TRANSPORT OF SOLAR FLARE PROTONS - COMPARISON OF A NEW ANALYTIC MODEL WITH SPACECRAFT MEASUREMENTS

J. E. Lupton and E. C. Stone

California Institute of Technology

Pasadena, California 91109

July 1971

SRL 71-3

TRANSPORT OF SOLAR FLARE PROTONS - COMPARISON OF A NEW ANALYTIC MODEL WITH SPACECRAFT MEASUREMENTS

J. E. Lupton and E. C. Stone

California Institute of Technology, Pasadena, California, U.S.A., 91109

An analytic solution has now been obtained to the complete Fokker-Planck equation including the effects of convection, interplanetary deceleration and acceleration, corotation, and anisotropic diffusion with $\kappa_{\parallel}$ constant and with $\kappa_{\perp} \propto r^2$. With the boundary of the diffusing region at 2.3 AU, a solar wind velocity of 400 km sec^{-1}, $\kappa_{\parallel} \sim 7 \times 10^{20}$ cm^2 sec^{-1}, and impulsive injection on the line of force connecting to the earth, the solution yields a time to maximum for the particle flux of ~10 h and an exponential decay time of ~25 h. Several solar flare particle events have been observed with the Caltech Solar and Galactic Cosmic Ray Experiment on OGO-6. Detailed comparisons of the calculated time dependence of the fluxes with these observations of 1-70 MeV protons show that the model adequately describes both the rise and decay times, indicating that $\kappa_{\parallel}$ = constant is a better representation of conditions inside 1 AU than is $\kappa_{\parallel} \propto r$.

1. Introduction. The propagation of energetic solar flare particles through interplanetary space has been studied both theoretically and experimentally for a number of years. Although Parker (1965) had included a term for adiabatic deceleration in his general formulation of particle propagation, analytical descriptions of solar flare particle propagation only recently have included adiabatic effects (Fisk and Axford, 1968; Forman, 1970; Forman, 1971).

Experimentally, the first evidence for energy-change processes in interplanetary space was reported by Murray, et al (1971) using data from the Caltech Solar and Galactic Cosmic Ray Experiment on OGO-6 (Althouse, et al, 1967). Since that report, additional flare events have been studied and compared with the above analytic descriptions of particle propagation. As expected, the predicted flux risetime was too slow or the predicted time dependence of the decay was other than the observed exponential dependence. Therefore, the following analytical solution for particle propagation was derived.

2. The New Solution. The Fokker-Planck equation, which describes the propagation of cosmic ray particles in interplanetary space, can be written:

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot \left\{ \vec{V} \left[n - \frac{1}{3} \frac{\partial}{\partial T} (\alpha T n) \right] - \underline{\underline{\kappa}} \cdot \vec{\nabla} n \right\} = - \frac{V}{3} \frac{\partial}{\partial r} \frac{\partial}{\partial T} (\alpha T n) \tag{1}$$

where n is the particle density, $\vec{V}$ the solar wind velocity, T the particle kinetic energy, $\alpha = (T + 2 m_0c^2) / (T + m_0c^2)$ and $\underline{\underline{\kappa}}$ is the diffusion tensor. If the solar wind velocity V is assumed to be independent of the spatial parameters, the equation then reduces to:

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{V}) - \vec{\nabla} \cdot (\underline{\underline{\kappa}} \cdot \vec{\nabla} n) = \frac{2V}{3r} \frac{\partial}{\partial T} (\alpha T n) \tag{2}$$

The right hand side, which treats the adiabatic deceleration caused by the solar wind expansion, can be generalized to include the effects of any energy change process which can be characterized by a time constant $\tau_E(r) \propto r$. The equation then becomes:

$$\frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{V}) - \vec{\nabla}\cdot(\underline{\underline{\kappa}}\cdot\vec{\nabla}n) = \frac{1}{\tau_o r}\frac{\partial}{\partial T}(Tn) \quad (3)$$

where $\tau_E(r) = \tau_o r$. This includes the effects of anisotropic diffusion, convection, and energy change. Note that adiabatic deceleration is a special case of Eq. 3 with $\tau_E\ (r) = 3r/4V$ and $\alpha(T) = 2$.

A solution to Eq. 3 has been found which describes solar flare particle transport, using the following simplifying assumptions:

1. The particle density n depends only on radial distance r, azimuthal angle θ, time t, and particle kinetic energy T.

2. The energy T is not treated as an independent variable.

3. The solar wind velocity V is radial and independent of r, θ, and t.

4. The density is a power law in kinetic energy $n(r,\theta,t,T) = n_o(r,\theta,t)T^{-\gamma}$.

5. The particles are impulsively injected at $r=r_s$ at time t=0.

6. The density n must remain finite as $r \rightarrow 0$ (this is a substitute for a more realistic but more complicated boundary condition specified at $r=r_s$).

7. A perfectly absorbing boundary exists at r=L so that $n(L,\theta,t,T) = 0$.

8. The diffusion tensor $\underline{\kappa}$, which is independent of T, is defined by $\kappa_\perp = \kappa_1 r^2$ and $\kappa_{\parallel} = \kappa$ = constant.

As demonstrated previously by Burlaga (1967) and Forman (1971), when $\kappa_\perp$ is assumed to vary as r^2, the equation can be separated as follows:

$$n(r,\theta,t,T) = Q(\theta,t)R(r,t)T^{-\gamma} \quad (4)$$

For δ-function injection at $\theta = 0$, the azimuthal part of the solution can be expanded as (Burlaga, 1967)

$$Q(\theta,t) = \sum_{\ell} \exp\left[-\kappa_1 \ell(\ell+1)t\right]\ (2\ell+1)P_\ell(\cos\theta) \quad (5)$$

The equation for the radial part of the solution becomes

$$\frac{\partial R}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\ r^2\left[\kappa\frac{\partial R}{\partial r} - VR\right] + \frac{1}{\tau_o r}\ \frac{\partial}{\partial T}(TR) \quad (6)$$

where the terms relating to diffusion, convection, and energy change are still clearly evident.

The new result presented here involves the following eigenvalue expansion for the solution to Equation 6:

$$R(r,t) = A \frac{\exp\left(V(r-r_s)/2\kappa\right)}{rr_s} \sum_{n=1}^{\infty} \frac{\mathrm{Fo}\left(\beta/2\alpha_n^{\frac{1}{2}},\alpha_n^{\frac{1}{2}} r_s\right)\ \mathrm{Fo}\left(\beta/2\alpha_n^{\frac{1}{2}},\alpha_n^{\frac{1}{2}} r\right)}{N_n} e^{-t/\tau_n} \quad (7)$$

where Fo(η,x) is the regular Coulomb wave function (Abramowitz and Stegun, 1964), and the α_n are defined by the eigenvalue equation Fo($\beta/2\alpha_n^{1/2}$, $\alpha_n^{1/2}$ L) = 0. The other parameters are defined as follows:

$$\beta = V(2C-1)/\kappa \quad (8)$$

$$C = 1 + (\gamma-1)/2V\tau_o \quad (9)$$

$$\tau_n = 4\kappa/(4\kappa^2\alpha_n + V^2) \quad (10)$$

$$N_n = \int_o^L \left[\mathrm{Fo}(\beta/2\alpha_n^{\frac{1}{2}},\ \alpha_n^{\frac{1}{2}}\ x)\right]^2 dx \quad (11)$$

The constant A is an arbitrary normalization determined by the number of particles injected. In the limit as $V \to 0$, this solution reduces to

$$R(r,t) = \frac{2A}{rr_sL} \sum_{n=1}^{\infty} \sin\left(\frac{n\pi r_s}{L}\right) \sin\left(\frac{n\pi r}{L}\right) \exp\left(-\frac{n^2\pi^2\kappa t}{L^2}\right) \quad (12)$$

which is identical to the ADB solution obtained by Burlaga (1967).

3. The Behavior of the Solution. Figure 1 shows the time profile of the solution at r = 1 AU for typical values of the parameters. The total solution n(t) is the product of the radial part R(t) and the azimuthal part Q(θ',t), with the transformation $\theta' = \theta_o + \Omega\, t$ included to describe the effects of corotation. In this example θ_o =-100°, which corresponds to a flare position of ~55° E solar longitude. Figure 1 clearly demonstrates that the radial part of the solution yields rise times of ~10 h and decay time constants of ~25 h using a reasonable value for $\kappa_{\parallel}$.

It can be seen from Equation 7 that at large times the first term in the expansion dominates and the time profile decays exponentially with $\tau_{DEC} = \tau_1$. This decay time constant is a function of the solar wind velocity V, the diffusion coefficient $\kappa_{\parallel}$, the outer boundary position L, and the energy-change parameter C. As expected, $\tau_{DEC} \propto 1/\kappa_{\parallel}$ for large values of $\kappa_{\parallel}$ as the solution approaches Burlaga's model.

4. Comparison with Spacecraft Measurements. A preliminary comparison of the new solution has been made with actual measurements of solar flare particle time profiles. Although we have not yet optimized the values of all the parameters

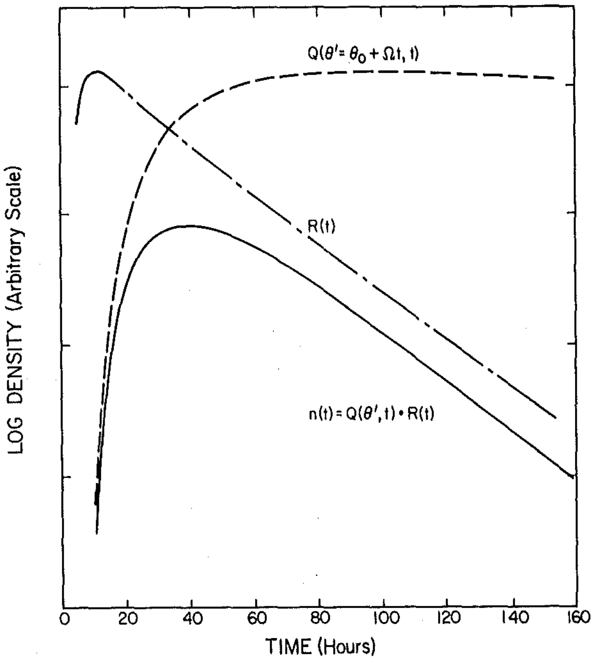

Fig. 1. The density n(t) predicted by the solution is shown as a function of time. The radial part R(t) of the solution and the azimuthal Q(θ',t) are shown separately.

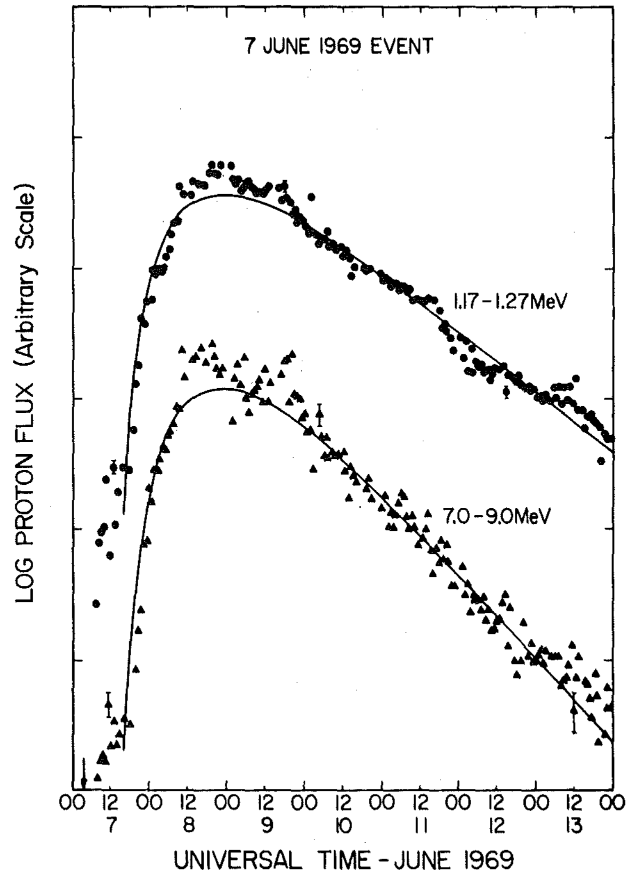

Fig. 2. Comparison of the calculated flux profile (solid lines) with solar flare event observations. The following values of the parameters were used: $\kappa_{\parallel}$ ∿7.5x10^{20} - 1.0x10^{21} cm^2/sec, $\kappa_{\perp}$ ∿3x10^{20} cm^2/sec, C ∿1.4 - 1.8, L = 2.3 AU, θ_o =-100°.

involved, reasonable estimates have been made, and the resulting fits to the actual data are shown in Figures 2 and 3.

The 7 June 1969 event, shown in Figure 2, has a time-to-maximum of ∿40 h, due to the ∿100° distance in solar longitude between the flare and the direct-connected field line. Consequently, the rise of the event is largely determined by the time profile of the Q(θ',t) function while the decay phase is defined by the radial function R(t) (see Figure 1). The model approximates the observed profiles quite well using L = 2.3 AU and values of $\kappa_{\parallel}$ ∿8x10^{20} cm^2/sec.

The 2 November 1969 event, which occurred at 90° W solar longitude, is separated by only ∿35° from the direct-connected field line and therefore has a much more rapid rise. Figure 3 demonstrates that reasonable fits can be achieved using L = 2.3 AU for energies from 1 to 70 MeV. It should be emphasized that for this November event the radial part of the solution alone determines the principal features of both the rise and decay, and that this event thus provides a critical test for the solution presented here.

5. Conclusion. The new work presented here consists of re-solving the differential equation for the radial part of the particle propagation, using $\kappa_{\parallel}$ = constant, and including the effects of convection and energy change that are known to be important at low energies. In every other respect the assumptions made are the same as those used by Burlaga and Forman. Although the detailed dependence of the solution on all of the parameters has not been completely investigated, the preliminary results reported here show that the solution can reproduce both the rise and decay phases of actual flare data quite well.

It has already been shown that a solution with $\kappa_{\parallel} \propto r$ describes the decay profile quite well, but predicts a rise which is longer than the 5 - 15 hours frequently observed (Forman, 1971). The rise profile is very sensitive to the value of $\kappa_{\parallel}$ near the sun, since diffusion is the principal mode of particle transport early in a flare event. Since the present solution describes both the rise and decay phase of a flare event, there is an indication that the actual behavior of $\kappa_{\parallel}$ inside 1 AU is better approximated by $\kappa_{\parallel}$ = constant than by $\kappa_{\parallel} \propto r$.

This solution also includes the effects of adiabatic deceleration, convection, and a non-adiabatic energy-change process, suggesting that further comparisons with data may provide a more thorough evaluation of interplanetary acceleration and deceleration processes.

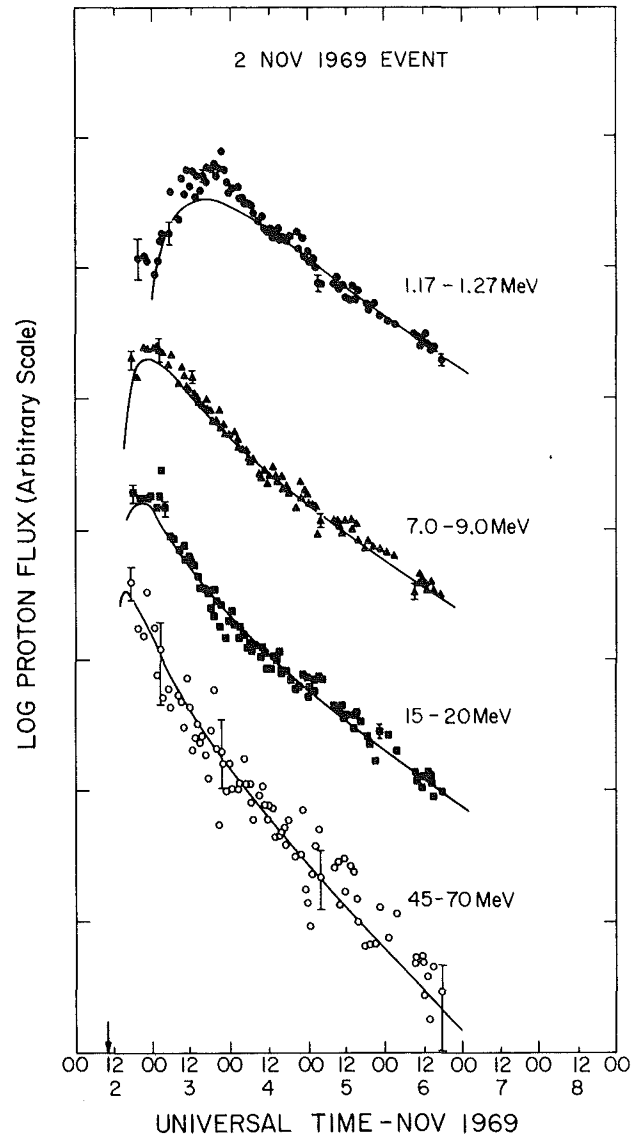

Fig. 3. Comparison of the calculated flux profile (solid lines) with solar flare event observations. The following values of the parameters were used:
$\kappa_{\parallel} \sim 2.5\times10^{20} - 1.6\times10^{21}$ cm^2/sec
$\kappa_{\perp} \sim 1.5\times10^{20} - 1.4\times10^{21}$ cm^2/sec,
$C \sim 1.1$, L = 2.3AU, $\theta_o = +35°$.

6. Acknowledgments. We are grateful to R. E. Vogt who has been intimately involved with the OGO-6 experiment. We also appreciate useful discussions with J. R. Jokipii and S. S. Murray. This work was supported in part by the National Aeronautics and Space Administration under Contract No. NAS5-9312 and Grant Nos. NGR-05-002-160 and NGL 05-002-007. One of us (J.E.L.) has received valuable support from an NDEA Fellowship and a NASA Traineeship.

7. References.

Abramowitz, M., and I. A. Stegun 1964. Handbook of Mathematical Functions, (Dover), 538.

Althouse, W. E., E. C. Stone, R. E. Vogt, and T. H. Harrington 1967. IEEE Trans.Nuc.Sci. 15, 229.

Burlaga, L. F. 1967. J.Geophys.Res. 72, 17, 4449.

Fisk, L.A., and W. I. Axford 1968. J.Geophys.Res. 73, 4396.

Forman, M. A. 1970. J.Geophys.Res. 75, 3147.

Forman, M. A. 1971. J. Geophys.Res. 76, 759.

Murray, S. S., E. C. Stone, and R. E. Vogt 1971. Phys.Rev.Letters 26, 663.

Parker, E. N. 1965. Planet. Space Sci. 13, 9.