}^{43}\text{Ca}$; and ${}^{40}\text{Ca}$, ${}^{43}\text{Ca}$ as a function of r_{ij} (which is assumed to be independent of the parent being considered). In obtaining the correlation coefficient ρ from equation (5), we have taken the individual cross section uncertainties to be 35% ($r = 0.35$), independent of parent species. The case of totally uncorrelated cross sections (to be considered below) corresponds to $\rho = 0$ ($r_{ij} = 0.35\sqrt{2}$). It can be seen from Figure 5 that if cross section ratios can be measured with reasonable uncertainties, say ~ 10 – 20% , our knowledge of the ratio of secondary production rates S_j/S_i can be improved by a factor ~ 2 – 3 . Similar calculations for cases (such as the isotopes of Ne) where a larger number of parents contribute significantly to the secondary production yield similar improvements. Thus cross-section ratio measurements, which are only weakly affected by the various systematic uncertainties (e.g., errors in the beam flux or the target thickness), which make precise measurements of absolute cross sections difficult, are of significant value.

For any combination of local flux ratios φ_i/φ_k and

φ_j/φ_k , it is possible to calculate the uncertainty produced in q_j/q_k by uncertainties in the spallation cross sections. Figures 3 and 4 show (dashed curves) the fractional uncertainties in q_j/q_k produced by uncorrelated uncertainties of 35% in all of the spallation cross sections.² The fractional uncertainties in q_j/q_k scale in proportion to the value which we assume as the typical uncertainty in the spallation cross sections, provided that this uncertainty is $\lesssim 35\%$. This scaling breaks down for larger cross-section errors since in this case the uncertainty in the secondary component of the flux of species k becomes, for the largest path length which we are considering, comparable to the primary component of this flux.

As observed above in the case of statistical uncertainties in the cosmic-ray observations, we see (Figs. 3 and 4) that for the case of a solar-like cosmic-ray source and propagation through 5.5 g cm^{-2} of interstellar material, the present level of cross-section uncertainties will only allow distinction between a solar-like source composition and a pure ${}^{32}\text{S}$ composition at a $\sim 1\frac{1}{2}\sigma$ level, and will not allow such a distinction to all in the case of the calcium isotopes.

c) Other Sources of Uncertainty

In addition to uncorrelated errors in the spallation cross sections, there are a number of other sources of uncertainty in the calculation of source abundances from locally observed fluxes. Since the overall uncertainty in the calculation is dominated by the spallation cross-section errors, we will confine our treatment to a brief comparison of the relative contributions of the various minor sources of error. In this way, it is

² In our treatment of cross-section uncertainties, we have considered only uncertainties in the effective cross section, $\sigma_{pi} + (n_{\text{He}}/n_{\text{H}})\sigma_{\text{alt}}$, and not individual uncertainties in σ_{pi} and σ_{alt} .

TABLE 3
MINOR SOURCES OF UNCERTAINTY IN THE CALCULATION OF SOURCE ABUNDANCES OF SULFUR ISOTOPES

SOURCE OF UNCERTAINTY	RELATIVE MAGNITUDE	
	Solar-like Cosmic-Ray Source	Range*
35% spallation cross-section errors (uncorrelated).....	$\equiv 1^\dagger$	$\equiv 1^\dagger$
Parent elemental abundances ‡	0.15	0.10–0.26
Randomization of parent isotopes §	0.15	0.12–0.16
± 0.2 uncertainty in spectral indices for S isotopes in the local interstellar medium.....	0.24	0.14–0.62
20% error in normalization of spallation cross sections.....	0.018	0.009–0.067
10% errors in total inelastic cross section (uncorrelated).....	0.015	0.008–0.048
10% error in normalization of total inelastic cross sections.....	0.012	0.006–0.048

* For local $^{33}\text{S}/^{32}\text{S}$ between 0.08 and 0.24 and source $^{34}\text{S}/^{32}\text{S}$ between 0.01 and 0.50.

† Normalization.

‡ Assuming composition uncertainties as given by Silberberg, Tsao, and Shapiro 1976, Table 6.

§ Isotopic composition of local elements randomized assuming all stable isotopes (including pure electron-capture species) are equally likely.

possible to determine the extent to which reduction of the spallation cross-section errors will reduce the uncertainty in the derived source ratios.

In Table 3 we show, for the case of the isotopes of sulfur, the magnitude of the uncertainty contributed by the various minor sources of error relative to the uncertainty caused by 35% uncorrelated errors in the spallation cross sections. We show both the value of this uncertainty ratio for the case of solar-like source composition and the range obtained when the source ratio $^{34}\text{S}/^{32}\text{S}$ is varied between 0.01 and 0.5 (compared to the solar system value of 0.046) and the local ratio $^{33}\text{S}/^{32}\text{S}$ (which parametrizes the amount of interstellar matter traversed) is varied between 0.08 and 0.24 (compared to a value of 0.16 obtained when source material of solar system composition is propagated through an exponential distribution of path lengths with a mean of 5.5 g cm^{-2}). The various errors considered are described below.

The local composition (both elemental and isotopic) of the parent species is needed for calculating the relative rates of secondary production of the sulfur isotopes. To evaluate the effect of elemental composition uncertainties, we held the local isotopic composition of each parent element fixed and varied the elemental abundances independently with Gaussian distributions with standard deviation obtained from the summary given by Silberberg, Tsao, and Shapiro (1976). For the more abundant elements, the rms uncertainties are typically $\sim 10\%$.

Since at present the information available concerning the isotopic composition of heavy parent elements is scanty, we evaluated the effect of uncertainties in the parents' isotopic composition by independently randomizing the isotopic composition of each parent element under the assumption that each stable isotope is equally likely. Table 3 shows the error produced in the $^{34}\text{S}/^{32}\text{S}$ source ratio by the randomization. In the case being considered, the value of the source ratio calculated using a local isotopic composition obtained

by propagating solar-like source material through an exponential distribution of path lengths with a mean of 5.5 g cm^{-2} differs from the mean value obtained using randomized isotopes by less than the standard deviation which results from the randomization.

In applying the tracer formalism to other cases, it is necessary to determine the suitability of employing random isotopic compositions of the parent elements. If, for the dominant parent elements, the relative cross sections for producing the isotopes of interest does not depend strongly on which isotope of the element is producing the secondaries, then detailed information about the isotopic composition of the parents is not essential, and randomization of the parent isotopes should be a reasonable approximation. There are cases, however (e.g., the production of the argon isotopes ^{36}Ar , ^{38}Ar , and ^{37}Ar from calcium and iron), where the cross-section ratios depend strongly on which isotope of the parent elements is responsible for the secondary production. In such cases, considerable reduction in the uncertainty in the calculation can be achieved if the isotopic composition of the dominant parent elements can be measured.

Local spectral indices for the three sulfur isotopes being considered cannot be neglected in equation (2) when the effects of ionization-energy loss are important. These spectral indices will be uncertain both because early isotope experiments will lack sufficient statistics for precisely determining the spectra of individual isotopes, and because there is a significant uncertainty involved in determining local interstellar spectra from the spectra which are observed at Earth after having undergone solar modulation. Investigations by a number of authors (Lezniak and Webber 1971; Urch and Gleeson 1972; Garrard 1973; Garrard, Stone and Vogt 1973) indicate that the uncertainties in the local hydrogen and helium spectra and in the level of solar modulation result in an uncertainty of approximately ± 0.2 in the index of the local interstellar spectra at the energy of interest here (700 MeV per amu). By

independently randomizing the spectral index of each sulfur isotope with a Gaussian distribution with a standard deviation of 0.2, we obtain the errors shown in Table 3 because of uncertainty in the local spectra. Note, however, that this level of uncertainty can be achieved only if the local spectra can be measured with high statistical accuracy.

In order to make a rough estimate of the uncertainty in the spectral index due to the finite statistical accuracy of the measurement, assume that N_1 events are accumulated at energy E_1 and N_2 events are accumulated at energy E_2 during a fixed observation time. If the spectral index is γ , then $N_2 \approx N_1(E_2/E_1)^{-\gamma}$. Also define $N \equiv N_1 + N_2$. Then the uncertainty in the spectral index is

$$\sigma_\gamma = [2 + (E_2/E_1)^\gamma + (E_2/E_1)^{-\gamma}]^{1/2} N^{-1/2} / \ln(E_2/E_1)$$

For the energy intervals and spectral indices being considered here, $E_2/E_1 \approx 1$, and σ_γ reduces to

$$\sigma_\gamma \approx 2N^{-1/2} / \ln(E_2/E_1).$$

This value of σ_γ is slightly smaller than that which would be obtained when γ is determined from N events distributed throughout the interval from E_1 to E_2 . If one is to obtain an uncertainty of $\sigma_\gamma \approx 0.2$ from measurements in an energy interval 100 MeV per amu wide at 700 MeV per amu, it is necessary to accumulate ~ 5000 events.

In addition to the uncorrelated errors discussed in the previous section, the spallation cross-section errors (especially those obtained from semiempirical formulae) may be subject to an overall normalization error. Assuming an average deviation of 20% between measured and calculated cross sections (Lindstrom *et al.* 1975), we have calculated the effect of such an error on the derived source abundances. Since the ratios S_j/S_i and S_k/S_i in equation (2) are not affected by this error (since it introduces the same factor in both numerator and denominator) its effect is quite small (see Table 3).

The final source of uncertainty which we have considered is due to errors in the total inelastic cross sections. As in the case of the partial cross sections treated above, we have considered both uncorrelated errors in the total inelastic cross sections for each of the sulfur isotopes and errors in normalization of all of these cross sections. Based on the work of Cheshire *et al.* (1974) and Lindstrom *et al.* (1975), we have adopted 10% as an upper limit to the magnitude of each of these cross-section errors. The effects of such errors on the determination of the source ratio, $^{34}\text{S}/^{32}\text{S}$, are shown in Table 3.

It should be noted that since the various sources of error which we have examined are independent, the overall uncertainty in the source ratio is a quadratic sum of the various contributions listed in Table 3. Therefore it is clear that at present the spallation cross-section errors contribute most of the uncertainty in calculating the source ratio.

IV. EFFECT ON THE DETERMINATION OF ASTROPHYSICAL PARAMETERS

The ultimate goal of experiments which attempt to determine the composition of the cosmic-ray sources is to obtain information about the astrophysical environment and processes by which the cosmic-ray source material is synthesized. Nucleosynthesis models predict the abundance ratios which will be produced under various physical conditions. Therefore, by comparing abundances derived from experimental observations with such predictions, it should be possible to determine the values of some of the parameters in these models. In order to determine the level of uncertainties which can be tolerated in an experiment, it is necessary to examine the sensitivity of the composition to changes in the parameters which one would like to determine.

As an example, we shall consider the production of the elements sulfur through calcium in an explosive oxygen-burning process. Woosley, Arnett, and Clayton (1973) have made a detailed analysis of the nuclear abundances resulting from such a process, as a function of the temperature, density, and neutron excess in the explosion. The neutron excess is defined as

$$\eta = (n_n - n_p)/(n_n + n_p),$$

where n_n and n_p are the number densities (both free and bound in nuclei) of neutrons and protons, respectively, in the region of the explosion. The isotopic composition resulting from explosive oxygen-burning is particularly sensitive to η (Woosley, Arnett, and Clayton 1973). We shall examine the extent to which uncertainties in cosmic-ray observations and in the spallation cross sections needed to correct those observations for secondary production limit our ability to distinguish among sources characterized by various values of η .

In Figures 1 and 2 we have plotted the abundance ratios at the source and in the local cosmic rays which would result from nucleosynthesis in environments having various levels of neutron excess, and subsequent propagation characterized by a mean path length $\Lambda = 5.5 \text{ g cm}^{-2}$. Notice that $\eta \approx 2 \times 10^{-3}$ corresponds to a solar-like source composition. The isotopic source abundances of the elements S through Ca are obtained from Woosley, Arnett, and Clayton (1973) while the source composition of elements heavier than calcium are assumed to be solar-like.

The closer the value of η for the sources which produce the cosmic rays is to the value at which solar-like abundances are produced, the more precise must be the observations in order to distinguish this difference. In Figures 6 and 7, we show the number of events which must be accumulated (right-hand scale) and the level of cross-section errors which can be tolerated (left-hand scale) if one wishes to distinguish, at a 2σ level, the abundances expected from a source of neutron excess η from the abundances expected from a solar-like source. We have assumed that the statistical errors and the cross-section errors contribute

FIG. 6.—Number of events (right-hand scale) required and level of uncertainty in the fragmentation cross sections (left-hand scale) required to distinguish a cosmic-ray source with neutron excess η from a solar-like source (arrow) for the case of sulfur isotopes.

equally to the overall uncertainty in the source ratio. By using the fact that the error due to statistics scales as $N^{-1/2}$, where N is the number of events observed, and the fact that the error due to cross-section uncertainties is proportional to the typical cross-section error, it is a simple matter to scale the curves in Figures 6 and 7 for other combinations of these uncertainties.

In the case of the isotopes of sulfur, the statistics available from present experiments (<1000 sulfur events over the entire observing time of an experiment) will not allow one to distinguish production in a source with the relatively large neutron excess of 5×10^{-3} from production in a solar-like source. Also, cross-section uncertainties of $\sim 35\%$ prevent one from distinguishing values of η less than about 5×10^{-3} from the solar value.

FIG. 7.—Number of events required (right-hand scale) and level of uncertainty in the fragmentation cross sections (left-hand scale) required to distinguish a cosmic-ray source with neutron excess η from a solar-like source (arrow) for the case of calcium isotopes.

It is possible to reduce both of these limits. In order for the statistical limit to be significantly reduced, it will be necessary to substantially increase the exposure factor of isotope experiments. This can be most easily accomplished if present-day balloon experiments (typical geometrical factor $\sim 500\text{--}1000 \text{ cm}^2 \text{ sr}$) can be exposed for times commonly attained with spacecraft observations ($\gtrsim 1 \text{ yr}$).

Reduction of the limit due to spallation cross-section uncertainties will require measurements of the more important cross sections for producing the isotopes in the region we are considering. The availability of an ^{56}Fe beam at the Bevalac should make such measurements possible in the near future. Also, as pointed out above, substantial reduction of this limit can be achieved merely by obtaining precise ratios of cross sections for production from individual parents.

V. EFFECTS OF SOLAR MODULATION

The preceding treatment assumes throughout that observations can be made of cosmic-ray abundances in local interstellar space. However, due to interplanetary propagation effects, the cosmic-ray fluxes which we observe in the vicinity of Earth can be significantly altered from those present in interstellar space. For this reason it is necessary to use near-Earth measurements along with solar modulation models for obtaining the interstellar flux. This process introduces additional uncertainty, since the level of modulation varies throughout the solar cycle, and at any given time the level of modulation is somewhat uncertain.

In attempting to assess the magnitude of the uncertainty introduced by this effect, we have adopted the "force field" approach (Gleeson and Axford 1968) to calculation of solar modulation effects. This approximation has been shown (Lezniak and Webber 1971) to adequately reproduce the observed effects of solar modulation on the spectra of cosmic-ray nuclei at the energies of interest here (above a few hundred MeV per amu). In addition, we have assumed that the cosmic-ray diffusion coefficient is of the form

$$\kappa(r, P) = k\beta P \quad (r < D), \\ = 0 \quad (r > D),$$

where P is the particle's magnetic rigidity, β is its velocity in units of the velocity of light, r is the distance from the sun, D is a constant characterizing the spatial extent of the modulation region, and k is a scale factor which varies slowly throughout the 11 year solar cycle. The precise form assumed for the radial and rigidity dependences of the diffusion coefficient does not strongly affect the present treatment.

At any given time in the solar cycle, one can characterize the level of modulation by a parameter ϕ , which is simply the average change in rigidity experienced by particles in penetrating from the local interstellar medium to the orbit of Earth. This change in rigidity can be used, along with the energy per nucleon and

mass per unit charge of a given species, to calculate Φ , the mean change in those particles' energy per nucleon. In the force-field approximation, the spectrum at Earth $J_{1\text{ a.u.}}$ is related to the spectrum in interstellar space J_{IS} by

$$J_{\text{IS}}\left(\frac{E}{M}\right)$$
$$= \frac{(E/M + m_p c^2)^2 - (m_p c^2)^2}{(E/M - \Phi + m_p c^2)^2 - (m_p c^2)^2} J_{1\text{ a.u.}}(E/M - \Phi).$$

Using this prescription for calculating the flux at a given energy per nucleon in interstellar space, it is possible to calculate the relative interstellar abundances of selected species at a selected energy per nucleon. Note that in this calculation the local fluxes for the different species are evaluated at slightly different energy per nucleon, since Φ depends on the particle's mass-to-charge ratio.

Using this model, we have investigated the sensitivity of the calculated interstellar ratios to the value of the modulation parameter ϕ . In Table 4 we show values of the quantity $R_{\text{IS}}^{-1} \cdot \partial R_{\text{IS}} / \partial \phi$, where $R_{\text{IS}} (\equiv \varphi_j / \varphi_k)$ is the interstellar abundance ratio of two species. In the table, we have assumed that the mass-to-charge ratio of the two species differs by 10%. Values are shown for interstellar energies ranging between 500 MeV per amu and 5 GeV per amu and for three values of ϕ (corresponding to no modulation, modulation characteristic of solar minimum, and modulation characteristic of solar maximum). We have assumed that the interstellar spectra are of the shape

$$J_{\text{IS}} \propto (E/M + \alpha m_p c^2)^{-2.65},$$
(6)

with $\alpha = 0.75$. The values given in the table will be reduced by $\sim 10\%$ for a power law in total energy ($\alpha = 1$), while for a power law in kinetic energy ($\alpha = 0$), they will be increased by up to $\sim 70\%$.

From Table 4 it can be seen that a 100 MV uncertainty in the modulation parameter ϕ will produce an uncertainty of $\lesssim 2\%$ in the calculated interstellar ratio between the isotopes ^{34}S and ^{32}S , which have mass-to-charge ratios differing by 6%. This uncertainty will result in an uncertainty in the source abundances of similar magnitude. Estimates of ϕ should be accurate to better than 200 MV, since a change of 200 MV

corresponds to $\sim 30\%$ of the total change of ϕ between solar minimum and solar maximum, and one expects ϕ to undergo a change of this size over a period of $\sim 1\frac{1}{2}$ yr. Thus the uncertainty in the solar modulation correction should not seriously degrade the determination of the source abundance ratio of $^{34}\text{S}/^{32}\text{S}$. However, determinations of source-abundance ratios involving an isotope whose local flux contains only a few percent of surviving primary particles (as is the case with ^{42}Ca from a solar-like source, as discussed above) will require improved knowledge of solar modulation effects.

VI. EFFECTS OF THE PATH-LENGTH DISTRIBUTION

In the development of the tracer formalism we have assumed that the propagation of cosmic rays can be accurately described by the leaky-box model (i.e., an exponential distribution of potential path lengths). We now wish to examine the extent to which the formalism can be applied if the path-length distribution is of some other form. Notice that since we obtain the secondary contribution to the flux of a given species by multiplying the total observed flux of the tracer (assumed to be entirely secondary) by the ratio of the production rates for these two species, our results ought to be approximately correct for an arbitrary distribution of path lengths. These results, however, are not strictly correct since for other path-length distributions the composition of the parents (used in calculating the production rates of the species being considered) vary depending on the grammage at which the secondaries are produced. However, if the path-length distribution is dominated by path lengths short enough so that the parent composition is not significantly altered in traversing them, then the tracer calculation should give a reasonable estimate of the source-abundance ratio. Alternatively, if the relative cross sections for producing the three isotopes being examined are approximately independent of the parent isotope, then the tracer results should be applicable, even if the path-length distribution does contain a substantial fraction of long path lengths.

In the case of the isotopes of sulfur, we have investigated the extent to which an analysis based on equation (2) can reproduce correct source-abundance

TABLE 4
EFFECT OF MODULATION UNCERTAINTY ON CALCULATED INTERSTELLAR ABUNDANCES

INTERSTELLAR ENERGY (MeV per amu)	100 · $R_{\text{IS}}^{-1} \partial R_{\text{IS}} / \partial \phi$ (% per MV)		
	No Modulation ($\phi = 0$)	Solar Minimum ($\phi = 400$ MV)	Solar Maximum ($\phi = 1100$ MV)
500.....	0.0160	0.0146	0.0100
750.....	0.0134	0.0127	0.0104
1000.....	0.0116	0.0112	0.0099
1500.....	0.0091	0.0090	0.0085
2000.....	0.0075	0.0074	0.0073
5000.....	0.0036	0.0036	0.0036

TABLE 5
APPLICATION OF TRACER FORMALISM TO LOCAL COMPOSITIONS FROM
NONEXPONENTIAL PATH-LENGTH DISTRIBUTIONS

Path-length Distribution*	Parameters† (g cm ⁻²)		Local ³⁴ S/ ³² S	Source Ratio Discrepancy††
Slab.....	0.5	...	0.024	1.00
Slab.....	1.5	...	0.069	1.03
Slab.....	2.5	...	0.121	1.10
Gaussian.....	0.0	1.0	0.036	1.00
Gaussian.....	1.0	1.0	0.057	1.01
Gaussian.....	2.0	1.0	0.090	1.04
Gaussian.....	3.0	1.0	0.145	1.12
Gaussian.....	0.0	2.0	0.067	1.01
Gaussian.....	1.0	2.0	0.084	1.02
Gaussian.....	2.0	2.0	0.108	1.04
Gaussian.....	3.0	2.0	0.145	1.09
Gaussian.....	3.5	2.0	0.170	1.14
Gaussian.....	0.0	3.0	0.095	1.02
Gaussian.....	1.0	3.0	0.111	1.03
Gaussian.....	2.0	3.0	0.133	1.05
Gaussian.....	3.0	3.0	0.164	1.10
Gaussian.....	3.5	3.0	0.184	1.13
Truncated Exponential.....	5.0	0.5	0.166	1.02
Truncated Exponential.....	5.5	0.5	0.185	1.06
Truncated Exponential.....	6.0	0.5	0.205	1.09
Truncated Exponential.....	5.0	1.0	0.178	1.03
Truncated Exponential.....	5.5	1.0	0.198	1.06
Truncated Exponential.....	6.0	1.0	0.218	1.10
Truncated Exponential.....	5.0	1.5	0.190	1.03
Truncated Exponential.....	5.5	1.5	0.209	1.07
Truncated Exponential.....	6.0	1.5	0.230	1.11

* For all pure exponential path-length distributions, the tracer technique reproduces the source abundances exactly.

† Parameters describing path-length distributions are as follows: for slabs, slab thickness; for Gaussians, mode and standard deviation; for truncated exponentials, characteristic grammage of exponential part and grammage at which linear and exponential are joined.

†† Ratio of source ³⁴S/³²S calculated using tracer formalism to Cameron's (1973) value of ³⁴S/³²S (assumed as the cosmic-ray source composition).

ratios if the propagation is described by various non-exponential path-length distributions. We have considered the following path-length distributions: slabs with thickness between 0.5 and 4.5 g cm⁻² (with He/H = 0.1 by number in the interstellar gas); Gaussians with most probable path lengths of between 0 and 4.0 g cm⁻² and standard deviations of 1.0, 2.0, and 3.0 g cm⁻²; and "truncated exponentials" (i.e., a linear rise from 0 to x_c g cm⁻²) characterized by $x_c = 0.5, 1.0,$ and 1.5 g cm⁻² and $x_0 = 5.0, 5.5,$ and 6.0 g cm⁻². For each path-length distribution, we calculated the source elemental composition required to produce the local elemental abundances compiled by Silberberg, Tsao, and Shapiro (1976) under the assumption that the isotopic composition of the source is identical to that compiled by Cameron (1973) for solar-system material. In some cases, such a calculation yields unphysical (i.e., negative) values of some source abundances. We set all such source abundances to zero. This set of source abundances were then propagated through the assumed path-length distribution to obtain local isotopic composition. The local elemental abundances obtained were compared with the Silberberg, Tsao, and Shapiro (1976) compilation, and, in cases where any element between S and Ni

showed in inconsistency of more than three standard deviations, we eliminated that path-length distribution from further consideration. For the path-length distributions yielding consistent elemental abundances, we performed a tracer calculation based on equation (2) of the source ratio, ³⁴S/³²S. The results of these calculations were compared with the assumed ratio (Cameron 1973).

In Table 5 we show, for selected path-length distributions, the local ³⁴S/³²S ratio (which is an approximate measure of the grammage traversed) and the ratio of the source ³⁴S/³²S value obtained from the tracer calculation to the Cameron value. It can be seen from this table that, for a variety of path-length distributions, the tracer calculation yields a source ratio (³⁴S/³²S) which agrees within 15% with the Cameron value which was assumed as a starting point for the calculation. The 15% difference is significantly smaller than the uncertainty in the determination of the source ratio due to errors discussed above. The agreement illustrates the utility of the secondary tracer approach for deriving source-abundance ratios in a way which is approximately model independent.

In cases where the ratio of surviving primaries to the total observed flux is at least as large as in the case

of ^{34}S from a solar-like source (i.e., surviving primary flux/total local flux $\gtrsim 0.15$) uncertainties concerning the exact form of the path-length distribution should not at present be a serious limit to our ability to obtain source-abundance ratios.

In a similar calculation for the case of the calcium isotopes where the observed ^{42}Ca flux contains only 1.5% of surviving primaries, the tracer calculation is not able to reproduce accurately the correct source ratio, $^{42}\text{Ca}/^{40}\text{Ca} = 0.006$ (Cameron 1973), when reasonable nonexponential path-length distributions are assumed. In such cases where only a small fraction of the observed flux is due to surviving primaries, the differences in the relative secondary production rates for exponential and nonexponential path-length distributions can produce errors in determining the surviving primary flux which are comparable to that flux.

VII. FURTHER APPLICATIONS

In the preceding sections we have examined in detail the uncertainties which affect the application of the tracer technique to the analysis of the isotopic source composition of sulfur and calcium. It is possible to generalize the results obtained in these cases in order to obtain some indication of the applicability of the tracer technique to other cases. By restricting the analysis to cases where the two primary species of interest are isotopes of the same element, one can avoid uncertainties due to possible chemical selection processes which may play a role in the acceleration of the cosmic rays. Also, since relative uncertainties in the

cosmic-ray observations and in the calculated secondary corrections are significantly magnified in the deduced source abundances in cases where the local flux consists largely of secondary particles, source abundances obtained by application of the tracer technique will be most accurate when the local fluxes of both species of interest are largely of primary origin.

The choice of purely secondary species to serve as tracer of the propagation can be made from isotopes of the same element as the primary species or of a nearby element (since the secondaries are produced at high energies and are unaffected by chemical selection processes). It is, however, important that the secondary species selected be produced predominantly by the same parents which produce the secondaries of the other two species being considered. First, such a choice of tracer will minimize the possibility that the interstellar path length indicated by the tracer could be different from the path length appropriate to the other species of interest. And second, as we have shown, the uncertainty in the calculated ratio of secondary production rates can be significantly reduced by measurement of spallation cross-section ratios if the parentage of the tracer and of the other species of interest is similar.

In Table 6 we indicate a number of cases to which the tracer technique should be applicable. We have assumed that the elemental composition of the cosmic-ray source is as given by Silberberg, Tsao, and Shapiro (1976) and that the isotopic compositions of individual elements are identical to the solar-system composition given by Cameron (1973). In addition, we

TABLE 6
UNCERTAINTIES IN SOURCE ABUNDANCE RATIOS OBTAINED WITH THE TRACER TECHNIQUE

ISOTOPES			SOURCE RATIO (percent) q_j/q_k	UNCERTAINTIES IN SOURCE RATIO (percent)			
Primaries		Tracer i		Statistics*	Cross-Section Errors†	Solar Modulation‡	Tracer Contaminations§
k	j						
^{20}Ne	^{22}Ne	^{19}F	12	21	25–31	1.5	0.3
^{20}Ne	^{22}Ne	^{21}Ne	12	22	24–30	1.8	2.2
^{24}Mg	^{25}Mg	^{19}F	13	15	14–19	0.6	0.2
^{24}Mg	^{25}Mg	^{21}Ne	13	16	13–17	0.4	1.2
^{24}Mg	^{25}Mg	^{31}P	13	18	13–17	0.2	12
^{24}Mg	^{26}Mg	^{19}F	14	12	6–9	0.7	0.1
^{24}Mg	^{26}Mg	^{21}Ne	14	12	5–8	0.8	0.5
^{24}Mg	^{26}Mg	^{31}P	14	13	5–8	1.3	4.7
^{28}Si	^{29}Si	^{33}S	5.1	26	9–10	0.1	2.3
^{28}Si	^{29}Si	^{35}Cl	5.1	26	9–10	0.2	6.3
^{28}Si	^{30}Si	^{33}S	3.3	39	16–17	1.5	4.1
^{28}Si	^{30}Si	^{35}Cl	3.3	38	17–18	1.6	11
^{32}S	^{34}S	^{33}S	4.4	185	87–92	3.2	22
^{32}S	^{34}S	^{37}Ar	4.4	208	93–99	3.6	0
^{36}Ar	^{38}Ar	^{33}S	19	213	100–127	2.6	25
^{36}Ar	^{38}Ar	^{37}Ar	19	238	112–147	3.1	0
^{56}Fe	^{54}Fe	^{55}Mn	6.3	23	19–20	1.9	6.1
^{56}Fe	^{54}Fe	^{55}Fe	6.3	22	19–20	0.6	0

* $10^{10} \text{ cm}^2 \text{ s sr MeV per nucleon.}$
† Assumes cross-section errors of 35%.
‡ Assumes modulation parameter $\phi = 400 \pm 100 \text{ MV.}$
§ Assumes solar-system abundances.

have employed equation (3) as an approximation to the full tracer equation (that is, we have neglected ionization-energy loss and the small differences in nuclear interaction length between the tracer and the other two species). For each case we show the nominal source ratio (Cameron 1973) and the percentage uncertainties in this ratio due to (1) the finite statistical accuracy of the cosmic-ray observations; (2) the uncertainties in the ratios of secondary production rates, S_j/S_i and S_k/S_i ; (3) the uncertainty in the level of solar modulation; and (4) the possibility of contamination of the tracer by small amounts of primary material.

The statistical errors are based on measurements by an instrument with $10^8 \text{ cm}^2 \text{ sr s}$ exposure over an energy interval 100 MeV per nucleon wide, and on cosmic-ray fluxes normalized to an oxygen flux of 5×10^{-7} particles per $\text{cm}^2 \text{ sr s MeV}$ per nucleon as reported at 500 MeV per nucleon under solar minimum conditions by Garcia-Munoz *et al.* (1977). The errors due to uncertainties in S_j/S_i and S_k/S_i assume uncorrelated cross-section errors of 35%. A range of uncertainties is indicated since we have not examined the extent to which the uncertainties in S_j/S_i and S_k/S_i are correlated. The limits given in the table correspond to perfect correlation and perfect anticorrelation of these uncertainties. The errors due to solar modulation uncertainties are based on interstellar fluxes at 700 MeV per nucleon from a spectrum of shape given by equation (6), and modulation characterized by $\phi = 400 \text{ MeV}$ (solar minimum conditions) with an uncertainty $\sigma_\phi = 100 \text{ MV}$. Finally, the errors due to primary contamination of the tracer isotope are based on nominal source abundances of these species as obtained from Silberberg, Tsao, and Shapiro (1976) in conjunction with Cameron (1973). In the case of ^{19}F we have used the solar system value $^{19}\text{F}/^{16}\text{O} \approx 10^{-4}$ to estimate the ^{19}F source abundance, since Silberberg, Tsao, and Shapiro (1976) find that the local cosmic-ray abundances are consistent with the complete absence of fluorine at the source. The source abundances of the K-capture species ^{37}Ar and ^{55}Fe will be zero, as indicated in the table, if a time large compared to the half-lives of these species (33 days and 2.6 years, respectively) elapses between the synthesis and the acceleration of the cosmic-ray nuclei.

VIII. SUMMARY

We have presented a formalism, based on the leaky-box model of cosmic-ray propagation, by which it is possible to derive the relative cosmic-ray source abundances of two nuclides from their local abundances and the local abundance of a purely secondary nuclide. Such a treatment is possible because of the equilibrium, position-independent character of the leaky-box model. The effects of production by the cosmic-ray source and by spallation reactions and of loss by nuclear destruction and escape, as well as the

effects of energy-changing processes, are easily incorporated in this formalism. We have shown that although the formalism is based on the leaky-box model, the source-abundances derived by the tracer technique are reasonably independent of detailed propagation models, whether typified by exponential, slab, Gaussian, or truncated exponential path-length distributions. This insensitivity to path-length distribution depends on the use of a secondary tracer which is produced predominantly by the same parents that produce the secondary contributions to the primary isotopes being measured. We have identified a number of candidate tracers which meet this criterion. The tracer formalism permits a quantitative evaluation of the effects of observational uncertainties on the uncertainty of the derived source abundances. To illustrate these effects, we have considered in detail the application of the tracer technique to the isotopes of sulfur and calcium. For a source of solar-system composition, one obtains in the former case a local flux of the less-abundant primary (^{34}S) which consists of $\sim 15\%$ of surviving primary material. In the latter case, the surviving ^{42}Ca composes $\sim 1.5\%$ of the local flux.

We have shown that the uncertainty in source-ratio determinations for these nuclides is at present limited by statistical uncertainties in the measurement of local cosmic-ray fluxes (when measured by high-resolution particle identifiers) and by uncertainties in the spallation cross sections needed for calculating the secondary contributions to the local fluxes. The effects of these uncertainties and the uncertainty in solar modulation have also been estimated for a number of other primary nuclides which are in general less sensitive to these uncertainties because the surviving primaries account for a larger fraction of the observed abundance. In general, the tracer formalism should permit the determination of the source abundances of those nuclides for which the primary contribution to the observed flux exceeds a few percent.

In order to investigate the possible evolutionary differences between cosmic-ray sources and solar-system material, however, it will be necessary to reduce the magnitude of both the statistical and cross-section uncertainties. The statistical uncertainty can be reduced by increasing the exposure factors of cosmic-ray instruments (as will be possible when instruments with geometrical factors of $\sim 100 \text{ cm}^2 \text{ sr}$ or larger can be exposed for extended periods on spacecraft). Significant reduction of the error introduced due to uncertainties in the spallation cross sections needed for calculating the secondary contributions to the observed fluxes can be achieved if measurements of ratios of cross sections for producing the isotopes of interest from individual parents can be made with moderate precision. A table listing the relative importance of various parents has been included as a guide to the most important cross-section ratios to be measured.

Finally, it should be noted that although the detailed application of the tracer technique is illustrated for source abundances similar to the solar system, the approach is generally applicable to the derivation of

source-abundance ratios from cosmic-ray isotope measurements.

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APPENDIX

In Tables 7–11 we show the percentage combinations to the secondary production,

$$S_i = \sum_l \varphi_l \frac{\sigma_{pli} + (n_{He}/n_H)\sigma_{all}}{M_H + (n_{He}/n_H)M_{He}},$$

of the cosmic-ray species considered above due to spallation of the dominant parents. The parent fluxes φ_l used are the result of propagating source abundances with elemental composition from Silberberg, Tsao, and Shapiro (1976) and isotopic composition from Cameron (1973) through an exponential distribution of path lengths with a mean of ~ 5.5 g cm⁻² and with $n_{He}/n_H = 0.1$. The cross sections used are evaluated at an energy of 700 MeV per nucleon from the semi-empirical formulae of Silberberg and Tsao (1973) as modified by Silberberg, Tsao, and Shapiro (1976). As discussed in the main text, the uncertainty in the derived source abundance ratio can be significantly reduced by the measurement of relative fragmentation cross sections for those parent nuclides which contribute a significant fraction of both tracer and the secondary contribution to the isotopes under study.

TABLE 7
CONTRIBUTIONS TO THE SECONDARY PRODUCTION OF ²⁰Ne, ²²Ne, ²¹Ne, AND ¹⁹F

PARENT ISOTOPE	PERCENTAGE CONTRIBUTION OF PARENT TO SECONDARY PRODUCTION OF DAUGHTER			
	²⁰ Ne	²² Ne	²¹ Ne	¹⁹ F
²⁰ Ne.....	...	...	...	39.4
²⁴ Mg.....	30.5	33.4	29.8	18.5
²⁸ Si.....	19.8	21.2	19.5	12.0
²¹ Ne.....	7.7	...	...	2.7
²² Ne.....	6.7	...	14.3	4.8
²³ Na.....	6.3	12.6	6.4	4.0
²⁵ Mg.....	5.5	6.3	5.6	3.4
²⁷ Al.....	4.7	5.4	4.8	3.0
²⁶ Mg.....	4.0	5.2	4.3	2.7
³² S.....	2.3	2.5	2.3	1.4
⁵⁶ Fe.....	1.7	1.7	1.9	1.4
All others.....	10.8	11.7	11.1	6.7

TABLE 8
CONTRIBUTIONS TO THE SECONDARY PRODUCTION OF ²⁴Mg, ²⁵Mg, ²⁶Mg, ¹⁹F, ²¹Ne, AND ³¹P

PARENT ISOTOPE	PERCENTAGE CONTRIBUTION OF PARENT TO SECONDARY PRODUCTION OF DAUGHTER					
	²⁴ Mg	²⁵ Mg	²⁶ Mg	¹⁹ F	²¹ Ne	³¹ P
²⁰ Ne.....	...	...	...	39.4	...	...
²⁴ Mg.....	...	...	...	18.5	29.8	...
²² Ne.....	...	...	...	4.8	14.3	...
²³ Na.....	...	...	...	4.0	6.4	...
²⁸ Si.....	38.1	41.6	47.8	12.0	19.5	...
²⁵ Mg.....	17.9	...	...	3.4	5.6	...
²⁷ Al.....	8.8	10.9	16.2	3.0	4.8	...
²⁶ Mg†.....	7.4	15.0	...	2.7	4.3	...
³² S.....	4.5	5.0	3.9	1.4	2.3	32.3
⁵⁶ Fe.....	2.8	3.3	5.1	1.4	1.9	11.7
²⁹ Si.....	2.8	3.3	3.5	0.9	1.5	...
²⁶ Al†.....	2.0	3.4	...	0.6	1.0	...
³⁰ Si.....	2.0	2.5	4.0	0.7	1.2	...
All others...	13.7	15.0	19.5	7.2	7.4	56.0

† Assumes that all ²⁶Al survives in cosmic rays.

TABLE 9
CONTRIBUTIONS TO THE SECONDARY PRODUCTION OF ²⁸Si, ²⁹Si, ³⁰Si, ³³S, AND ³⁵Cl

PARENT ISOTOPE	PERCENTAGE CONTRIBUTION OF PARENT TO SECONDARY PRODUCTION OF DAUGHTER				
	²⁸ Si	²⁹ Si	³⁰ Si	³³ S	³⁵ Cl
²⁸ Si.....	17.0	...	...	...	...
³² S.....	15.5	17.8	22.9	...	...
⁵⁶ Fe.....	9.9	11.0	13.3	16.3	17.2
³⁰ Si.....	7.2	14.5	...	...	...
⁴⁰ Ca.....	4.9	4.7	5.1	8.2	10.1
³¹ P.....	4.7	6.5	9.0	...	...
³⁶ Ar.....	4.4	4.3	4.7	11.0	13.7
³⁴ S.....	3.1	4.1	3.9	10.1	...
³⁵ Cl.....	2.8	2.8	3.2	4.5	...
³⁶ Ar.....	2.7	2.9	3.4	5.0	6.6
³³ S.....	2.5	3.2	2.8	...	...
All others.....	25.3	28.2	31.7	44.9	52.4

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TABLE 10
CONTRIBUTIONS TO THE SECONDARY PRODUCTION OF
³²S, ³⁴S, ³⁶Ar, ³⁸Ar, ³³S, AND ³⁷Ar

PARENT ISOTOPE	PERCENTAGE CONTRIBUTION OF PARENT TO SECONDARY PRODUCTION OF DAUGHTER					
	³² S	³⁴ S	³⁶ Ar	³⁸ Ar	³³ S	³⁷ Ar
⁵⁶ Fe.....	17.3	21.3	20.9	26.0	16.3	15.3
⁴⁰ Ca.....	8.4	8.0	11.9	11.4	8.2	25.7
³⁶ Ar.....	7.9	7.2	...	...	11.0	...
³³ S.....	7.1	...	...	...	...	...
³⁴ S.....	5.6	...	...	...	10.1	...
³⁵ Cl.....	5.4	6.4	...	...	4.5	...
³⁸ Ar.....	4.7	5.4	6.5	...	5.0	9.8
³⁹ K.....	3.3	3.5	5.6	5.8	3.4	6.1
⁴⁷ Ti.....	2.8	3.4	4.0	4.8	2.9	3.4
⁴⁸ Ti.....	2.8	3.6	3.9	4.5	2.7	2.6
⁵² Cr.....	2.8	3.6	3.8	4.4	2.7	2.6
⁴⁶ Ti.....	2.7	3.1	3.8	4.3	3.0	4.2
⁴⁵ Sc.....	2.6	3.3	3.6	4.2	2.5	2.4
³⁷ Ar.....	2.1	2.9	4.3	...	2.2	...
⁴² Ca.....	2.1	2.4	2.8	3.1	2.2	2.9
⁴⁹ V.....	1.9	2.2	2.7	3.0	2.1	2.9
⁵⁴ Fe.....	1.8	2.1	2.5	2.9	2.0	2.6
All others...	18.7	21.6	23.7	25.6	19.2	19.5

TABLE 11
CONTRIBUTIONS TO THE SECONDARY PRODUCTION OF
⁵⁶Fe, ⁵⁴Fe, ⁵⁵Fe, AND ⁵⁵Mn

PARENT ISOTOPE	PERCENTAGE CONTRIBUTION OF PARENT TO SECONDARY PRODUCTION OF DAUGHTER			
	⁵⁶ Fe	⁵⁴ Fe	⁵⁵ Fe	⁵⁵ Mn
⁵⁶ Fe.....	...	85.0	92.6	96.3
⁵⁸ Fe.....	...	8.3	...	...
⁵⁷ Fe.....	36.9	1.4	1.6	1.6
⁵⁸ Ni.....	31.5	4.0	4.5	0.7
⁶⁰ Ni.....	18.3	0.7	0.8	0.7
All others.....	13.3	0.6	0.5	0.7

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