

COSMOLOGICAL SPECTRAL DECONVOLUTION

ROLAND DE PUTTER

NASA Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, MS 169-237, Pasadena, CA, 91109, U.S.A.
and
California Institute of Technology, MC 249-17, Pasadena, California, 91125 U.S.A.

GILBERT P. HOLDER

Department of Physics, McGill University, 3600 Rue University, Montreal, Quebec H3A 2T8, Canada

TZU-CHING CHANG

IAA, Academia Sinica, P.O. Box 23-141, Taipei 10617, Taiwan

OLIVIER DORÉ

NASA Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, MS 169-215, Pasadena, CA, 91109, U.S.A.
and
California Institute of Technology, MC 249-17, Pasadena, California, 91125 U.S.A.

Draft version March 18, 2014

ABSTRACT

One of the main goals of modern observational cosmology is to map the large scale structure of the Universe. A potentially powerful approach for doing this would be to exploit three-dimensional spectral maps, i.e. the specific intensity of extragalactic light as a function of wavelength and direction on the sky, to measure spatial variations in the total extragalactic light emission and use these as a tracer of the clustering of matter. A main challenge is that the observed intensity as a function of wavelength is a convolution of the source luminosity density with the rest-frame spectral energy distribution. In this paper, we introduce the method of spectral deconvolution as a way to invert this convolution and extract the clustering information. We show how one can use observations of the mean and angular fluctuations of extragalactic light as a function of wavelength, assuming statistical isotropy, to reconstruct jointly the rest-frame spectral energy distribution of the sources and the source spatial density fluctuations. This method is more general than the well known line mapping technique as it does not rely on spectral lines in the emitted spectra. After introducing the general formalism, we discuss its implementation and limitations. This formal paper sets the stage for future more practical studies.

1. INTRODUCTION

As spectral and imaging capabilities evolve, it is becoming increasingly common in astronomy to think of large three-dimensional data cubes, the specific intensity distribution of sources along many lines of sight that span a range of wavelengths. For studies of internal dynamics of galaxies or molecular clouds, the frequency dimension generally corresponds to elements at different line-of-sight velocities, while for cosmological studies the frequency dimension (ignoring peculiar velocities) corresponds to elements at different cosmological redshifts, and therefore at different distances.

Cosmological large scale structure surveys routinely use the spatial distribution of extragalactic light emission or absorption as a tracer of the matter density to constrain cosmological models. Such surveys typically exploit a subset of the three-dimensional data cube of the aggregate intensity of extragalactic light as a function of direction on the sky and of wavelength¹, for instance by working with a catalog of positions of a set of bright objects. In this article, we instead consider

the scenario where a full spectral map is available, such as could be obtained² from integral field spectroscopy (e.g. HETDEX³ Hill et al. 2008) or narrow band imaging surveys (e.g. J-PAS⁴ Benítez et al. 2009; Moles et al. 2010, PAU⁵ Benítez et al. 2009, or the Alhambra Survey Moles et al. 2008). In other words, the data are treated at the level of a continuous three-dimensional map as opposed to a discrete catalog.

The spectral intensity as a function of wavelength along a given line of sight in such a map is in general a superposition of the contributions of multiple sources (and absorbers). The goal is then to reconstruct the distribution of these sources as a function of redshift. The difficulty here is that, generically, sources emit over a wide range of wavelengths, so that the mapping from redshift to wavelength is degenerate. One common strategy to evade this degeneracy is to focus on objects that are bright enough that they stand out from the background

¹ The caveat to this description is that it ignores polarization and time-domain information (and cosmological information beyond the electromagnetic spectrum).

² In realistic applications, this map would of course still be limited by the sky coverage, wavelength range, and angular and spectral resolutions of the survey(s) providing the data.

³ <http://hetdex.org>

⁴ <http://j-pas.org/>

⁵ <http://www.pausurvey.org/>

and dominate the signal along a given line-of-sight (such as bright galaxies or quasars), i.e. the “catalog-based” approach mentioned above. Depending on how well the redshift direction is sampled, this approach has led to measurements of 2-dimensional (angular projected) clustering (e.g. Hauser & Peebles 1973; Efsthathiou & Moody 2001; Scranton et al. 2002; Tegmark et al. 2002; Frith et al. 2005; Cooray et al. 2010; Donoso et al. 2013), “2 + 1-dimensional” clustering using photometric redshifts (e.g. Padmanabhan et al. 2007; Ross et al. 2011; Ho et al. 2012, 2013) and of full 3-dimensional clustering using spectroscopic redshifts (e.g. Feldman et al. 1994; Cole et al. 2005; Eisenstein et al. 2005; Beutler et al. 2011; Parkinson et al. 2012; de la Torre et al. 2013; Anderson et al. 2013). An alternative approach to reconstructing the distribution of sources is line mapping, where the signal from well-known spectral lines, e.g. the HI 21-cm transition, allows a direct mapping between wavelength and redshift (e.g. Chang et al. 2008; Loeb & Wyithe 2008; Visbal & Loeb 2010; Chang et al. 2010; Pritchard & Loeb 2012; Switzer et al. 2013).

Here, we study the reconstruction of the source luminosity density in the more general case where we do not restrict ourselves to bright sources nor rely on spectral lines only. Instead, we consider reconstruction of the total luminosity density of all sources contributing to the extragalactic signal, optimally exploiting both the spectral and angular variations in the spectral map. While for simplicity we will assume the signal is described in terms of a single effective rest-frame spectral energy distribution (SED) shape, this method does not rely on prior knowledge of this SED form, but instead measures it from the data itself.

Formally, our work stems from the following observation. Generally, the specific intensity, I , at a given observed wavelength, λ , can be written as the weighted sum along the line of sight of the contribution of sources of spectrum, s , at a variety of cosmological redshifts:

$$I(\ln \lambda) = \int dz w(z) s(\ln \lambda, z), \quad (1)$$

where $w(z)$ is a weight function proportional to the density of emitters. If we assume that the spectral energy distribution is non-evolving with redshift and write it in terms of the emitted SED s_{rest} , the specific intensity can be written as the convolution

$$I(\ln \lambda) = \int d \ln(1+z) [(1+z)w(z)] s_{rest}(\ln \lambda - \ln(1+z)). \quad (2)$$

The observation that the specific intensity is a pure convolution, where the rest-frame SED has been convolved with the redshift distribution, bears important implications that we explore in this work. Since both the SED and the redshift distributions are unknown quantities of cosmological interest, we study in particular how this mathematical structure can be fully exploited to reconstruct these quantities.

Throughout this work, we will develop an explicit procedure for the spectral deconvolution of Eq. (1) and will elaborate on its domain of validity. We will introduce a general formalism for single type sources and discuss the case of an emitter of known SED in Sec. 2. In Sec. 3, we discuss the use of angular fluctuations in the intensity

map, taking advantage of the assumption of statistical isotropy to express the observables in terms of angular (cross-)power spectra. In Sec. 4, we then show how measuring fluctuations over a sufficiently large range of angular scales and wavelengths can potentially allow us to reconstruct both the a priori unknown SED and the underlying density fluctuations. We conclude in Sec. 5 and discuss in more detail the range of scales at which spectral deconvolution is feasible in Appendix A.

2. OBSERVED SPECTRA AND DECONVOLUTION FROM A KNOWN EMITTED SPECTRUM

We describe the observed specific intensity in terms of two quantities. On the one hand, the normalized rest-frame SED $s_{rest}(\ln \lambda)$ quantifies the power as a function of wavelength λ (to simplify later notation we write it in terms of the logarithm of wavelength $\ln \lambda$) emitted by a source, normalized by the luminosity of the source. It could be measured in $\text{W}/\text{\AA}/L_{\odot}$ (i.e. per solar luminosity). In other words, the observed (redshifted) differential flux per unit wavelength for a single source at redshift z with luminosity L , is

$$S_{\lambda}(\ln \lambda) = \frac{L s_{rest}(\ln \lambda - \ln(1+z))}{4\pi(1+z)D_L^2(z)} \quad (\text{single source}), \quad (3)$$

where D_L is the luminosity distance to the object. The factor $(1+z)$ in the denominator arises because this is the flux per unit wavelength, and wavelength intervals $d\lambda$ are redshifted. The second ingredient is the luminosity density of sources as a function of direction \hat{n} and redshift, $\mathcal{L}(z, \hat{n})$ (in units solar luminosity L_{\odot} per comoving volume), which is given by an integral over the luminosity function,

$$\mathcal{L}(z, \hat{n}) = \int dL L \phi(L, z, \hat{n}). \quad (4)$$

Here the luminosity function $\phi(L, z, \hat{n})$ gives the number of objects per comoving volume per luminosity interval dL . For the study of cosmological large scale structure, our main interest is in the spatial variations of $\mathcal{L}(z, \hat{n})$, as they are expected to be a biased tracer of the underlying dark matter distribution, thus providing information on the bias of sources and the shape of the matter power spectrum.

Putting these two ingredients together, the specific intensity I_{λ} (measured in, say, $\text{W m}^{-2} \text{\AA}^{-1} \text{sr}^{-1}$) can be written as the superposition of the emission along the line of sight at a range of cosmological redshifts,

$$I_{\lambda}(\ln \lambda, \hat{n}) = \int d \ln(1+z) \frac{\mathcal{L}(z, \hat{n})}{4\pi(1+z)^2 H(z)} s_{rest}(\ln \lambda - \ln(1+z)). \quad (5)$$

The above expression will be the basis for the work presented in this article.

The main assumption that goes into Eq. (5) is that the emitted SED is independent of position (i.e. of both z and \hat{n}), up to a normalization, so that angular variations are due to anisotropy in the source luminosity density and the wavelength dependence of the specific intensity is a simple convolution. This assumption would hold if all sources had the same SED *shape*, independent of location. Alternatively, the SED shape s_{rest} can be interpreted as a mean SED averaged over different objects

and luminosities. Our description would then be an appropriate one if the relative contributions to s_{rest} from different individual SED's were independent of position. The level to which Eq. (5) is a realistic description depends on the wavelength range considered. While nature will typically be more complicated than this, it serves as a good starting point, which can later be refined.

As noted above, a key property of the specific intensity is that Eq. (5) can be written as a convolution of the redshift distribution of emitters with the rest frame SED. Let us first simplify the expressions by defining

$$g(N, \hat{n}) \equiv \frac{\mathcal{L}(z, \hat{n})}{4\pi(1+z)^2 H(z)}, \quad (6)$$

where we have defined $N \equiv \ln(1+z)$, the number of e-foldings of expansion. The function g will be used extensively in the remainder of this article and can be interpreted as a rescaled luminosity density of sources (for simplicity, we will often refer to it as luminosity density). Eq. (5) thus reads

$$I_\lambda(\ln \lambda, \hat{n}) = \int dN g(N, \hat{n}) s_{rest}(\ln \lambda - N). \quad (7)$$

If the rest-frame SED is known, then one can simply perform the deconvolution by working in Fourier space. Indeed, neglecting the position dependence for now, Fourier transforming with respect to $\ln \lambda$, and denoting the conjugate variable as r (an indicator of spectral resolution) and using \tilde{X} to indicate Fourier transformed quantities, we find for the source luminosity density,

$$\tilde{g}(r) = \frac{\tilde{I}_\lambda(r)}{\tilde{s}_{rest}(r)}. \quad (8)$$

This is a cosmological analog of the Fourier quotient method of stellar kinematics (Simkin 1974; Sargent et al. 1977).

From this form, we see that it is beneficial for the reconstruction of the luminosity density to have a rest-frame SED with broad support in the Fourier domain. For example, a delta function in frequency can be seen to be useful, corresponding to the well-known case of line emission (e.g., 21cm, CO or CII). However, it is not required that the SED be a single line; a uniform SED in frequency leads to an inability to reconstruct luminosity density for $r \neq 0$, but any SED with non-trivial structure (e.g., a ‘‘break’’, as is commonly used in photometric redshift surveys) will allow the deconvolution to be done.

3. EXPLOITING ANGULAR VARIATIONS WHEN THE EMITTED SPECTRUM IS NOT KNOWN

If the normalized rest-frame SED function s_{rest} is not known, then it must be estimated from the same data. In a single line of sight, this is not possible: it is impossible to differentiate between density or luminosity differences at different redshifts and differences in intrinsic emitted SED at the same redshift, but at different wavelengths. However, one can do this measurement at multiple locations on the sky. While different locations on the sky will have different source densities at a given redshift, we have assumed the normalized rest-frame SED s_{rest} to not vary strongly on sufficiently large scales (see the brief discussion in Section 2). We illustrate below how to use this fact in the case of a constant rest-frame SED.

Let us thus consider the spectral density as a function of wavelength *and* angular position on the sky, $I_\lambda(\ln \lambda, \hat{n})$. Fourier transforming Eq. (5) with respect to $\ln \lambda$ then gives,

$$\tilde{I}_\lambda(r, \hat{n}) = \tilde{s}_{rest}(r) \times \tilde{g}(r, \hat{n}). \quad (9)$$

The luminosity density function $g(N, \hat{n})$ is proportional to the source density and we can thus describe the statistics of the spatial fluctuations in this function in terms of the statistics of the clustering of the source galaxies. However, we cannot assume that the fluctuations in $g(N, \hat{n})$ obey homogeneity because the background value $g(N) \equiv \langle g(N, \hat{n}) \rangle$ has a redshift dependence, and because, as we look towards higher z , the clustering amplitude of the sources may vary. In general, the only applicable symmetry is isotropy. The underlying reason is that we are studying fields defined on our past light cone, that are a function of angle on the sky and of redshift. Their statistics are invariant under rotations, but not under shifts in the radial direction. It makes sense then to analyze the perturbations in the observed intensity field in terms of spherical harmonics, which we do below.

Let us write for the (rescaled) luminosity density,

$$g(\hat{n}, N) = g(N) [1 + b(N)\delta_m(\vec{x}, N) + \epsilon]. \quad (10)$$

Here, $g(N)$ is the background value, and $\delta_m(\vec{x}, N)$ is the homogeneous (at a fixed cosmic time) three-dimensional matter overdensity field in comoving coordinates and the second argument denotes the time dependence of the field. The position \vec{x} is implicitly chosen to be on the past light cone, i.e. $\vec{x} = D(N)\hat{n}$ for a spatially flat universe, where $D(N)$ is the comoving distance to redshift z with $N = \ln(1+z)$. The quantity $b(N)$ is the luminosity weighted bias of the source density relative to the matter density⁶. This bias may evolve both due to variation with redshift of the source luminosity function, and due to evolution of the galaxy bias at fixed luminosity. Finally, ϵ is a stochastic shot noise contribution due to the finite number of sources contributing to the signal. We will ignore its contribution from here on.

Restricting the analysis for simplicity to large scales which are in the linear regime, we define the statistics of the matter overdensity field in Fourier space by

$$\langle \delta_m(\vec{k}, N)\delta_m(\vec{k}', N') \rangle = (2\pi)^3 \delta^D(\vec{k} + \vec{k}') P_0(k) T(N) T(N'), \quad (11)$$

where $P_0(k)$ is the matter power spectrum at $z = 0$ and $T(N)$ is the linear growth rate of matter perturbations relative to $z = 0$ (assumed to be scale-independent).

We can now write variations in the Fourier-space intensity field in Eq. (9) as

$$\delta \tilde{I}_\lambda(r, \hat{n}) = \tilde{s}_{rest}(r) \int dN e^{iNr} g(N) b(N) \delta_m(\hat{n}, N). \quad (12)$$

For a given r , this is thus a line-of-sight integral of the overdensity field, multiplied by some kernel that only

⁶ $b(z) = \int dL L \phi(L, z) b_s(L, z) / (\int dL L \phi(L, z))$, where ϕ is the luminosity function of sources and $b_s(L, z)$ is the bias relative to matter of the density of sources with luminosity L at redshift z . For simplicity, we consider large, linear scales so that a scale-independent bias is appropriate.

depends on N . This is analogous to other cosmological observables that are functions of \hat{n} , like the cosmic shear/convergence field, or the overdensity of galaxies in a redshift bin. We can thus expand the \hat{n} -dependence in terms of spherical harmonics, and apply the usual machinery to derive an expression for the angular power spectrum. This leads to

$$C_\ell(r, r') = \tilde{s}_{rest}(r) \tilde{s}_{rest}^*(r') \frac{2}{\pi} \int dk k^2 P_0(k) W_\ell(k, r) W_\ell^*(k, r'), \quad (13)$$

$$\text{with } W_\ell(k, r) = \int dN j_\ell[kD(N)] e^{iNr} g_0(N) b(N) T(N)$$

where j_ℓ is the spherical Bessel function and $D(N)$ is comoving distance. While we will neglect this term in the following, for completeness we note that, if we assume the sources to be Poisson tracers of the underlying matter distribution, there will be a shot noise contribution

$$C_\ell^{sn}(r, r') = \tilde{s}_{rest}(r) \tilde{s}_{rest}^*(r') \times \int dN \frac{e^{iN(r-r')}}{D^2(N)D'(N)} g^2(N) \left(\frac{\int dL L^2 \phi(L, N)}{(\int dL L \phi(L, N))^2} \right), \quad (14)$$

where ϕ is the luminosity function.

Returning to the contribution due to large scale clustering, in the limit that the scale $\sim r^{-1}$ of radial/wavelength fluctuations is much larger than the transverse fluctuation scale $\sim \ell^{-1}$, we can apply the Limber approximation, leading to

$$C_\ell(r, r') = \tilde{s}_{rest}(r) \tilde{s}_{rest}^*(r') \times \int dN P_0 \left(\frac{\ell + \frac{1}{2}}{D(N)} \right) \frac{e^{iN(r-r')}}{D^2(N)D'(N)} g^2(N) b^2(N) T^2(N) \quad (15)$$

where $D' \equiv dD/dN$. We will discuss the range of validity of the Limber approximation in more detail in the Appendix.

We have ignored redshift space distortions in the above discussion. While these are in general non-negligible, they are small in the same limit where the Limber approximation is valid. The reason for this is simply that in the Limber approximation only transverse density modes contribute to the observed angular power spectra and that peculiar velocities due to transverse modes do not have a line-of-sight component and thus do not cause redshift space distortions. Since the main result of this paper, i.e. the procedure (discussed in the next section) for simultaneously extracting the distribution of sources and the rest-frame SED from the data itself, is only applied on scales where the Limber approximation is valid, it is a justified approximation to neglect redshift space distortions.

By angle-averaging, we can also estimate the mean intensity

$$\tilde{I}_\lambda(r) = \tilde{s}_{rest}(r) \times \tilde{g}(r). \quad (16)$$

We thus fundamentally have two observables: the mean, Eq. (16), and the angular cross- and power-spectra,

Eq. (13). In the next section, we will discuss how to extract the clustering of sources from these data, without assuming prior knowledge of either the rest-frame SED $\tilde{s}_{rest}(\ln \lambda)$ or the mean luminosity density $g(N)$.

4. JOINTLY RECONSTRUCTING A CONSTANT REST-FRAME SED AND THE CLUSTERING OF SOURCES

Using the tools developed in the previous section, we can now study to what extent we can extract interesting physical quantities from the observables $C_\ell(r, r')$ and $\tilde{I}_\lambda(r)$. We would especially like to obtain a measurement of the clustering of sources, i.e. the information contained in the matter power spectrum $P_0(k)$, and the biased transfer function $b(N)T(N)$. It turns out it is possible to indeed isolate this clustering information, while simultaneously estimating the normalized rest-frame SED $s_{rest}(\ln \lambda)$ and the mean luminosity density $g(N)$. We explain this below.

Let us consider first the diagonal angular spectra⁷, $r' = r$. The observed spectrum is then a separable function of r and ℓ ,

$$C_\ell(r) \equiv C_\ell(r, r) = A(\ell) |\tilde{s}_{rest}(r)|^2, \quad (17)$$

with

$$A(\ell) \equiv \int dN P_0 \left(\frac{\ell + \frac{1}{2}}{D(N)} \right) \frac{g^2(N) b^2(N) T^2(N)}{D^2(N)D'(N)}. \quad (18)$$

This separability property will prove to be crucial for the argument below.

It is straightforward to obtain clustering information that is independent of the rest-frame SED $s_{rest}(\ln \lambda)$ from the diagonal spectra $C_\ell(r)$. For instance, we can take the ratio with the squared mean intensity defined in Eq. (16),

$$\frac{C_\ell(r)}{|\tilde{I}_\lambda(r)|^2} = \frac{A(\ell)}{|\tilde{g}(r)|^2}. \quad (19)$$

This quantity contains an integral over the clustering of sources (inside $A(\ell)$) and otherwise only depends on the unknown function $g(N)$. Alternatively, $A(\ell)$ could have been directly estimated using the separability of Eq. (17). In either case, the next challenge is to estimate $g(N)$ from the data. This can be done in several ways and we outline a step-by-step procedure below.

- First, we estimate $\tilde{s}_{rest}(r)$, up to an r -independent normalization. We fix this normalization by specifying $\tilde{s}_{rest}(r)$ at some $r = r_0$, i.e. $\tilde{s}_0 \equiv \tilde{s}_{rest}(r_0)$. It turns out that this normalization does not matter for the reconstruction of the clustering information so it is not a problem if it remains unknown. The first step is to use the separability of $C_\ell(r)$ to estimate the norm of $\tilde{s}_{rest}(r)$ for all r (we will discuss the range of validity of this procedure in more detail in Appendix A). Explicitly,

$$|\tilde{s}_{rest}(r)|^2 = |\tilde{s}_0|^2 \sum_\ell W_\ell^{(\tilde{s})}(r) \frac{C_\ell(r)}{C_\ell(r_0)}. \quad (20)$$

⁷ We expect a large fraction of the information to reside in the $r = r'$ configurations as the terms other than the $e^{iN(r-r')}$ in the integrand on the right hand side of Eq. (15) are slowly varying, so that the signal quickly declines for $r \neq r'$.

Here, for each ℓ , the quantity $|\tilde{s}_0|^2 C_\ell(r)/C_\ell(r_0)$ is an estimator of $|\tilde{s}_{\text{rest}}(r)|^2$ so a general estimator is written as a sum over all multipoles with a set of weights $W_\ell^{(\tilde{s})}(r)$. In principle, these weights can be adjusted to optimize the signal-to-noise ratio of the estimator, but we leave this question for future work.

- The above calculation only gives us the norm of $\tilde{s}_{\text{rest}}(r)$ as a function of r , but not the phase. To extract the phase information, the angular *cross* $r' \neq r$ power spectra can be used. For example, we can take ratios of the following kind to estimate

$$\frac{\tilde{s}_{\text{rest}}(r')}{\tilde{s}_{\text{rest}}(r)} = \frac{C_\ell(r', \frac{1}{2}(r+r'))}{C_\ell(r, \frac{1}{2}(r+r'))}. \quad (21)$$

Using this relation for various pairs of r and r' gives the evolution of the phase of \tilde{s}_{rest} (relative to the assumed phase $\tilde{s}_0 = \tilde{s}_{\text{rest}}(r_0)$). In principle, these ratios also give information on the norm, but we expect that the diagonal angular power spectra discussed in the previous step carry more information.

- Now that we know $\tilde{s}_{\text{rest}}(r)$, we can use the mean intensity $\tilde{I}_\lambda(r) = \tilde{s}_{\text{rest}}(r) \tilde{g}(r)$ to estimate $\tilde{g}(r)$, i.e.

$$\tilde{g}(r) = \frac{\tilde{I}_\lambda(r)}{\tilde{s}_{\text{rest}}(r)}. \quad (22)$$

Since we had to assume \tilde{s}_0 for our estimate of $\tilde{s}_{\text{rest}}(r)$, we have really estimated $\tilde{g}(r) \times \tilde{s}_0$.

- We can now insert the estimate of $\tilde{g}(r)$ (and therefore $g(N)$) into, e.g., Eq. (19). We see that the normalization \tilde{s}_0 conveniently drops out and we have now isolated the effect of $P_0(k)$ and $b(N)T(N)$.

Of course, we still only measure the projected power spectrum (see Eq. (18)), and uncertainty in the background cosmology, which determines the projection through $D(N)$ and $D'(N)$, still needs to be taken into account.

Moreover, the above relies crucially on the Limber approximation, as in general (see Eq. (13)), the ℓ and r dependences do not factorize. We therefore discuss in the Appendix for what range of multipoles ℓ and wavelength Fourier conjugate r the above procedure can be applied. The main conclusion (but see the Appendix for details) is that the reconstruction described above is valid for multipoles $\ell \approx 30 - \ell_{\text{max}}$, where ℓ_{max} is the largest multipole that can be observed given the angular resolution (and noise and non-linear cutoff) of the experiment. For a given multipole ℓ , the approach can be applied to line-of-sight/wavelength Fourier modes $r \approx r_{\text{min}} - \min(\ell/10, r_{\text{max}})$, where r_{min} is the minimum observed mode, set approximately by $1/\Delta \ln \lambda$, with $\Delta \ln \lambda$ the wavelength range covered, and r_{max} is the maximum resolved mode, set by the spectral resolution R . Put simply, the approach sketched in this work can be applied to mostly transverse modes, i.e. those that vary more rapidly in the transverse (angular) direction than in the line-of-sight/wavelength direction.

In this section, we have outlined an explicit series of steps that can be followed to obtain information on the

clustering of sources and on the rest-frame SED simultaneously. When applied to data, it may make more sense to either model or parametrize the functions that are to be reconstructed, i.e. $g(N)$, $b(N)T(N)$, $s_{\text{rest}}(\ln \lambda)$ and $P_0(k)$, and to simultaneously fit the resulting parameters to the 3D spectral mapping observables discussed above. The explicit reconstruction method discussed in this section then shows that, because of statistical isotropy, the degeneracy between these parameters can be broken when the mean and angular variations in the map are used, so that parameters describing the clustering on the one hand, and parameters describing the mean luminosity and the rest-frame SED on the other hand, can in principle be constrained independently.

5. CONCLUSIONS

We have introduced a formalism for measuring the projected two-point function of the luminosity density of extragalactic sources from spectral mapping data, i.e. from a three-dimensional data cube of specific intensity as a function of wavelength and line of sight direction. This method does not rely on the use of spectral lines and works for an arbitrary rest-frame SED form. In fact, the source SED does not even need to be known in advance, but is reconstructed from the data itself.

This spectral deconvolution technique makes use of both the mean intensity as a function of wavelength and its angular variations. The wavelength dependence of the rest-frame SED and the redshift dependence of the luminosity density of sources can be disentangled because the observed specific intensity is a convolution of these two quantities. This allows a straightforward spectral deconvolution in Fourier space. The spatial variations in the reconstructed luminosity density contain valuable information on the large scale clustering of matter.

While one might expect this approach to only work if the (mean) rest-frame SED is known a priori, we have shown in Section 4 how to use the statistical isotropy of the observed signal to simultaneously measure the rest-frame SED and the clustering of the luminosity density when this is not the case. Specifically, by first Fourier transforming the spectral map with respect to $\ln \lambda$ (with Fourier conjugate r), and then considering the angular power spectrum for mostly transverse modes (multipoles $\ell \gg r$), the term describing the clustering of extragalactic sources to a good approximation only depends on multipole ℓ , while the rest-frame SED term only depends on the wavelength direction r . It is this separability of variables that allows for their independent reconstruction.

The purpose of this paper has been to give a rather formal presentation of the method of spectral deconvolution, leaving more concrete explorations of how to apply the technique to realistic data for future work. A particularly strong assumption we have made throughout this work is that the extragalactic signal can be described in terms of a single effective rest-frame SED (up to a free normalization). It will be interesting to generalize the method to scenarios where multiple populations with distinct SED's need to be factored in.

One additional motivation for studying the information content of three-dimensional spectral maps, beyond the fact that such data will be available from spectral mapping experiments, is that, as discussed in the Introduction, such maps provide a unifying description for a

large range of cosmological surveys, with different probes accessing different subsets of the data cube. It would thus be useful to build a more general understanding of all the information that can in principle be extracted from the full data cube. This can then be a guide towards identifying how to optimally exploit these data in the future.

We acknowledge the Aspen Center for Physics where part of this work originated. The center is supported by NSF grant 1066293. TC acknowledges support from the Simons Foundation. Part of the research described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

APPENDIX

ANGULAR AND WAVELENGTH VALIDITY RANGE

Section 4 in principle gives us a step-by-step approach to reconstructing the clustering and rest-frame SED from the spectral mapping data. We here estimate for what range of multipoles ℓ and line of sight wave numbers r this procedure is applicable. We assume the angle averaged intensity $\bar{I}(r)$ and the anisotropies in the intensity $\delta\bar{I}_\lambda(r, \hat{n})$ can be reliably measured for some range

$$r \in [r_{\min}, r_{\max}], \quad (\text{A1})$$

$$\ell \in [\ell_{\min}, \ell_{\max}]. \quad (\text{A2})$$

While it goes against the spirit of this paper to precisely quantify these bounds for a realistic experiment, their interpretation in terms of survey properties is easily understood. The lower bound r_{\min} is related to the range of wavelengths for which we can observe the spectrum, $r_{\min} \sim 1/\Delta \ln \lambda$ (with $\Delta \ln \lambda$ the observed spectral range in $\ln \lambda$), while r_{\max} is related to the spectral resolution of the instrument $r_{\max} \sim 1/d \ln \lambda \sim R$ (with $d \ln \lambda$ the wavelength resolution). For the multipole range, ℓ_{\min} is given by the sky coverage of the survey, while ℓ_{\max} would be determined by the resolution and by the angular scale at which the noise becomes large. Moreover, restricting the analysis to clustering in the linear regime places a constraint on the maximum multipole that can be used.

Another important restriction on the range of phase space that can be used comes from the Limber approximation, which was a crucial assumption in the derivations of the previous section. The Limber approximation is valid for multipoles larger than a critical scale, which may depend on r , i.e. $\ell > \ell_L(r)$. To quantify $\ell_L(r)$, consider the analysis presented in [Loverde & Afshordi \(2008\)](#). Here, an explicit expression for the leading order correction to the Limber approximation is presented and used to quantify the range of validity of the Limber approximation. In their notation, the integrated kernel relevant for the spectral mapping scenario studied in the present paper can be written as

$$f_{A/B}(\chi) = \frac{aH}{\sqrt{\chi}} g(\chi) b(\chi) T(\chi) e^{\pm iN(\chi)r} \equiv \bar{f}(\chi) e^{\pm iN(\chi)r}. \quad (\text{A3})$$

To avoid confusion with the ‘‘resolution parameter’’ r (the Fourier conjugate of $\ln \lambda$), we have used the letter χ for comoving distance instead of r (which is used in [Loverde & Afshordi 2008](#)). We have explicitly expressed ‘‘line-of-sight’’ quantities in terms of χ instead of $N = \ln(1+z)$.

In Eq. (13) of [Loverde & Afshordi \(2008\)](#), the correction to the Limber approximation is expressed as an integral over $d \ln f_A/d \ln \chi d \ln f_B/d \ln \chi$. The correction is small, and therefore the Limber approximation good, if the product of this quantity with ℓ^2 is small, which in our case means

$$\begin{aligned} & \ell^{-2} d \ln f_A/d \ln \chi d \ln f_B/d \ln \chi = \\ & \ell^{-2} \left((d \ln \bar{f}/d \ln \chi)^2 + (aH\chi)^2 r^2 \right) \ll 1. \end{aligned} \quad (\text{A4})$$

We require that both terms in the parentheses satisfy the condition individually. The first term in the parentheses then gives the usual condition appropriate for a kernel $\bar{f}(\chi)$, say $\ell > \ell_{L,0}$. We expect the kernel $\bar{f}(\chi)$ relevant for spectral mapping to be relatively wide because the luminosity density $g(\chi)$, the product of bias and transfer function $b(\chi)T(\chi)$, and the geometric factors in Eq. (A3), should all vary slowly with distance. Assuming therefore that the mean distance $\bar{\chi}$ of the kernel and its width $\Delta\chi$ are of the same order of magnitude, we expect $(d \ln \bar{f}/d \ln \chi)^2$ to be of order unity. To be conservative, we then use $\ell_{L,0} \sim 30$. The second term in the parentheses gives an r -dependent requirement, $\ell > \ell_{L,r} r$ and we estimate $\ell_{L,r} \sim 10$ (assuming the typical redshift for which the kernel is large to be $z \sim 1$). Summarized, we get the following constraint:

$$\begin{aligned} \ell > \ell_L(r) & \equiv \max\{\ell_{L,0}, \ell_{L,r} r\} \\ \text{with } \ell_{L,0} & \sim 30 \quad \text{and} \quad \ell_{L,r} \sim 10. \end{aligned} \quad (\text{A5})$$

Equations (A1), (A2) and (A5) list the restrictions on the range of scales for which we can use $C_\ell(r)$. To continue, we assume

$$\ell_{\min} < \ell_{L,0} < \ell_{\max}, \quad (\text{A6})$$

$$r_{\min} < \frac{\ell_{L,r}}{\ell_{L,0}}. \quad (\text{A7})$$

These are realistic assumptions and simplify the following expressions. For a given r in the range (A1), we can use the $C_\ell(r)$'s in the range $\ell \in [\ell_L(r), \ell_{\max}]$. Requiring that this range is non-zero gives the range of r that is usable,

$$\begin{aligned} r &\in [r_{\min}, \min\{\ell_{\max}/\ell_{L,r}, r_{\max}\}] \\ \ell &\in [\ell_L(r), \ell_{\max}], \end{aligned} \quad (\text{A8})$$

or equivalently

$$\begin{aligned} \ell &\in [\ell_{L,0}, \ell_{\max}] \\ r &\in [r_{\min}, \min\{\ell/\ell_{L,r}, r_{\max}\}]. \end{aligned} \quad (\text{A9})$$

The usable range for the angle-averaged intensity is simply given by Eq. (A1).

Given the range of scales where we can use the observables $C_\ell(r)$ and $\tilde{I}_\lambda(r)$, we can now write the estimator for $|\tilde{s}_{\text{rest}}(r)|$, given in Eq. (A10) in the previous section, as

$$|\tilde{s}_{\text{rest}}(r)|^2 = |\tilde{s}_0|^2 \sum_{\ell \in [\ell_L(r), \ell_{\max}]} W_\ell^{(\tilde{s})}(r) \frac{C_\ell(r)}{C_\ell(r_0)}, \quad (\text{A10})$$

where we have now explicitly specified the range of multipoles to be summed over. For the optimal range of ℓ to be available, we need to choose

$$r_0 \in \left[r_{\min}, \frac{\ell_{L,0}}{\ell_{L,r}} \right]. \quad (\text{A11})$$

Following the entire series of steps detailed in Section 4, and applying the restrictions on the range of scales that can be used, as given in Eq. (A8) or (A9), we conclude that the projected power spectrum of the source density, $A(\ell)$ can be measured for all ℓ given in Eq. (A9). The background luminosity density $\tilde{g}(r)$ and rest-frame SED $\tilde{s}_{\text{rest}}(r)$ can both be reconstructed for all r in Eq. (A8).

To extract the clustering information from $A(\ell)$, we want to insert the function $g(N)$, which is obtained by Fourier transforming the reconstructed function $\tilde{g}(r)$. The range of r , for which this reconstruction is possible tells us that we can only reconstruct variations in $g(N)$ on scales smaller than $(\Delta N)_{\max} \sim 1/r_{\min} \sim \Delta \ln \lambda$ (the wavelength range covered by the spectral mapper). Moreover, we cannot recover variations on scales smaller than $(\Delta N)_{\min} \sim \max(\ell_{L,r}/\ell_{\max}, 1/r_{\max} \sim R^{-1})$, which is the maximum of the wavelength resolution and $\sim 10\times$ the angular resolution scale (in radians). This will limit how well one can recover the power spectrum information.

REFERENCES

- Anderson, L., *et al*, 2013, arXiv:1312.4877
- Benítez, N., Gaztañaga, E., Miquel, R., Castander, F., Moles, M., Crocce, M., Fernández-Soto, A., Fosalba, P., Ballesteros, F., Campa, J., *et al*, 2009, ApJ, 691, 241
- Bentler, F., Blake, C., Colless, M., Jones, D. H., Staveley-Smith, L., Campbell, L., Parker, Q., Saunders, W., & Watson, F. 2011, MNRAS, 416, 3017
- Chang, T.-C., Pen, U.-L., Bandura, K., & Peterson, J. B. 2010, Nature, 466, 463
- Chang, T.-C., Pen, U.-L., Peterson, J. B., & McDonald, P. 2008, Physical Review Letters, 100, 091303
- Cole, S., Percival, W. J., Peacock, J. A., Norberg, P., Baugh, C. M., Frenk, C. S., Baldry, I., Bland-Hawthorn, J., Bridges, T., Cannon, *et al*, 2005, MNRAS, 362, 505
- Cooray, A., Amblard, A., Wang, L., Arumugam, V., Auld, R., Aussel, H., Babbedge, T., Blain, A., Bock, J., Boselli, A., Buat, V., Burgarella, D., Castro-Rodriguez, N., Cava, A., Chianial, P., *et al*, 2010, A&A, 518, L22
- de la Torre, S., Guzzo, L., Peacock, J. A., Branchini, E., Iovino, A., Granett, B. R., Abbas, U., Adami, C., Arnouts, S., Bel, J., *et al*, 2013, A&A, 557, A54
- Donoso, E., Yan, L., Stern, D., & Assef, R. J. 2013, arXiv:1309.2277
- Efstathiou, G. & Moody, S. J. 2001, MNRAS, 325, 1603
- Eisenstein, D. J., Zehavi, I., Hogg, D. W., Scoccamarro, R., Blanton, M. R., Nichol, R. C., Scranton, R., Seo, H.-J., Tegmark, M., Zheng, *et al*, 2005, ApJ, 633, 560
- Feldman, H. A., Kaiser, N., & Peacock, J. A. 1994, ApJ, 426, 23
- Frith, W. J., Outram, P. J., & Shanks, T. 2005, MNRAS, 364, 593
- Hauser, M. G. & Peebles, P. J. E. 1973, ApJ, 185, 757
- Hill, G. J., Gebhardt, K., Komatsu, E., Drory, N., MacQueen, P. J., Adams, J., Blanc, G. A., Koehler, R., Rafal, M., Roth, M. M., *et al*, 2008, in Astronomical Society of the Pacific Conference Series, Vol. 399, Panoramic Views of Galaxy Formation and Evolution, ed. T. Kodama, T. Yamada, & K. Aoki, 115
- Ho, S., Agarwal, N., Myers, A. D., Lyons, R., Disbrow, A., Seo, H.-J., Ross, A., Hirata, C., Padmanabhan, N., O'Connell, R., *et al*, 2013, arXiv:1311.2597
- Ho, S., Cuesta, A., Seo, H.-J., de Putter, R., Ross, A. J., White, M., Padmanabhan, N., Saito, S., Schlegel, D. J., Schlafly, E., *et al*, 2012, ApJ, 761, 14
- Loeb, A. & Wytthe, J. S. B. 2008, Physical Review Letters, 100, 161301
- Loverde, M. & Afshordi, N. 2008, Phys. Rev. D, 78, 123506
- Moles, M., Benítez, N., Aguerri, J. A. L., Alfaro, E. J., Broadhurst, T., Cabrera-Cañó, J., Castander, F. J., Cepa, J., Cerviño, M., Cristóbal-Hornillos, D., *et al*, 2008, AJ, 136, 1325
- Moles, M., Sánchez, S. F., Lamadrid, J. L., Cenarro, A. J., Cristóbal-Hornillos, D., Maicas, N., & Aceituno, J. 2010, PASP, 122, 363
- Padmanabhan, N., Schlegel, D. J., Seljak, U., Makarov, A., Bahcall, N. A., Blanton, M. R., Brinkmann, J., Eisenstein, D. J., Finkbeiner, D. P., Gunn, J. E., *et al*, 2007, MNRAS, 378, 852
- Parkinson, D., Riemer-Sørensen, S., Blake, C., Poole, G. B., Davis, T. M., Brough, S., Colless, M., Contreras, C., Couch, W., Croom, S., *et al*, 2012, Phys. Rev. D, 86, 103518
- Pritchard, J. R. & Loeb, A. 2012, Reports on Progress in Physics, 75, 086901
- Ross, A. J., Ho, S., Cuesta, A. J., Tojeiro, R., Percival, W. J., Wake, D., Masters, K. L., Nichol, R. C., Myers, A. D., de Simoni, F., Seo, H. J., Hernández-Montegudo, C., Crittenden, R., Blanton, M., Brinkmann, J., *et al*, 2011, MNRAS, 417, 1350

- Sargent, W. L. W., Schechter, P. L., Boksenberg, A., & Shortridge, K. 1977, *ApJ*, 212, 326
- Scranton, R., Johnston, D., Dodelson, S., Frieman, J. A., Connolly, A., Eisenstein, D. J., Gunn, J. E., Hui, L., Jain, B., Kent, S., *et al*, 2002, *ApJ*, 579, 48
- Simkin, S. M. 1974, *A&A*, 31, 129
- Switzer, E., Masui, K., Bandura, K., Calin, L. M., Chang, T. C., *et al*. 2013
- Tegmark, M., Dodelson, S., Eisenstein, D. J., Narayanan, V., Scoccimarro, R., Scranton, R., Strauss, M. A., Connolly, A., Frieman, J. A., Gunn, J. E., *et al*, 2002, *ApJ*, 571, 191
- Visbal, E. & Loeb, A. 2010, *JCAP*, 11, 16