THE ECONOMIC JOURNAL

MAY 1992

The Economic Journal, **102** (*May* 1992) 437–460
Printed in Great Britain

MARSHALLIAN VS. WALRASIAN STABILITY IN AN EXPERIMENTAL MARKET*

Charles R. Plott and Glen George

The experiments discussed below are an attempt to examine concepts of stability as found in economic textbooks.[1] Two concepts of stability, which stem from two different concepts of market adjustment, seem to have dominated thinking. Whilst these two concepts are typically called Walras stability and Marshall stability, some controversy exists over the extent to which these two models represent their respective thinking.[2] No doubt the current formal statements of the theories reflect an evolution of the ideas through the work of many theorists. The terminology is retained for convenience. Regardless of their origins, these two concepts lead to competing hypotheses about the conditions under which market instability will be observed so the subject is a natural one for experimental investigation. Furthermore, since this is the first experimental examination of the stability of equilibria, the strategy is to inquire about stability in the context of these two classical models and to avoid the temptation to attempt to extend them or integrate them with more modern theory. The old models have not been checked. They seem to be an appropriate place to start.

The Walrasian model views price as changing in response to excess demand at that price. The Marshallian model views volume as adjusting in response to the difference between demand price and supply price at that volume. These two models hold different implications for general theories of disequilibrium and market adjustment, and they make different statements about the conditions under which markets will exhibit instability. If supply is negatively sloped and if demand cuts supply from above, then the equilibrium is Walrasian unstable and Marshallian stable. If supply is negatively sloped and if demand cuts supply from below, then the equilibrium is Walrasian stable and Marshallian unstable. In Fig. 1 the downward sloping supply function is the

* The financial support of the National Science Foundation is gratefully acknowledged as well as support from the California Institute of Technology Laboratory for Experimental Economics and Political Science. This project was first assigned as a project in an experimental economics class. Stephen Pitts contributed significantly to the development of instructions and to finding parameters of the continuous model that yielded acceptable integer solutions. The comments of John Ledyard and Jeffrey Dubin influenced the experimental design and data analysis. Comments by Gary Becker and Eskander Alvi were useful in helping us understand the theories and the literature. Special thanks go to Jessica Goodfellow for her help as a research assistant.

[1] A standard treatment can be found in Henderson and Quandt, 1980, p. 160.

[2] See a good summary of the controversy in Takayama (1974).

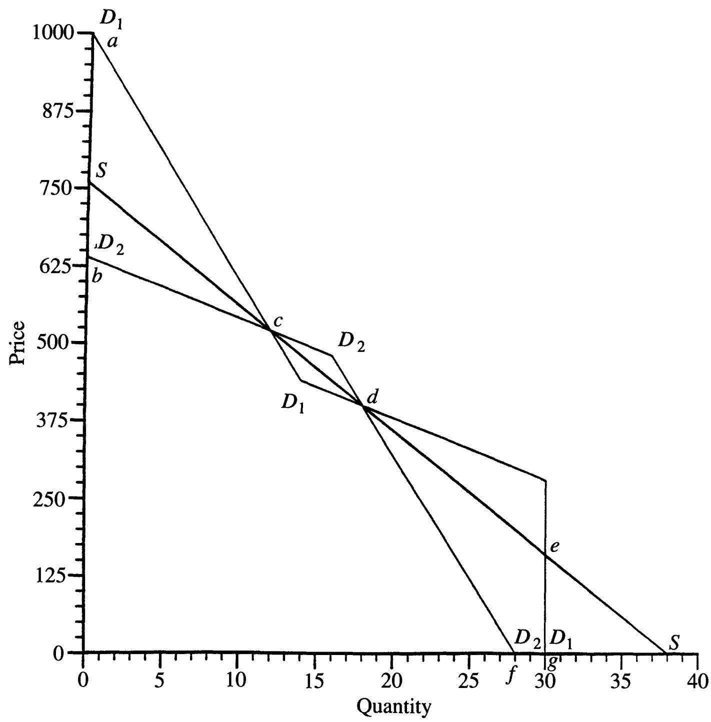

Fig. 1. Supply and demand curves (continuous).

curve SS. Given the demand function $D_1 D_1 D_1$, three equilibria exist excluding the boundaries. Points c and e are Walrasian unstable and Marshallian stable. Point d is Walrasian stable and Marshallian unstable. The vertical boundary contains an additional stable Walrasian (but not Marshallian) equilibrium and the horizontal boundary contains an additional stable Walrasian (but not Marshallian) equilibrium. If the demand function is $D_2 D_2 D_2$, point c is Walrasian (Marshallian) stable (unstable) and point d is Walrasian (Marshallian) unstable (stable). Point e is no longer an equilibrium. The vertical axis contains a Marshallian (but not Walrasian) stable equilibrium and the horizontal axis contains a Walrasian (but not Marshallian) stable equilibrium.

The curves in Fig. 1 are continuous approximations of the parameters actually used in the experimental markets. The markets first exist under conditions D_1 and according to theory the emerging prices should be near to one of the stable equilibria. Presumably prices will converge only to the stable equilibria. If demand is shifted to D_2 then prices and volume should move to one of the other equilibria because every stable equilibria under D_1 is unstable under D_2. This relationship among the equilibria is the key to the experiments reported in the paper.

Two major problems present themselves to anyone who attempts to conduct experiments suggested by these stability concepts. The first and most difficult is determining a method for experimentally inducing a negatively sloped supply. The markets created for this study all have 'forward falling' as opposed to 'backward bending' supply curves. The distinction is important because some believe that the nature of stability will change depending upon the conditions that generate the downward sloping supply. The decision to study the forward falling case reflects the existence of some fundamental limitations on the experimental methodology necessary for studying the backward bending case. The second problem involves a choice of market institutions. Three different market organisations are studied: the double auction, the sealed bid/offer, and the secant tâtonnement.[3]

Three basic questions are posed by the research. The first deals with the ability of the law of supply and demand to predict market behaviour. Downward sloping supply functions have not been studied experimentally so whether or not equilibration occurs is an open question. The forward falling case, which involves an externality, is especially problematic. The theory of market supply in this case is constructed from elements of a theory of consistent expectations. The question is: does the market demand and supply model predict price and volume? The second question is: does either concept of stability have predictive power? It is not obvious that these classical notions of stability, which originated by analogy to physical phenomena (Walras 1954, p. 112; Guillaband 1961, p. 346), as opposed to direct observations of markets, have any predictive power at all in a market context. The third question asks which stability concept is more appropriate. The fourth and final question is an inquiry about the sensitivity of answers to all of the above questions to market organisation. Certainly the usual description about Marshallian vs. Walrasian stability leans heavily on the existence of a price taking process such as tâtonnement. Does the relevance of a stability concept depend upon how the market is organised? The examination of this question required the first study of a new class of tâtonnement processes.

A controversy exists in the literature motivated by the interpretation of original texts. Some feel that the Marshallian concept should apply only in the presence of production which takes time and that the Walrasian model is more appropriate for exchange. Production in the economic environment studied in this paper does not have any obvious time dimension so one might conjecture that Marshallian stability is less likely to be observed. In that sense one could consider the environment to be biased in favour of the Walrasian concept. Thus, if Walrasian stability had been observed, subsequent experiments would have focused on environments with production lags to isolate the boundaries of the model. Since Marshallian stability is observed, the results are strong because the Marshallian concept has been found operating under circumstances that a branch of theory suggests that it should not be.

[3] So many alternative organisations exist that the narrowing of the options to only three necessarily involved some subjective decisions and does not mean that the alternative forms should be neglected. These three were chosen because previous experiments with them exist and would be valuable in case the downward sloping supply induced substantially inexplicable behaviour. Some practical considerations existed as well. For example, posted prices have been studied considerably but posted prices are related to sealed bid auctions which are related to the sealed bid/offer process.

Table 1

Redemption Value Table for Demanders

	Condition D_1			Condition D_2		
	Agent			Agent		
Unit	1 and 6	2 and 5	3 and 4	1 and 6	2 and 5	3 and 4
1	960	880	800	590	610	630
2	600	640	720	570	530	530
3	440	410	410	480	510	510
4	350	390	390	200	280	320
5	330	310	290	120	40	0
6	0	0	0	0	0	0

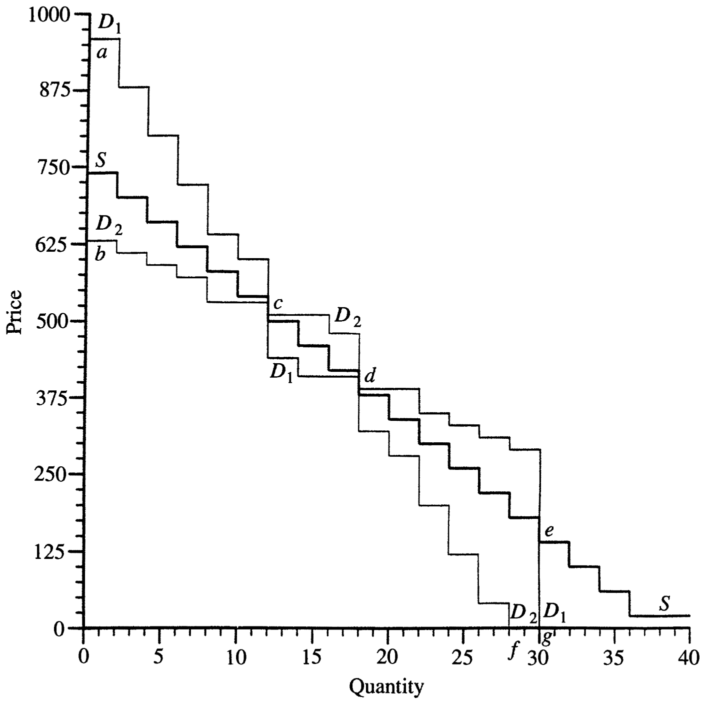

Fig. 2. Supply and demand curves (discrete).

I. PARAMETERS

The market demand and market supply functions based upon the actual parameters used in the experiment are shown in Fig. 2. Subjects were primarily inexperienced undergraduate students and graduate students from the California Institute of Technology. Preferences and costs were established by application of standard financial inducement techniques (see Plott (1982) or Smith (1982)). The actual instructions are included as an appendix. The franc/dollar conversion rate was 0·4 cent per franc for demanders and 1·1 cent per franc, 1·5 cent per franc and 1·5 cent per franc for the three types of sellers, respectively. After the first three experiments, which operated very slowly relative to the other experiments, the rates were 25 % less for buyers and 50 % less for sellers. The conversion rates were chosen to provide adequate incentives near the system equilibria.

A. *The Demand Curve*

Six agents were designated as demanders. These demanders were partitioned into two demanders of each of three types of agents. The redemption values are in Table 1. Buyers were given two tables. One contained their marginal redemption values for all periods, and the other contained totals.

Two different aggregate demands were used. These are labelled condition D_1 and condition D_2 as shown in Fig. 2. In most experiments a shift of demand occurred in period 10. The first ten periods were under condition D_1 and then the demand was shifted to D_2. However, in some experiments conditions D_1 were maintained longer than ten periods to see if convergence to a theoretical equilibrium would become 'closer' with more periods under the same demand conditions.

B. *The Supply Curve*

The literature discusses three forms of negatively sloped supply. The 'backward bending' curve is usually associated with 'backward bending' individual supplies derived from negative income effects. The backward bending labour supplies are the typical example. The second form is the 'forward falling' supply curve associated with the existence of an externality. According to the forward falling concept, specialisation in resource functions and resource supplies has the effect of lowering costs as an industry grows. Competition is maintained in the forward falling case because individual firms have upward sloping marginal costs, given the volume of others. However, costs fall as the volume of others increases. Lowered costs are translated to lower prices through competition. The third form of negative sloped supply involves falling average cost at the individual firm level over the whole range of demand. This latter case theoretically results in monopoly.

Our choice was to study the second form. The first, backward bending, necessitated the existence of at least two commodities. Conceptually, the appropriate two commodities could be created in a laboratory setting but practical problems associated with experimental methodology precluded an immediate examination of this case. The third form is interesting and has received some attention experimentally. The monopoly case, however, is not in the full spirit of the Marshallian vs. Walrasian stability issue. Our decision was to study the second 'forward falling' form.

The forward falling supply curve is usually attributed to 'external economies' of scale. As market volume increases, the cost to each firm decreases even though an individual firm's cost increases with an increase in its own volume (volume of other firms held constant). Technically speaking, this is an externality in the cost function of each firm. That is, a firm's costs depend upon its own output and the output of all other firms.

In general, each firm has a cost function of the form $C^i(X_i, \Sigma_{j \neq i} X_j)$. The cost function of firm i depends upon its own output, X_i, and the output of all other firms, $\Sigma_{j \neq i} X_j$. The latter is the key externality.

The theory of competitive behaviour yields an individual supplier's behavioural equation of the form

$$P = \frac{\partial C(X_i, q_i^e)}{\partial X_i}, \tag{1}$$

where P = market price

q_i^e = individual i's expectation about the quantity sold by others; i.e., $\Sigma_{j \neq i} X_j$.

Application of the theory of competitive supply yields an individual supply function of the form

$$X_i = s^i(P, q_i^e). \tag{2}$$

The theory of consistent conjectures yields

$$q_i^e = \sum_{j \neq i} X_j \quad \text{for all } i \tag{3}$$

at equilibrium.

Together the theory of competitive market supply gives a summation of (2), over n firms, and substitution from (3) yields a market supply function

$$X_s = s(P). \tag{4}$$

The continuous approximations of the actual parameters used in the experimental markets will make the model precise. Let α_i be parameters specific to seller i. By assumption β and γ are parameters common to all sellers. The cost function of each firm is given by:

$$C^i(X_i, \sum_{j \neq i} X_j) = \frac{1}{2}\left(\frac{1-\gamma}{\beta}\right) X_i^2 - \left(\frac{\alpha_i + \gamma \Sigma_{j \neq i} X_j}{\beta}\right) X_i. \tag{5}$$

The data were given to subjects in total cost tables which were evaluations of (5) and also in the form of marginal cost tables which were evaluations of

$$\frac{\partial C^i(\cdot, \cdot)}{\partial X_i} = \left(\frac{1-\gamma}{\beta}\right) X_i - \left(\frac{\alpha_i + \gamma \Sigma_{j \neq i} X_j}{\beta}\right). \tag{6}$$

The actual tables seen by subjects are included as an appendix. Of course, each subject was informed of only his (her) own parameters. By applying consistent conjectures (3) and from (6), individual supply functions are characterised by the relation

$$X_i = \left(\frac{1}{1-\gamma}\right)(\alpha_i + \beta P + \gamma \sum_{j \neq i} X_i). \tag{7}$$

Aggregate supply satisfies the relation

$$X_s = \sum_i X_i = \sum_i \left[\left(\frac{1}{1-\gamma} \right) (\alpha_i + \beta P + \gamma \sum_{j \neq i} X_j) \right] \tag{8}$$

which reduces to the supply function where X_s is the market supply quantity

$$X_s = \left(\frac{1}{1-n\gamma} \right) (\sum_i \alpha_i + Pn\beta). \tag{9}$$

The objective of having a negatively sloped supply curve places some obvious parametric restrictions on (9). In particular we chose

$$\beta = y\gamma \quad \text{so} \quad y > 1 \quad \text{implies} \quad \frac{\partial X_i}{\partial P} > \frac{\partial X_i}{\partial X_j} \quad \forall j \neq i,$$

$$\beta > 0, \quad \sum_i \alpha_i < 0.$$

Under the above assumption the aggregate supply function is of the general form

$$X_s = \frac{\sum_i \alpha_i}{1-n\gamma} + \frac{yn\gamma P}{1-n\gamma}. \tag{10}$$

The actual parameters chosen for the supply function were

$$n = 6; y = 6; \gamma = \tfrac{1}{3}; \alpha_a = -6\tfrac{2}{3}; \alpha_b = -6\tfrac{1}{3}; \alpha_c = -6.$$

The indexes a, b, c, can be recognised as seller 'types'. Each seller of a given type had identical cost parameters. For all experiments there were two sellers of each given type (e.g. two sellers had parameters α_a, etc.). A graph of this continuous approximation is in Fig. 1 and the actual parameters are in Fig. 2 as the curve *SS*. The equation with a 240 scaling factor for P is $X_s = 38-(1/20)P$. The continuous model is not a particularly good approximation for the actual parameters used in the experiment and seen by subjects. The actual parameters are contained in the Appendix.

II. MARKET ORGANISATION

While the stability notions themselves require no qualifications regarding market organisation, much recent experimental work leaves no doubt that market organisation is potentially important. The stability ideas themselves give hints about the form of organisation that might favour one theory over the other. The Walrasian stability concept, for example, would seem to be most appropriate when a tâtonnement process is used.

[4] This is true for the computerised MUDA of experiment 4 but not for the first three experiments. In the single unit oral auction, trading within the final fifteen seconds automatically added thirty seconds to the clock. In the single unit case the variable-length period was thought to be necessary to avoid biasing the results toward one of the equilibria. If the time period was short, the bias could be in favour of a small volume equilibrium. If it was long, a large volume equilibrium might be favoured as individuals trade to avoid boredom, attempt to collude, etc. in the extra time. The feature was unnecessary in the MUDA because of the speed at which a volume can move.

Three different organisations were studied. The first is the double auction. In this process buyers tender bids and sellers tender asks publicly. Trading is open for a limited period of time[4] with a large number of potential bids and asks possible. One important feature of this organisation is that market volume; i.e. the volume of others, is observed during a period. Participants are able to make decisions during a period contingent upon the volume that has already occurred in that period. Sellers also know that their own volume will affect the costs of others in ways that might induce others to sell more. Thus, from a practical point of view, the double auction has potentially important features.

The second organisation studied was a sealed bid/offer market (see Smith *et al.*, 1982). Buyers and sellers each list the prices at which they wish to buy or sell each unit. That is, each participant submits a demand or supply function. These functions are aggregated in the ordinary way and the equilibrium is computed. The last accepted and first excluded demand and supply units determine the price. The market price is computed to be midway between the minimum of the last accepted bid and first rejected offer and the maximum of the last accepted offer and first rejected bid.

The second organisation is interesting for two reasons. First, because of the 'excluded price auction' feature, demand revealing aspects exist, though this is not to imply that this mechanism is demand revealing.[5] Second, a special case of the sealed offer institution is that in which suppliers bring a fixed quantity to the market that they will sell whatever the price. In any case, suppliers cannot make their offers contingent upon the actions of others in these markets so one might expect the stability if not the equilibrating properties of this process to differ from the double oral auction.

The third process is the secant tâtonnement which is studied for the first time in this paper. In this process the experimenter (price adjuster) announces a price. Agents respond with quantity offers at that price. If excess demand is zero, or if some other stopping rule is involved, the process stops and trade takes place at the announcement price. If conditions for stopping are not satisfied, then the secant method for finding a new price is used and a new price is announced.

Price changes were determined by the secant formula

$$P_{t+1} = P_t + \left| \frac{P_t - P_{t-1}}{ED_t - ED_{t-1}} \right| ED_t$$

where ED_t is the excess demand at t; i.e. it is the sum of amounts demanded minus the sum of amounts supplied at P_t. If $ED_t - ED_{t-1} = 0$ then $P_{t+1} - P_t = P_t - P_{t-1}$. Notice that the process is potentially unstable. If excess demand is positive, then prices will increase or if excess demand is negative, prices will decrease. This quality is exactly the unstable behaviour that underlies the Walrasian concept.

The process stopped according to two rules:

(1) If excess demand is zero, then the price change is zero and the process stops.

[5] Participants are trading multiple units. Consequently, extramarginal units can have a strategic use.

(2) If $|(P_t - P_{t-1})/(ED_t - ED_{t-1})|$ is small enough, then a rounding procedure will treat the quantity as zero and the process stops even though excess demand is not zero. This latter property can be important in cases of horizontal demand or supply curves. The formula and stopping rules were on the chalkboard with examples for all to see and study.

Very little is known about the behaviour of tâtonnement processes. Operational problems involved with implementing such a process abound. The secant process as opposed to a process often referenced in the literature as the 'proportional adjustment rule' (i.e. $\dot{X} = \alpha ED$) was chosen as the first to study because the natural stopping rules listed above could be applied in the presence of discontinuities.[6]

III. MODELS

Two aspects of the economic environment are of interest. The first is the set of equilibria and the second is the detail of the dynamic models. The classical analysis will be maintained throughout the paper. That is, only the market demand and supply functions will be used as the basic parameters of the environment. We will indicate those points where we are aware that this classical model produces results inconsistent with modern game theory.

The Walrasian definition is not the same as the Marshallian definition. The Walrasian definition has a price to be an equilibrium if the quantity demanded at that price equals the quantity supplied at that price. The Marshallian definition has a quantity to be in equilibrium if the demand price and supply price are equal at that quantity. Both authors also defined equilibria as limit points of a dynamic process. This latter distinction is not so important if the curves are continuous as they are in Fig. 1 but special problems occur when the curves are discrete as in Fig. 2.

The literature appears to contain no discussion of problems caused by discontinuities so latitude remains to apply these theories in ways that appear reasonable. In this paper an equilibrium is either a point that satisfies a static definition or it is the limit point of a dynamic process as defined by one of the two dynamic theories.

Table 2 lists the equilibria. The letters correspond to the points as located in Fig. 2. Under both demand conditions, D_1 and D_2, the interior points c, d, e are equilibria according to both theories. At the indicated price, quantity demanded equals quantity supplied. Some ambiguity can result if the Marshallian definition of equilibrium, that demand price equals supply price, is used. Strictly speaking, demand price and supply prices are never equal in the environment given the discrete curves as drawn. However, the Marshallian dynamics would 'push' the system to the Marshallian stable points in the set $\{c, d, e\}$ if the Marshallian model of dynamics is correct.

The boundary points do not have the symmetry of the interior points. The equilibrium properties of these points change according to the demand conditions and the theory.

[6] The proportional rule for adjustment has been studied by Patrick Joyce (1984).

Table 2

Equilibria and Stability Properties

			Condition D_1		Condition D_2	
Point	Price*	Quantity	Marshall	Walras	Marshall	Walras
a	960	0	Non eq.	Stable	Non eq.	Stable
b	630	0	Non eq.	Non eq.	Stable	Non eq.
c	500–540	12	Stable	Unstable	Unstable	Stable
d	380–410	18	Unstable	Stable	Stable	Unstable
e	140–180	30	Stable†	Unstable	Non eq.	Non eq.
f	0	28	Non eq.	Non eq.	Non eq.	Stable
g	0	30	Non eq.	Stable	Non eq.	Non eq.

* Price reflects conditions D_1. Under D_2 the range would be narrower.

† Strictly speaking, this point is not an equilibrium because the demand price at this quantity is 290 and is therefore above the supply price which is 180. However, quantities on both sides have dynamics that move the system in the direction of this quantity according to Marshall.

In summary, under condition D_1, Marshallian theory would predict equilibration near points $\{c, e\}$ and Walrasian would predict $\{a, d, g\}$. Under conditions D_2 Marshallian theory would predict points $\{b, d\}$ while Walrasian would predict $\{c, f\}$. That point a is a Walrasian equilibrium under condition D_1 is a departure from a game theoretic analysis. At prices that high, some producers would be willing to supply units even if others produced zero. Thus at prices that high, the support of game theory is lost.

We will now turn to the dynamic models. Since the continuous approximation of the parameters leads to a piecewise linear model, the analysis will be in terms of a linear model. Let the demand equation be

$$X_d - a_d - b_d P_d = 0 \tag{11}$$

and the supply equation be

$$X_s - a_s - b_s P_s = 0, \tag{12}$$

where X_d and X_s are the demand and supply quantities, respectively, and when P_d and P_s are the demand price and supply price, respectively.

The Marshallian theory of dynamics is

$$X = X_d = X_s, \quad \frac{dX}{dt} = G(P_d - P_s), \quad G'(\cdot) > 0. \tag{13}$$

The Walrasian theory of dynamics is

$$P = P_s = P_d, \quad \frac{dP}{dt} = F(X_d - X_s), \quad F'(\cdot) > 0. \tag{14}$$

A problem exists in making these theories operational. The following assumptions will be used to facilitate a direct and operational comparison of the two adjustment models. The assumptions are convenient tools for purposes of exposition and do not affect the substantive conclusions of the paper.

Assumption 1. The time t refers to an experimental period.

Assumption 2. X_t = the observed number of transactions in period t.

Assumption 3. $X_t = X_d$. That is, the observed number of transactions can be interpreted as X_d in the demand equation (11).

Assumption 4. Disequilibrium movements lie on the demand curve. This means that equation (11) is always satisfied.

Assumption 5. The speeds of adjustment functions $G(\cdot)$ and $F(\cdot)$ in (13) and in (14) are linear. That is $G(P_d - P_s) = \alpha(P_d - P_s)$ and $F(X_d - X_s) = \beta(X_d - X_s)$.

Assumptions 1 to 5 together with (11) and (12) yield the following implications of (13) and (14). The exercise is a context in which the theories can be said to be 'opposites'.

Marshallian Theory:

$$\frac{dX_t}{dt} = \alpha\psi_{\text{Marshall}}(X_t) = \alpha\left(\frac{X_t - a_d}{b_d} - \frac{X_t - a_s}{b_s}\right), \quad \alpha > 0 \tag{15}$$

Walrasian Theory:

$$\frac{dp}{dt} = \frac{1}{b_d}\frac{dX_t}{dt} = \beta\left\{X_t - \left[a_s + b_s\left(\frac{X_t - a_d}{b_d}\right)\right]\right\}, \tag{16}$$

$$\frac{dX_t}{dt} = \beta b_d \psi_{\text{Walras}}(X_t) = \beta b_d\left\{X_t - \left[a_s + b_s\left(\frac{X_t - a_d}{b_d}\right)\right]\right\}, \quad \beta > 0. \tag{17}$$

Under the assumptions listed above and the maintained hypothesis of the theory regarding the market demand and supply, the only unobserved variables in (15) and (17) are the speed of adjustment parameters α and β.

Under the linearity assumption the relationship between the two theories is

$$\left.\frac{dX}{dt}\right|_{\text{Walras}} \equiv -\frac{\beta}{\alpha} b_d b_s \left.\frac{dX}{dt}\right|_{\text{Marshall}}. \tag{18}$$

Since $b_d b_s > 0$ in the downward sloping case and since $\beta/\alpha > 0$ by the maintained hypotheses of the two theories, the two theories give almost diametrically opposed predictions. The predicted direction of movement will always be the opposite. However, the quantitative relationship can be affected by boundaries and by the nonlinearity of the demand function. Nevertheless, because the functions are piecewise linear, the relationship (18) is true for large segments of the price and quantity space.

Alternative ways of comparing the models exist. The analysis above treated quantity as the only observable variable (Assumption 3) and left the price to be determined by theory (Assumption 4). Alternatively, the price could have been taken to be the only observable variable while allowing quantity to be determined by theory, i.e. $\min\{X_d, X_s\}$. Since both prices and quantities are actually observable, no need exists to impose any such structure for purposes of estimation. The two equations in (13) can be estimated independently without imposing assumptions like (3) or (4).

IV. RESEARCH DESIGN

A total of twelve experiments were conducted. Table 3 summarises the treatment conditions. The first four experiments were double auctions (DA).

Table 3

Treatment Conditions for All Experiments

Experiment	Market Organisation	Demand Conditions
1	DOA	D_1 all 6 periods
2	DOA	D_1 all 10 periods (period 11 deleted – had been preceded by announcement)
3	DOA	D_1 all 10 periods (periods 11–12 deleted – had been preceded by announcement)
4	MUDA	D_1 periods 1–9
		D_1 all 10 periods, D_2 periods 10–24
		Announce $P^* = 400$, $X^* = 18$ after period 11, reiterate announcement after period 13
5	SBO†	D_1 periods 1–9
		D_1 periods 10–14
6	SBO‡	D_1 periods 1–9
		D_2 periods 10–15
7	SBO‡	D_1 periods 1–14
		D_2 periods 15–20
8	SBO‡	D_1 periods 1–14
		D_2 periods 15–24
		Announce $P^* = 400$, $X^* = 18$ after period 20
9	Tâtonnement	D_1 periods 1–9
		D_2 periods 10–13
10	Tâtonnement	D_1 periods 1–9
		D_2 periods 10–12
11	Tâtonnement	D_1 periods 1–9
		D_2 periods 10–15
		Announce $P^* = 400$, $X^* = 18$ after periods 11 and 12
12	Tâtonnement	D_1 periods 1–9
		D_2 periods 10–19
		Announce $P^* = 400$, $X^* = 18$ after periods 11 and 12

† Public information at the end of a period included the highest bid, lowest bid, lowest accepted bid, and highest accepted ask, price, and volume.
‡ The highest rejected bid and lowest rejected ask were announced along with the information described in footnote †.

The first three of these four were conducted under constant demand conditions as single unit double oral auctions (DOA). The DOA process required more time than was anticipated. In addition the system was not equilibrating as anticipated so a decision was made to keep demand constant. Since the multiple unit double auction (MUDA) used in experiment 4 is faster,[7] a demand shift was implemented in that market.

Experiments 5, 6, 7 and 8 were conducted as sealed bid offer (SBO) auctions. For these, because of the speed of the market, we were able to implement the shifting demand design as planned.

Experiments 9, 10, 11, and 12 were all secant tâtonnement. Demand was held at D_1 for the first nine periods and was shifted to D_2 beginning the tenth period. Demand condition D_2 was maintained for two to four periods after which the experiment terminated or an experimenter intervention occurred as explained below.

[7] The MUDA was computerised. The program can be found in Plott (1991).

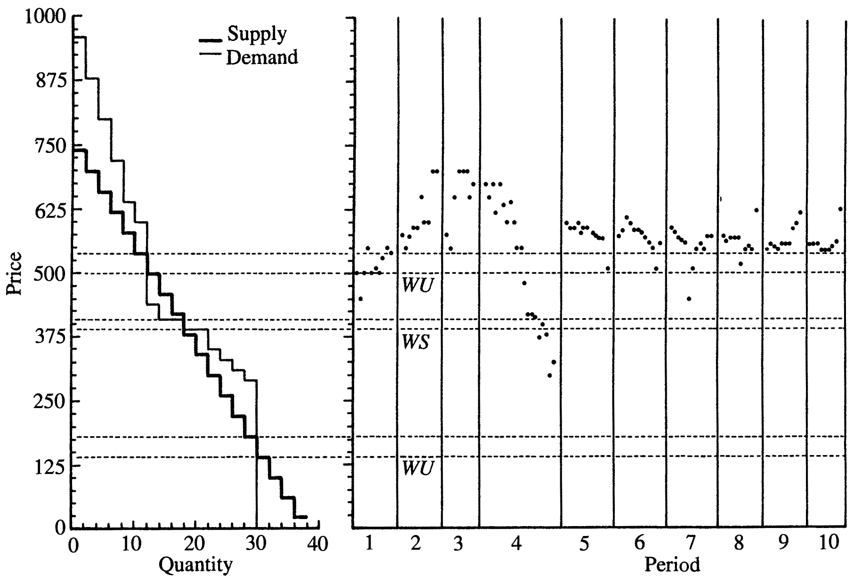

Fig. 3. All contracts from experiment 3, double oral auction.

In some experiments an experimenter intervention occurred in the form of a public announcement that demonstrated[8] the existence of an equilibrium (Marshallian stable and Walrasian unstable) at $P^* = 400$, $X^* = 18$. These interventions came at times after the primary objectives of the research had already been completed and an opportunity existed to explore secondary objectives at a very low marginal cost. The intervention represented an exploratory check to see if the market would stay at a Marshallian stable (Walrasian unstable) point if the market was 'placed' there. The data from these interventions were not used in the analysis but are reproduced for the interested reader.

V. RESULTS

Figs. 3, 4 and 5 contain an example of a time series from each of the three different types of market organisation. Table 4 contains the average price and volume for all periods of all experiments. The table indicates when the shift from D_1 to D_2 occurred and when information about equilibria was announced. In the first four experiments under the double auction processes, prices vary within a period so the average price does not contain all of the information. In all other experiments all transactions took place at the same price.

The first conclusion and remark address the general question of equilibration

[8] Each seller received a slip of paper that contained a statement and question of the form 'Assume the volume of others was ____'. Subjects were then asked to reveal the amount they would supply or demand at a price of $400. The responses were public so the consistency of the responses with the slip revealed the equilibrium to all. The volume of others would be as expected and demand and supply would be equal. In experiments (8, period 21), (11, periods 12, 13), (12, periods 12, 13) the announcement and responses were actually counted as periods.

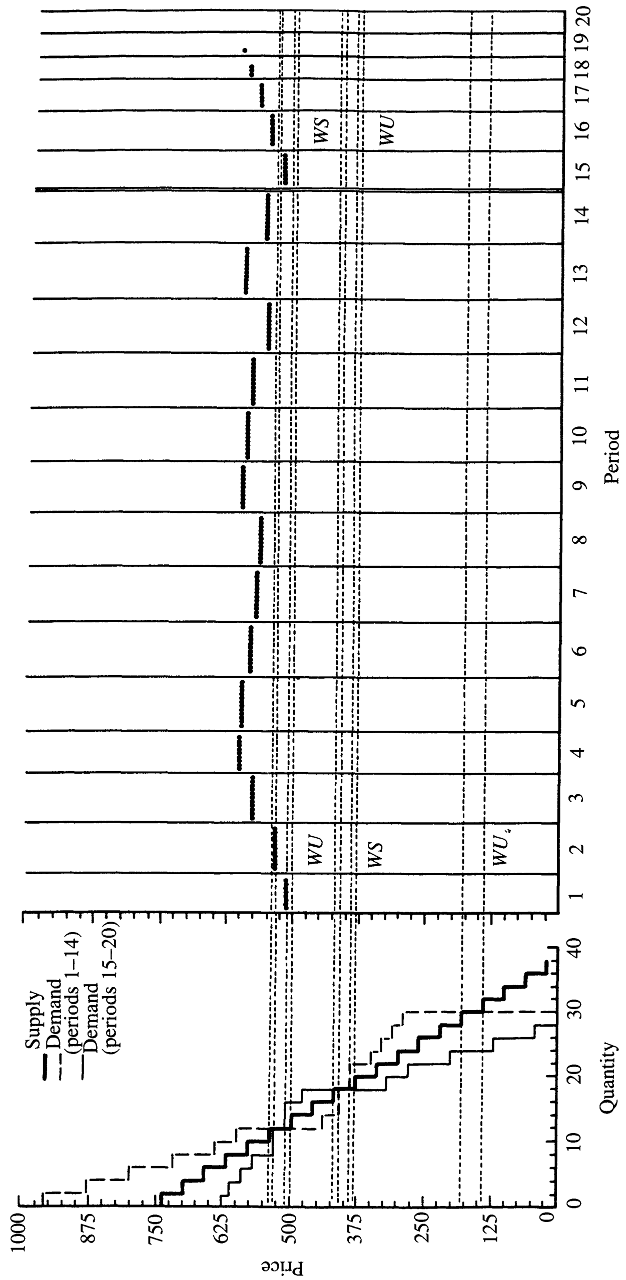

Fig. 4. All contracts from experiment 7, sealed bid offer.

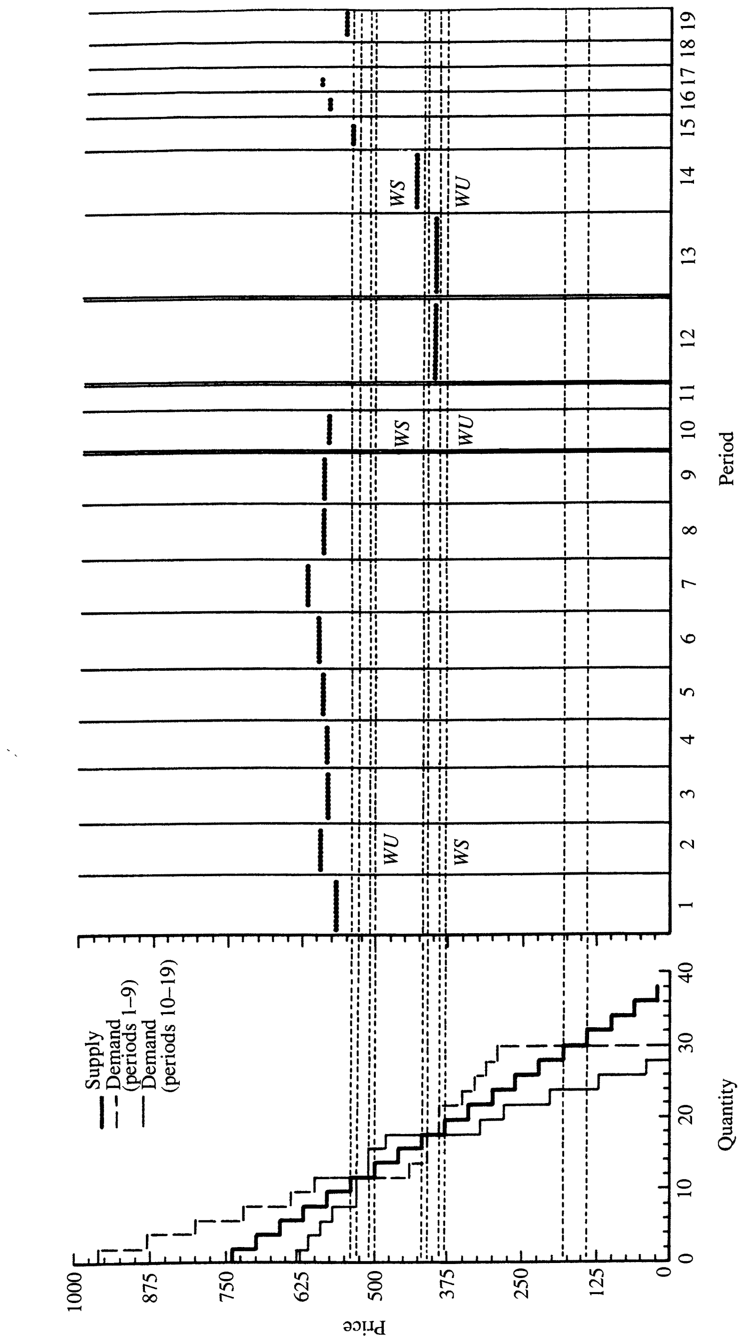

Fig. 5. All contracts from experiment 12, tâtonnement.

Table 4

Volume and Average Price for All Periods of All Experiments

Experiment	Period	Vol.	Av. Price
1 DA*	1	13	602·3
	2	11	549·4
	3	8	564·5
	4	4	622·5
	5	4	639·8
	6	7	627·9
2 DA	1	10	558·0
	2	12	562·9
	3	9	569·4
	4	12	561·2
	5	12	556·7
	6	12	553·3
	7	11	553·6
	8	12	553·8
	9	13	539·5
	10	12	541·6
3 DA	1	10	513·0
	2	10	612·5
	3	8	650·0
	4	20	519·2
	5	12	578·8
	6	12	572·5
	7	12	552·9
	8	10	565·0
	9	10	570·0
	10	9	564·4
4 MUDA†	1	8	627·5
	2	8	632·3
	3	6	631·7
	4	6	633·7
	5	8	648·9
	6	7	646·6
	7	7	649·0
	8	7	653·1
	9	8	659·0
	Treatment Change		
	10	0	
	11	0	
	Treatment Change		
	12	19	456·1
	13	1	450·0
	Treatment Change		
	14	20	403·9
	15	17	419·4
	16	14	438·9
	17	10	455·5
	18	7	467·1
	19	4	500·0
	20	6	450·0
	21	18	461·1
	22	18	452·6
	23	15	464·7
	24	17	472·3
5 SB/O‡	1	13	440.0
	2	13	440·0
	3	10	500·0
	4	11	600·0
	5	11	600·0
	6	10	610·0
	7	10	609·0
	8	10	599·0
	9	10	599·0
	Treatment Change		
	10	7	560·0
	11	6	585·0
	12	4	594·0
	13	3	600·0
	14	1	600·0
6 SB/O	1	6	600·0
	2	12	525·0
	3	12	500·0
	4	12	500·0
	5	12	500·0
	6	12	500·0
	7	11	503·0
	8	11	530·0
	9	11	545·0
	Treatment Change		
	10	7	533·0
	11	7	538·0
	12	7	550·0
	13	5	575·0
	14	3	591·0
	15	4	580·0
7 SB/O	1	8	512·0
	2	11	534·0
	3	11	577·0
	4	9	602·0
	5	12	598·0
	6	12	583·0
	7	12	572·0
	8	12	565·0
	9	11	599·0
	10	12	591·0
	11	12	782·0
	12	12	555·0
	13	12	600·0
	14	12	560·0
	Treatment Change		
	15	8	530·0
	16	8	554·0
	17	6	575·0
	18	3	595·0
	19	1	610·0
	20	0	

Table 4 (*cont.*)

Experiment	Period	Vol.	Av. Price
8 SB/O	1	10	475·0
	2	11	500·0
	3	11	500·0
	4	7	550·0
	5	8	599·0
	6	9	630·0
	7	10	629·0
	8	10	615·0
	9	10	600·0
	10	10	620·0
	11	10	622·0
	12	9	611·3
	13	10	623·0
	14	10	619·0
	Treatment Change		
	15	6	559·0
	16	5	590·0
	17	3	600·0
	18	2	615·0
	19	1	629·0
	20	0	
	Treatment Change		
	21	18	400·0
	22	15	500·0
	23	11	511·0
	24	9	530·0
9 T§	1	8	546·0
	2	10	577·0
	3	9	568·0
	4	10	578·0
	5	10	583·0
	6	12	579·0
	7	10	580·0
	8	11	573·0
	9	10	583·0
	Treatment Change		
	10	3	572·0
	11	3	567·0
	12	1	591·0
	13	3	564·0
10 T	1	12	454·0
	2	12	528·0
	3	6	645·0
	4	8	605·0
	5	9	571·0
	6	9	614·0
	7	6	626·0
	8	5	622·0
	9	5	635.0
	Treatment Change		
	10	1	630·0
	11	1	602·0
	12	0	631·0
11 T	1	8	633·0
	2	9	615·0
	3	10	632·0
	4	10	607·0
	5	10	608·0
	6	10	605·0
	7	9	602·0
	8	10	609·0
	9	11	600·0
	Treatment Change		
	10	2	619·0
	11	0	602·0
	Treatment Change		
	12	18	400·0
	13	18	400·0
	14	17	371·0
	15	18	386·0
12 T	1	12	567·0
	2	10	592·0
	3	11	581·0
	4	9	584·0
	5	10	590·0
	6	11	598·0
	7	10	617·0
	8	11	590·0
	9	10	590·0
	Treatment Change		
	10	7	582·0
	11	0	615·0
	Treatment Change		
	12	18	400·0
	13	18	400·0
	14	13	435·0
	15	6	541·0
	16	3	586·0
	17	2	600·0
	18	0	631·0
	19	6	552·0

* Double auction.
† Multiple unit double auction.
‡ Sealed bid/offer.
§ Tâtonnement.

in such a complicated environment. The conclusion is important because the degree to which the secant tâtonnement process would settle on any price at all was completely unknown. The remark that follows the conclusion is important because the model of the market supply function is based on behavioural principles that are not used in models of supply that traditionally have been studied in experimental markets.

Conclusion 1. The (secant) tâtonnement process operated to determine a market price.

The tâtonnement process employed in experiments 9, 10, 11, and 12 always terminated according to the rules. A market price and corresponding exchanges were always established.

Remark 1. The demand and supply model is not as accurate in the downward sloping supply case as it has been in other studies involving upward sloping supply functions.

In order to understand the remark, study the final periods of D_1 for experiments 1, 4 and 10. In these markets volume is about midway between two equilibria and the prices are off by 20% or more. Usually, under the double auction or under the sealed bid offer, the accuracy of the price predictions of the competitive model falls within 5% or so. The problem is seen again in the volume predictions. In all but two experiments volume was below predictions in the last period of D_1 and it was never above. Thus the errors of the model are systematic and occasionally large.[9] This phenomena is present under all three institutions.

Conclusion 2. For all experiments under conditions D_1 before the demand shift, prices and volume converge nearer to point *c* than to any other equilibrium. This point is Marshallian stable and Walrasian unstable.

Again observe the data in the last period of condition D_1. For all experiments the stable Marshallian equilibrium nearest the data is point *c* and the nearest Walrasian stable equilibrium is point *a*. All but five of the twenty-four data are closer to *c* than to *a*. The exceptions are prices in experiments 1, 8, 10, and 13 and volume in experiment 10. On average, the price predictions of the (stable) Marshallian model has a 14% of price and a 3 unit volume error. By contrast the (stable) Walrasian model exhibits price predictions that are on average off by 20% and volume predictions that are off by 9·6 units. The equilibria of the Marshallian stable model are better predictors of the data.

Conclusion 3. For all experiments under conditions D_2 after the demand shift, the data are nearer point *b* than to any other equilibrium. This point is Marshallian stable and Walrasian unstable.

Consider the final period of condition 2 before any announcements are made about the locations of equilibria. The data are closer to the stable Marshallian equilibrium *b* than to the stable Walrasian equilibrium *c* in seventeen of the eighteen cases. The exception is the price data in experiment 9. On average the

[9] A referee conjectures that this phenomena might be sensitive to the magnitude of 'excess demand' in the neighbourhood of the equilibria. The conjecture is that a wider 'diamond' between D_1 and D_2 would facilitate tighter convergence to the Marshallian stable equilibrium.

price predictions in the stable Marshallian model are off by 3%[10] and the quantity predictions are off by 0·9 units. By contrast the price predictions of the stable Walrasian are off by 20% on average and the volume predictions are off by 11 units on average.

The Marshallian and Walrasian models are fundamentally theories of dynamics rather than theories of equilibria. Comparison of equilibria does not explore the nature of price movements and thus does not get to the essence of the difference in approach. In order to give a direct test of the two models, discrete variations of (15) and (17) were used. Specifically, the estimated models were

$$\Delta X = X_{t+1} - X_t = a_k + b_k \psi_k(X_t) + \epsilon_{kt},$$

$$X_t = \text{observed market volume in period } t$$

$$k \in \{\text{Marshall, Walras}\}, \tag{19}$$

$$\psi_{\text{Marshall}}(X_t) = (P_d - P_s)|_{X_t}, \tag{20}$$

$$\psi_{\text{Walras}}(X_t) = (X_d - X_s)|_{X_t}. \tag{21}$$

The values of $\psi_{\text{Marshall}}(X_t)$ are observable without additional theory of the adjustment process. So the econometric measurements can be made directly without additional maintained hypotheses. The value of $\psi_{\text{Walras}}(X_t)$ can be observed as a result of assumptions (1) through (5) as was demonstrated in equation (17).[11]

The magnitudes can be estimated by application of equation (19). Both dynamic theories predict

$$a_k = 0 \quad b_k > 0.$$

However, by virtue of the experimental design, it is unlikely to observe $b_k > 0$ for both values of k since over wide ranges of the variables the models differ only by a negative constant of proportionality. However, the models are estimated separately because differences can occur in certain areas of the observation space because of the nonlinearities. The models were estimated over several restricted data sets. First all of the data prior to announcements were used. Then the models were checked using only first period data. The models were also estimated with the inclusion of data after announcements. These perturbations of the data resulted in no changes in the conclusions. The estimates in Table 5 are those for the data before any parameter announcements were made. Since the announcements were not implemented sufficiently systematically to be considered as treatment variables for purposes of statistical analysis, the data after announcements were not used to obtain the regression results reported in Table 5.

[10] These figures assume that the prices are at the Marshallian equilibrium when zero quantity and no prices are observed. This makes the Marshallian model look better but does not change conclusions. In a process like tâtonnement, prices are observed without trade.

[11] Since market prices are observable, other tests not involving these assumptions could be devised. In particular, mean price changes could be used as the dependent variable for the Walras model. However, the direct comparison of models would be lost.

Table 5

Estimated Coefficients, t-statistic, Durbin–Watson Statistic, and Adjusted R^2.

$$\Delta X = a + b\psi_k$$

		ψ measured by Marshall			
	N	$\hat{a}$	$\hat{b}$	DW	R^2
Double Auction	48	−1·64 (−2·33)	0·038 (5·62)	2·03	0·459
Sealed bid/offer	69	−0·198 (−0·73)	0·031 (11·12)	P^* = 0·243	0·651
Tâtonnement	47	−1·20 (−2·24)	0·034 (7·48)	P^* = 0·233	0·528
		ψ measured by Walras			
	N	$\hat{a}$	$\hat{b}$	DW	R^2
Double Auction	48	0·623 (0·677)	−1·62 (−2·64)	2·14	−0·027
Sealed bid/offer	69	−0·198 (−0·73)	−12·37 (−11·12)	P^* = 0·243	0·651
Tâtonnement	47	−0·103 (−0·173)	−4·36 (−1·105)	1·754	0·004

* An AR(1) was run to correct for autocorrelation.

Conclusion 4. The Walrasian model can be rejected in favour of the Marshallian model in all three organisations.

In Table 5 the estimate of adjustment term, $\hat{b}$, is positive and significant under the Marshallian model. Contrary to the predictions of the Walrasian model, the $\hat{b}$ is always negative and significant except in the tâtonnement where it is not significant. Adjusted R^2 are never worse under the Marshallian model. The size of the estimated coefficient, $\hat{b}$, seems small under the Marshallian adjustment theory but in reality the size is simply a result of the units involved in the independent variable, $\psi_{\text{Marshall}}(X_t)$. A change of units would change the slope without affecting the intercept.

The support for the Marshallian model is not all positive. Both theories predict that the intercept term will be zero. As shown in Table 5, $\hat{a}_k$ is significantly negative for two of the three processes according to the Marshallian model. However, the magnitudes are small. We conjecture that they result from nonlinearities in the adjustment process or perhaps from the fact that the markets tended to equilibrate with volumes less than the equilibrium.

VI. CONCLUSIONS

This paper explored the two classic theories of stability found in the literature. Several new experimental issues were involved with the study. Downward sloping supplies had never been studied under any circumstances. The forward falling case rests on the existence of an externality and the theory of market supply requires a consistent conjectures component. Consequently even equilibration itself was in need of study. Classical discussions of stability seem to presuppose the existence of a tâtonnement process so in order to facilitate the study within that type of organisation, a special type of (secant) tâtonnement was designed and implemented. The stylised results are as follows.

1. The law of supply and demand works with reasonable accuracy in the case of forward falling supply. Even though the data fall substantially short of strong support for the model (see Remark 1), the degree of success of the model is surprising because the supply model involved a rational expectations component that has not been present in other studies.

2. The (secant) tâtonnement process reliably equilibrated the markets. The data reported here involve the first investigation of the operation of this class of processes.

3. The dynamics of market equilibration suggest that stability is a property of equilibria. Shifts in the demand which cause an equilibrium to become unstable are followed by movement away from the equilibrium. Whether or not equilibration at an unstable point can actually be observed occurring in a market remains an open question.

4. The Marshallian stability model captures the observed phenomena and the Walrasian model does not. Between the two models the Marshallian is the appropriate one for the forward falling supply case.

5. The conclusions above are independent of market organisation. The Marshallian model is the appropriate model even when the market is organised by tâtonnement.

The most important question has not been answered. Does the Marshallian model have a firm foundation in modern theory? Or, was it an accident resulting from a choice of parameters and procedures that made the Marshallian model look good? Will the result hold up in the backward bending supply case? The prospect was advanced years ago that the two conditions of supply, forward falling and backward bending, might be characterised by different stability properties.[12]

For some reason the Marshallian concept of stability has almost been dropped from consideration by the profession. Only the Walrasian model appears in textbooks as a suggestion for applications. While it is much too early to claim that the practice is wrong, these data suggest that unanswered questions exist about the circumstances under which these competing concepts might be used reliably.

California Institute of Technology

Date of receipt of final typescript: August 1991

[12] I am indebted to Eskander Alvi for calling to my attention Kahn (1933).

Appendix

Instructional Material and Parameters

Buyers were given standard instructions such as those found in Plott (1989). The only differences are the numbers used in the examples. Seller instructions are special because of the nature of the externality. These are reproduced in this appendix. A quiz and a period zero were both administered to check subjects' understanding of the accounting system. The *Cost Sheet* contains the parameters of sellers. These are important because slight adjustments from the continuous model were made in order to obtain the quantified incentives used in the experiment. In addition to a sheet with marginal cost information, sellers were given sheets with total costs.

Specific Instructions to Sellers

During each market period you are free to sell as many units as you might be able to. The profit from each sale (which is yours to keep) is computed by taking the difference between the price at which you sold the unit and the cost of the unit. Note that you *may* sell a unit at a price below the cost of the unit. Therefore

$$[\text{your profit} = (\text{sale price}) - (\text{cost})].$$

Your cost depends upon **your volume** and the **volume of others.** This means that when you sell units you will not know your costs with certainty. Your costs will be known only at the end of a period when the total volume of sales is known. Examine your *Cost Sheet*. If the **volume of others** is zero, that is, you were the only one who sold units, then the cost of each of your units is found in the column labelled **0**. If the **volume of others** is 23 then the cost of each of your units is found in the column labelled **23**.

Suppose, for example, that you sold two units in a market in which a total of ten units were sold. Find the appropriate column in your *Example Cost Sheet* (as illustrated on the chalkboard). Since the **volume of others** is 8 units, the cost to you of the first unit is 1,500 and the cost of the second unit is 2,000. If you sold each unit for 3,500, your profit is:

$$\text{Profit from first unit} = 3500 - 1500 = 2000,$$
$$\text{Profit from second unit} = 3500 - 2000 = 1500,$$
$$\text{Total Profit} = 2000 + 1500 = 3500.$$

The blanks on the *Record Sheet* will help you record your profit. The sale price of the first unit you sell during the first period should be recorded in row (1). The same should be done (in the appropriate rows) for any additional units sold in this period. At the end of the period, enter the **market volume** of the period in row (A), enter **your volume** in row (B) and subtract row (B) from row (A) to determine the resulting **volume of others** which is entered in row (C). Then look on your *Cost Sheet* to find your unit costs. On the *Record Sheet* enter the cost of the first unit in row (2). You should then record the profit on

Cost sheet

	Volume of Others																															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1st UNIT	780	700	670	640	610	550	500	470	440	410	380	300	270	240	210	180	100	70	40	20	20	20	20	20	20	20	20	20	20	20	20	20	20
2nd UNIT	560	780	750	720	690	660	580	550	520	490	460	380	350	320	290	260	180	150	120	90	60	20	20	20	20	20	20	20	20	20	20	20	20
3rd UNIT	940	860	830	800	770	740	660	630	600	570	540	460	430	400	370	340	260	230	200	170	140	60	30	20	20	20	20	20	20	20	20	20	20
4th UNIT	1020	940	910	880	850	820	740	710	680	650	620	540	510	480	450	420	340	310	280	250	220	140	110	80	50	20	20	20	20	20	20	20	20
5th UNIT	1100	1020	990	960	930	900	820	790	760	730	700	620	590	560	530	500	420	390	360	330	300	220	190	160	130	100	20	20	20	20	20	20	20
6th UNIT	1180	1100	1070	1040	1010	980	900	870	840	810	780	700	670	640	610	580	500	470	440	410	380	300	270	240	210	160	190	70	40	20	20	20	20
7th UNIT	1260	1180	1150	1120	1090	1060	980	950	920	890	860	780	750	720	690	660	580	550	520	490	460	380	350	320	290	260	180	150	120	90	60	20	20
8th UNIT	1340	1260	1230	1200	1170	1140	1060	1030	1000	970	940	860	830	800	770	740	660	630	600	570	540	460	430	400	370	340	260	230	200	170	140	60	30
1st UNIT	820	790	760	730	700	620	590	560	530	500	420	390	360	330	300	220	190	60	130	100	120	20	20	20	20	20	20	20	20	20	20	20	20
2nd UNIT	900	870	840	810	780	700	670	640	610	580	500	470	440	410	380	300	270	240	210	160	100	70	40	20	20	20	20	20	20	20	20	20	20
3rd UNIT	980	950	920	890	860	780	750	720	690	660	580	550	520	490	460	380	350	320	290	260	180	150	120	90	60	20	20	20	20	20	20	20	20
4th UNIT	1060	1030	1000	970	940	860	830	800	770	740	660	630	600	570	540	460	430	400	370	340	260	230	200	170	140	60	30	20	20	20	20	20	20
5th UNIT	1140	1110	1080	1050	1020	940	910	880	850	820	740	710	680	650	620	540	510	480	450	420	340	310	280	250	220	140	110	80	50	20	20	20	20
6th UNIT	1220	1190	1160	1130	1100	1020	990	960	930	900	820	790	760	730	700	620	590	560	530	500	420	390	360	330	300	220	190	160	130	100	20	20	20
7th UNIT	1300	1270	1240	1210	1180	1100	1070	1040	1010	980	900	870	840	810	780	700	670	640	610	580	500	470	440	410	380	300	270	240	210	180	100	70	40
8th UNIT	1380	1350	1320	1290	1260	1180	1150	1120	1090	1060	980	950	920	890	860	780	750	720	690	660	580	550	520	490	460	380	350	320	290	260	180	150	120
1st UNIT	800	770	740	660	630	600	570	540	460	430	400	370	340	260	230	200	170	140	60	30	20	20	20	20	20	20	20	20	20	20	20	20	20
2nd UNIT	880	850	820	740	710	680	650	620	540	510	480	450	420	340	310	280	250	220	140	110	80	50	20	20	20	20	20	20	20	20	20	20	20
3rd UNIT	960	930	900	820	790	760	730	700	620	590	560	530	500	420	390	360	330	300	220	190	160	130	100	20	20	20	20	20	20	20	20	20	20
4th UNIT	1040	1010	980	900	870	840	810	780	700	670	640	610	580	500	470	440	410	380	300	270	240	210	180	100	70	40	20	20	20	20	20	20	20
5th UNIT	1120	1090	1060	980	950	920	890	860	780	750	720	690	660	580	550	520	490	460	380	350	320	290	260	180	150	120	90	60	20	20	20	20	20
6th UNIT	1200	1170	1140	1060	1030	1000	970	940	860	830	800	770	740	660	630	600	570	540	460	430	400	370	340	260	230	200	170	140	60	30	20	20	20
7th UNIT	1280	1250	1220	1140	1110	1080	1050	1020	940	910	880	850	820	740	710	680	650	620	540	510	480	450	420	340	310	280	250	220	140	110	80	50	20
8th UNIT	1360	1330	1300	1220	1190	1160	1130	1100	1020	990	960	930	900	820	790	760	730	700	620	590	560	530	500	420	380	360	330	300	220	190	160	130	100

Scheme 1. Cost sheet (Type a, b, c, respectively)

this sale as directed in row (3). After computing the profit for each unit sold, record the total profit for that period in the last row on the page, row (41). Subsequent periods should be recorded similarly in the appropriate column (period 1 in column (1); period 2 in column (2); etc.).

Record sheet

Unit sold		Trading period number	1	2	3	4	5	6	7	8	9	10	11	12
	A	Market volume												
	B	Own volume												
	C	Volume of others (row A–row B)												
1	1	Selling price												
	2	Cost of first unit												
	3	Profit (row 1–row 2)												
	4													
2	5	Selling price												
	6	Cost of second unit												
	7	Profit (row 5–row 6)												
	8													

References

Guillaband, C. W., ed. (1961). *Marshall's Principles of Economics*, vol. 1. London: Macmillan.

Henderson, James M. and Quandt, Richard E. (1980). *Micro-economic Theory; A Mathematical Approach.* 3rd edn. New York: McGraw-Hill.

Joyce, Patrick (1984). 'The Walrasian tâtonnement mechanism and information.' *Rand Journal of Economics*, vol. 15, pp. 416–25.

Kahn, R. F. (1933). 'The elasticity of substitution and the relative share of a factor.' *Review of Economic Studies* vol. 1, pp. 72–8.

Plott, Charles R. (1982). 'Industrial organization theory and experimental economics.' *Journal of Economic Literature*, vol. 20 (December), pp. 1485–1587.

—— (1989). 'An updated review of industrial organization: applications of experimental methods.' In *Handbook of Industrial Organization*, vol. II, (R. Schmalensee and R. D. Willig eds.). Elsevier Science Publishers B.V.

—— (1991). 'A computerized laboratory market system and research support systems for the multiple unit double auction.' Social Science Working Paper no. 783. Pasadena: California Institute of Technology.

Smith, Vernon L. (1982). 'Microeconomic systems as an experimental science.' *American Economic Review*, vol. 72 (December), pp. 923–55.

——, Williams, Arlington W., Bratton, W. Kenneth and Vannoni, M. G. (1982). 'Competitive, market institutions: double auctions vs. sealed bid-offer auctions.' *American Economic Review*, vol. 72 (March), pp. 58–77.

Takayama, Akira. (1974). *Mathematical Economics.* Hinsdale, Illinois: The Dryden Press.

Walras, Leon (1954). *Elements of Pure Economics.* Translated by William Jaffe. Homewood, Illinois: Richard D. Irwin.