RELATION OF THE RADIAL GRADIENT OF COSMIC-RAY PROTONS TO THE SIZE OF THE SOLAR-MODULATION REGION

T. L. Garrard, E. C. Stone, and R. E. Vogt

Division of Physics, Mathematics, and Astronomy

California Institute of Technology

Pasadena, California (USA) 91109

The radial intensity-gradient of cosmic-ray protons has been calculated for a range of values of the distance to the boundary of a spherically symmetric solar-modulation region. We find that the radial dependence of the gradients may be described in terms of two characteristic domains of the modulation region: (a) an "inner region" where the gradients are relatively small and constant, and (b), an "outer region" where the gradients are large and show a strong radial dependence. The magnitude of the gradient in the inner region is small for reasonable values for the physical parameters of the modulation mechanism.

1. Introduction. The propagation of galactic cosmic rays in the solar system may be described in terms of an equilibrium between (a) their outward convection by the solar wind, which reduces their intensity at all energies, (b) their diffusion into the solar system, and (c) their adiabatic deceleration in the expanding solar wind, which, depending on their spectral shape, may flatten or steepen the spectrum. In equilibrium, the galactic cosmic-ray intensity in the solar system is reduced, and shows a radially dependent gradient.

2. Solution of the Cosmic-ray Transport Equation. Under the assumption of spherical symmetry, the transport of galactic cosmic-ray particles in the interplanetary medium (see recent review, Jokipii, 1971) may be described in terms of the transport equation

$$-\frac{V}{r^2}\frac{\partial}{\partial r}(r^2U) + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\kappa\frac{\partial U}{\partial r}) + \frac{2V}{3r}\frac{\partial}{\partial T}(\alpha TU) = 0 \qquad (1)$$

where V is the solar wind velocity, assumed independent of heliocentric radius, r, $U = 4\pi j/\beta c$ is the cosmic-ray particle density/unit energy, j is the intensity, βc the particle velocity, T kinetic energy, κ the cosmic-ray diffusion coefficient, and $\alpha = (W + m)/W$, where W is total energy and m the rest energy of a particle. We have solved equation (1) with help of the numerical Crank-Nicholson technique, first used by Fisk (1971), for the following model of the interplanetary medium. We assume κ to be a separable function of radius and rigidity,

$$\kappa = \beta\, g(r)\, f(R) \qquad (2)$$

where $R = pc/e$, the proton magnetic rigidity, and p is proton momentum (Gleeson and Urch, 1972). Illustrative examples of solutions to the transport equation will be discussed for

$$g(r) = k = \text{const.} \qquad \text{for } r < D \qquad (3a)$$

where D is the distance to the modulation boundary, and

$$f(R) = \begin{cases} R^{1.5}\, R_0^{-0.5} & \text{for} \quad R > R_0 \\ R^{0.5}\, R_0^{0.5} & \phantom{\text{for}} \quad R < R_0 \end{cases} \tag{3b}$$

with

$$\begin{aligned} R_0 &= 1200 \text{ MV} \\ V &= 400 \text{ km/sec} \\ \frac{V(D-1)}{k} &= 3100 \text{ MV} \end{aligned} \tag{3c}$$

The parameters in equations (3a-c) are derived from studies of the solar modulation of electrons (Cummings, 1973) for the 1969 period of the solar cycle. The results of our calculations are relatively insensitive to this particular choice of parameters (see also Cummings et al., 1973, Garrard et al., 1973). The transport equation is solved for $r \leq D$, with the boundary condition $j(r{=}D,T) = j_D(T)$, the interstellar proton spectrum represented by

$$j_D = j_0 (W-m/4)^{-2.65} \tag{3d}$$

(Garrard et al., 1973). The assumed interstellar proton spectrum is experimentally verified at high energies (see review, Meyer, 1969), while the uncertainties at lower energies cannot affect the gradient except near the modulation boundary (see Garrard et al., 1973).

3. Results. Figure 1 shows the radial gradient of the differential proton intensity, $\frac{\partial \ell n\, j}{\partial r}(T)$ calculated for heliocentric radii of 1 AU and 18 AU for a boundary distance of D = 20 AU. Near 1 AU, the gradient is relatively small and relatively independent of energy, while near the boundary it is large with a strong dependence on energy.

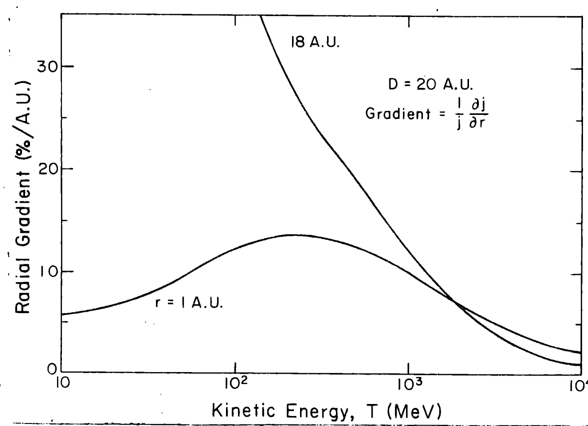

Fig. 1. Radial gradient of the differential proton intensity vs. energy for two selected heliocentric radii and a boundary distance D = 20 AU.

Figure 2 shows the effect of different boundary distances upon the heliocentric radial dependence of the proton differential-intensity gradient at T = 25 MeV. Similar to Figure 1, the modulation regions have two major domains, an inner region with relatively small gradients, and an outer region with large gradients near the boundary.

The radial gradient for the proton intensity at selected energies is shown at 1 AU as a function of boundary distance. It is evident that, except for relatively small boundary distances, e.g., $\lesssim$ 5 AU, the expected gradients near 1 AU are small. This result reflects the fact that a 1 AU location lies well within the inner modulation region for $D \gtrsim 5$ AU. Figure 3 shows that near 1 AU the largest gradients would occur near energies of ~ 250 MeV. (see also Fig. 1).

Figure 4 is analogous to Figure 3 for the gradient of the integral proton intensity, $\frac{\partial \ell n\ J}{\partial r}$ (> 25 MeV), with a similar qualitative behaviour. Since the integral intensity is dominated by the high-energy flux, its threshold dependence for $T \lesssim 250$ MeV is relatively insignificant.

4. Discussion. The results of the above calculations suggest that there are two rather different domains within the modulation region. In the outer domain, which extends several AU inside the boundary, diffusion and convection processes dominate. A large intensity gradient is required to establish equilibrium between these two processes at energies below a few hundred MeV. In the inner domain, convection and adiabatic deceleration processes dominate at low energies and large gradients are not required to maintain equilibrium. The low-energy particles, which essentially all derive from higher-energy particles by deceleration, in effect reflect the low intensity gradients characteristic of energies from several hundred to ~ 1000 MeV. However, the intensity gradient near 1 AU is sufficiently sensitive to the boundary-distance parameter so that its precise measurement can give useful information on the boundary distance. Although measurements of high statistical accuracy would be easiest to achieve for the integral intensity gradient, the integral gradient is poorly suited, since its calculation depends on knowledge of the diffusion coefficient over a very large energy range. For this reason, the low-energy differential gradient may be more useful in deducing boundary distances from measurement within the inner solar-modulation region.

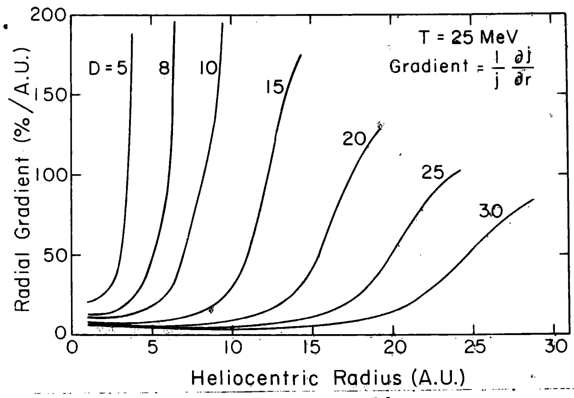

Fig. 2. Radial gradient vs. heliocentric radius, at 25 MeV, for selected boundary distances.

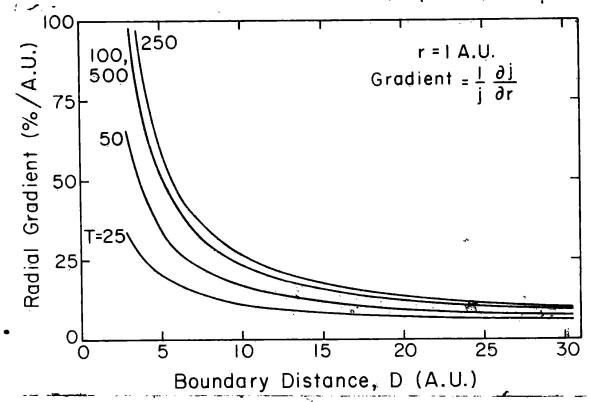

Fig. 3. Radial gradients vs. boundary distance, for selected energies, at 1 AU.

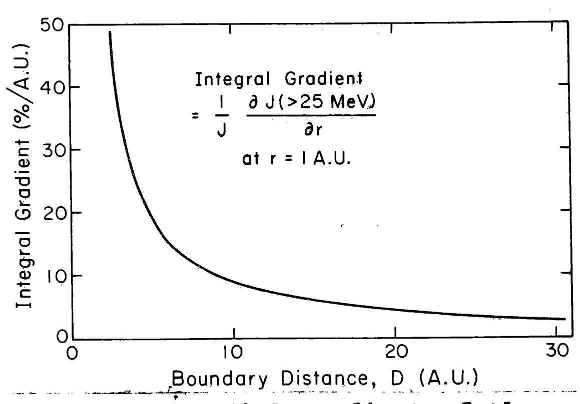

Fig. 4. Radial gradient of the integral ($\gtrsim$ 25 MeV) proton intensity near 1 AU as function of boundary distance.

5. References.

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