nitrogen of the pyridine and the hydrogen of water. The band for water in pyridine lies at 2.719μ ($3678\ cm^{-1}$). For water in carbon tetrachloride (an inactive solvent) a double band, (ν_σ, $3705\ cm^{-1}$ and ν_π, $3614\ cm^{-1}$) is found, but for water in pyridine ν_π is not found. The band in fused quartz is at a somewhat higher frequency than is found for hydroxyl groups bound to an atomic structure, namely, 3600 to 3400 cm^{-1} (cf. Pauling[11] or, for example, the 3643 cm^{-1} O–H band in mica). It is suggested that the 2.72μ band in fused quartz is due to water in a solid, closely-packed solution perturbed by the oxygens of the $Si\,O_2$ group. Water appears to be more probable than carbon dioxide as an impurity in view of the studies on silicates, and since the 2.72μ band does not show the double structure characteristic of CO_2 in this region.

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On the Large Scale Distribution of Matter in the Universe

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The following considerations represent an attempt to analyze those effects which govern the large scale distribution of matter in the universe. It is proposed to discuss the problems of: (a) the frequency of occurrence of *clusters of nebulae* which contain different numbers of nebulae, (b) the relative numbers of *nebulae* in dependence of their intrinsic luminosity and mass, and (c) the frequency distribution of various types of *stars* in dependence of their mass and intrinsic luminosity. It is found observationally that clusters of nebulae become the more frequent the fewer nebulae they contain. On the other hand the data so far available indicate that the luminosity function for nebulae exhibits a frequency maximum at the absolute magnitude $M_0 = -14.2$. Reasons are advanced which suggest that this result which was obtained from the purely observational approach to the problem of the general distribution of matter cannot be final. The theoretical analysis of the frequency distribution of various types of stars is largely complicated because of the interference of effects due to light pressure and the generation of energy through subatomic processes and cannot at the present be carried through. The analogous problems for the nebulae and the clusters of nebulae, however, lend themselves to a simple qualitative analysis which, starting from the assumption of a stationary universe, makes use of the principle of the conservation of energy, the virial theorem and the application of statistical considerations to distributions of the Boltzmann and the Smoluchowski types. The results obtained are in good agreement with the observed distribution of the clusters of nebulae. In addition an understanding is arrived at of the important fact that the velocities of field nebulae on the average are only about half as large as those of cluster nebulae. The theoretical considerations given, however, are in *complete contradiction with the luminosity function now generally adopted for the nebulae inasmuch as the existence of a large number of intrinsically faint nebulae* representing more than half of all of the matter in the visible universe *is predicted* which to date must have been overlooked. On the basis of this prediction various criteria are developed to facilitate the discovery of intrinsically faint stellar systems and a systematic search for such systems utilizing the 18-inch Schmidt telescope on Palomar Mountain is described. On the basis of the results achieved so far it is shown that the new distribution function of nebulae in the so-called local group of nebulae deviates markedly in the direction of the theoretical expectations from the luminosity function derived by previous observers. Practical methods are discussed, the application of which should make possible the construction of a more representative distribution function for nebulae over a large range of luminosities. In the final section some of the known observational facts are briefly reviewed which favor the assumption of a stationary rather than an expanding universe.

A. STATEMENT OF THE PROBLEM

One of the most conspicuous aspects of the distribution of matter in space lies in the existence of a great variety of types of condensations. Starting from the elementary particles, conglomerations of ever increasing dimensions and material content may be found until we arrive by the way of the stars and the stellar systems (or the nebulae) at the clusters and clouds of nebulae which at the present time are the largest known aggregations of matter which possess individual characteristic structures.

We shall here be concerned with the analysis of the *frequency distribution of various types of large scale condensations of matter*. So far the so-called purely *observational approach* has been made use of exclusively to determine the relative numbers of various types of stars, of nebulae and of clusters of nebulae. Through this approach the accumulation of important data during the recent decades has been rapid and frequency distributions for stars and for nebulae were arrived at which at first sight seem trustworthy. A closer scrutiny of the problem however reveals that the purely observational approach has, of necessity, become the victim of some rather serious oversights. This is nothing new or surprising since the purely observational approach, because of the interference of the ever present demon of selectivity, has often been restricted to the discovery of partial truths only, just as the pure theory has often become sterile because of lack of recognition of the simple and perhaps best established fact that the totality of all natural phenomena is beyond the imagination of any theory starting from *a priori* premises. It is the purpose of this investigation to show through the analysis of concrete examples that even the "geographical" aspects in astronomy are as yet by no means satisfactorily established. New discoveries promise to be abundant if the purely observational approach is supplemented by constructive theory which does not stop with the analysis of already known phenomena but which insists upon checking its conclusions through new types of observations.

The particular problems with which we shall deal here are:

(a) The frequency distribution of *clusters of nebulae* of varying mass and luminosity.

(b) The frequency distribution of *nebulae* or of stellar systems of varying mass and luminosity.

(c) The frequency distribution of *stars* of varying mass and luminosity.

Although the problems of the frequency distribution of nebulae and of clusters of nebulae will be particularly favored, problem (c) is at least of equal importance, but its solution has not yet been pushed sufficiently far to be discussed in greater detail at the present time.

B. SOME OF THE RESULTS OF THE PURELY OBSERVATIONAL APPROACH

(α) The Clusters of Nebulae

Nebulae appear either as isolated objects (field nebulae) or in groups whose membership runs from as few as two to as many as several thousand presumably physically related nebulae. The largest of these groups are called *clusters* if they show relatively high central condensations and *clouds* of nebulae if the accumulation resembles a swarm of objects without any particularly marked central condensation. The formation of groups of nebulae may be partly due to chance but in most cases it will be caused by the mutual attraction of the nebulae. The radial distribution of nebulae in large spherically symmetrical clusters is such that it is in agreement with conclusions drawn from statistical mechanics based on the assumption that between the nebulae the ordinary Newton inverse square law of attraction is operative.[1]

Few systematic investigations are as yet available about the frequency of groups containing different numbers of nebulae.[2] From the material on hand, however, the important fact is fairly apparent that isolated nebulae are the most numerous and that in a given large volume of space the frequency ν of the groups steadily decreases as the number N of nebulae in the groups increases. The frequency function $\nu(N)$ may therefore be assumed to be monotonously decreasing with increasing N. The morphology of the clusters of nebulae and the frequency function $\nu(N)$ present problems which can adequately be solved with large Schmidt telescopes but the quantitative discussion of these problems must be postponed until more data are available.

(β) The Nebulae

The investigations on the frequency distribution of nebulae of various types are first of all concerned with the number $n(M)$ of nebulae in a given volume of a given absolute magnitude M expressing the absolute luminosity L. From the standpoint of statistical mechanics the frequency function $n'(\mu)$ would be more interesting where μ is the mass of a nebula. Although these masses are not yet adequately known we may assume that the functions $n'(\mu)$ and $n(M)$ or rather $n(L)$ are similar in character.

Through extensive investigations by Hubble[3]

[1] F. Zwicky, Astrophys. J. **86**, 217 (1937).
[2] E. Holmberg, Astrophys. J. **92**, 200 (1940).
[3] E. Hubble, Astrophys. J. **84**, 158 (1936).

and others a frequency distribution $n(M)$ for extragalactic nebulae was established which is of the type of a Gaussian error curve with the frequency maximum located at the absolute photographic magnitude* $M_o = -14.2$. The dispersion in magnitude was found to be about $\sigma = 0.85$. With the customary normalization

$$\int_{-\infty}^{+\infty} n(M)dM = 1, \tag{1}$$

the distribution function now generally adopted is

$$n(M) = \exp\left[-(M-M)^2/2\sigma^2\right]/(2\pi)^{\frac{1}{2}}\sigma$$
$$= 0.469 \exp\left[-(M-M_o)^2/1.445\right]. \tag{2}$$

The continuous curve in Fig. 3 represents this function which is based on the statistical evaluation of a large number of nebulae both in the general field and in clusters of nebulae. The distribution in the local group of nebulae according to Hubble[4] slightly favors the fainter nebulae. It should here be mentioned that until recently the local group was known to include the large and small Magellanic clouds, the Andromeda nebula (Messier 31) and its two companions (M_{32}, NGC 205), the spiral nebula M_{33} and the two irregular systems NGC 6822 and IC 1613. If we properly include our own galactic system the mean luminosity for the local group is about $M_o = -14.0$, a value which agrees very closely with the value $M_o = -14.2$ furnished by field nebulae and clusters. How secure the observational basis for the distribution function (2) seemed until recently is well illustrated by Hubble's discussion of the slight discrepancy between the luminosity function of the local group and that of the larger sample collection of nebulae respectively, when he states that "the slight discrepancies are largely accounted for by the presence of three very faint dwarfs in the local group—IC 1613, and NGC 6822 and 205. The results suggest that there might be, in the general field, many similar dwarfs so faint that they would be overlooked in general surveys. A careful reexamination of the surveys demonstrates that such nebulae would be detected if they existed in considerable numbers, and, therefore they must be regarded as relatively rare objects.* Their presence in the local group appears to be a unique feature of the group, and they detract from its significance as a fair sample of nebulae in general."

In spite of the strong observational support of the distribution function (2) certain facts, *both observational and theoretical* have recently come to light which indicate that the *distribution function (2) is really quite incorrect* inasmuch as it is based on a particular *observational selection* and does not embrace the totality of all nebulae. The principal reasons which suggest that the true function $n(M)$ must be very different from the function (2) are as follows.

One objection against the luminosity function of the type (2) for nebulae derives from the fact that this function possesses a maximum for a luminosity roughly equivalent to the absolute magnitude $M_o = -14.2$ while for the frequency distribution of clusters of nebulae no such maximum exists. It is very difficult to see why the results should be so radically different in the two cases of the grouping of nebulae into clusters and the grouping of stars into nebulae. If both of these groupings are due either to random accumulation or to Newton's law of attraction and if the dissolution of these groups is the result of close encounters with other groups, the frequency distributions $\nu(N)$ and $n(M)$ or rather $\bar{n}(L)$ should be similar in character unless some additional effect is operative which suppresses the existence of small nebulae while no such effect hinders the formation of small clusters of nebulae. It will be shown in the following that probably no such effect exists and that therefore the luminosity function $n(M)$ for nebulae derived at the present from the purely observational approach is incorrect. In support of this contention the following qualitative considerations may be advanced.

(1) During the past few years further *faint*

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* If L_1 and L_2 are the energies radiated per second by two bodies 1 and 2, the difference in their absolute magnitudes is given by $M_1 - M_2 = 2.5 \log_{10} L_2/L_1$. The absolute photographic magnitude of the sun is $M(\odot p) = +5.3$. Therefore the luminosity L_{0p} (in the photographic range) of an average nebula of the photographic absolute magnitude $M_{0p} = -14.2$ is $L_{0p} = 65 \times 10^6 L(\odot p)$.

[4] E. Hubble, *The Realm of the Nebulae* (Yale University Press, 1936), p. 149.

* Baade in Astrophys. J. **88**, 112 (1938) also states that the luminosity function (2) may be considered as "well established."

members were added to the local group. These are the *Sculptor* and *Fornax* systems[5] discovered by the staff of the Harvard College Observatory. Also, according to Baade,[6] the *Wolf-Lundmark nebula* which is not contained in Hubble's original list must be added to the local group.*

(2) Nebulae can reach statistically stationary states characterized by a Boltzmann distribution of coordinates and of velocities only so long as the mean free paths Λ of the constituent particles (stars) are smaller than the linear dimensions D of the system. With decreasing mass the ratio Λ/D for some nebulae may become so great that these nebulae never can reach a Boltzmann steady state but remain indefinitely of an irregular extended (Smoluchowski) type.[7] For low masses we therefore expect a great variety of types of nebulae ranging from the relatively extended Smoluchowski type (S) of low surface brightness to the compact type of high surface brightness represented by a globular star cluster (G). In nebulae of very great mass the characteristics of the types S and G co-exist in the same individual nebulae, applying to the center disk and to the peripheral parts, respectively. The identification and determination of the properties of individual nebulae of small mass, because of the small surface brightness of the type S and the compactness of type G will be more difficult than the analysis of the larger nebulae. There are therefore good reasons to believe that the frequency function (2) is based on the selection of material which favors the brighter nebulae of larger surface brightness.

(3) Investigations on the morphology of nebulae and of clusters of nebulae rather strongly support the contention that these objects form a statistically stationary ensemble.[8] In such an ensemble one must expect the heavier and presumably more luminous nebulae to show a higher tendency[1] toward clustering than the fainter nebulae. The faintest nebulae will be distributed most uniformly throughout space. If, as might be expected in a statistically stationary ensemble the less massive nebulae tend less toward clustering than the more massive nebulae the necessity of covering large fields in order to locate the least massive nebulae and to analyze their properties adds to the difficulty of establishing equally representative collections of nebulae of small and of large masses respectively.

From the preceding considerations the case against the luminosity function (2) for nebulae seems strong enough to justify (1) an attempt on the basis of a simple theory of the large scale distribution of matter to estimate the approximate number of faint nebulae actually existing but not represented by the function (2), and (2) a determined search for such faint nebulae. In Section C we shall therefore discuss some of the theoretical aspects of the luminosity function of nebulae while in Section D we present some preliminary results of the search for very faint nebulae obtained through the utilization of the 18-inch Schmidt telescope on Palomar Mountain and the large reflectors of the Mount Wilson Observatory.

(γ) The Stars

The frequency distribution of stars in dependence of their absolute brightness and their mass has long occupied the attention of astronomers. From the observational standpoint the problem is obviously one of great difficulty, because the fainter stars become, intrinsically, the more difficult it is to secure representative collections of such stars. It is interesting to note that about seventeen years ago a change of outlook took place regarding the luminosity function of stars which is very similar to the change of outlook which we may expect to take place in the near future regarding the luminosity func-

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[5] W. Baade and E. Hubble, Pub. Astr. Soc. Pac. **51**, 40 (1939).

[6] Private communication by Dr. Baade.

* After it was found that these three systems belong to the local group of nebulae and that they are intrinsically faint Hubble and Baade stated in the Pub. Astr. Soc. Pac. **51**, 44 (1939) that "Hitherto it has been assumed that the two branches (of the luminosity function) are symmetrical, although the brighter branch alone has been reliably determined. Although the information now available suggests that the local group is not a fair sample of nebulae in general the new data emphasize the importance of a thorough reexamination of the luminosity function." However according to the considerations given in the following it cannot be expected that the local group contains a larger percentage of faint nebulae than the general field and neither is it likely that the correctly formulated luminosity function exhibit any maximum at all.

[7] F. Zwicky, Phys. Rev. **58**, 478 (1940); Astrophys. J. **93**, 411 (1941).

[8] F. Zwicky, *Th. von Karman Anniversary Volume* (1941), pp. 137–153.

tion of nebulae. Some twenty years ago, from the data then available astronomers concluded that in a large region of our galaxy centered around our sun the absolute number of stars rapidly increases as the brightness declines from stars of the absolute magnitude $M=-5$ (stars which are about 10,000 as bright as the sun whose absolute magnitude is approximately $M(\odot)\cong+5$) to stars of $M=+7$. For such stars which are about ten times fainter than the sun the luminosity function seemed to exhibit a maximum. In 1924 however Seares[9] showed that intrinsically faint stars are far more numerous than had been previously supposed and that if the luminosity function has any maximum at all it must lie at stars $M>+13$ which are at least one thousand times fainter than the sun.

Unfortunately no satisfactory theory of the frequency distribution of stars of different luminosity and mass is available. While the theory of the frequency distribution of clusters of nebulae and of nebulae is simplified by the fact that no good reason is known at the present why the clusters of nebulae and the nebulae should not exist as aggregations of matter of continuously increasing mass up to a certain point, the problem presented by the stars is entirely different, since because of the interference of the energy generation through subatomic processes and the action of the light pressure certain types of stars whose existence in any statistically stationary ensemble might otherwise be expected do not occur because they do not satisfy the laws of dynamic stability.

At the present the frequency distribution of various types of stars can only be approached through a more systematic search. This search is essentially confined to the neighborhood of the sun but has at its disposal some powerful means such as the search for large proper motion stars and the spectroscopic method of determining parallaxes as well as a number of additional criteria which it is not necessary to discuss here in detail. For the nebulae all of these criteria are not available and new criteria therefore had to be found, some of which will be discussed in Section D.

[9] F. H. Seares, Astrophys. J. **59**, 310 (1924).

C. SOME STATISTICAL MECHANICAL CONSIDERATIONS ON THE LARGE SCALE DISTRIBUTION OF MATTER

(α) On the Clustering of Nebulae

We shall first apply a few simple theorems to the problem of the clustering of nebulae. As a first approximation we assume that the universe is in a stationary state of statistical equilibrium* and that the nebulae are more or less permanent objects whose identity or whose number can be materially changed only through rare and most violent processes such as direct collisions, the effects of which we shall touch upon later.

The Principle of Conservation of Energy

Suppose we start from a uniform distribution of nebulae for which the average potential energy $\bar{E}_{po}$ may arbitrarily be set equal to zero. The average kinetic energy per unit mass in this distribution we denote with $\bar{E}_{ko}$. For $\bar{E}_{ko}=0$ any static distribution of nebulae is unstable according to Earnshaw's theorem. Clustering with subsequent aquisition of kinetic energy by the nebulae will therefore result from this instability. For $\bar{E}_{ko}\neq0$ a uniform distribution of nebulae results if

$$\bar{E}_{ko}\gg-\bar{E}_p, \tag{3}$$

where $\bar{E}_p$ is the average potential energy per unit mass in any possible state of clustering. In actuality condition (3) is not fulfilled since clustering obviously exists and we therefore shall have to replace the inequality (3) by a more accurate relation which we may obtain from the energy principle. For simplicity we shall bunch together on the one hand all nebulae and groups of nebulae for which $\bar{E}_p$ is negligible and on the other hand the larger clusters of nebulae for which $\bar{E}_p$ is appreciable. The nebulae in the first group we shall denote with the index e (external with respect to the clusters) and the nebulae in the second group with the index i (internal). The total mass of the external field nebulae in a given large volume is M_e while the total mass of the nebulae in larger clusters is M_i. If the average kinetic and potential energies per unit mass of

* In the final section E, at the suggestion of the editors some of the evidence now available is briefly reviewed, which favors the assumption of a stationary rather than an expanding universe.

the cluster nebulae are $\bar{E}_{ki}$ and $\bar{E}_{pi}$ and the average kinetic energy per unit mass of the field nebulae is $\bar{E}_{ke}$ we have under the assumptions stated*

$$M_e\bar{E}_{ke}+M_i(\bar{E}_{ki}+\bar{E}_{pi})=(M_i+M_e)\bar{E}_{ko}\geqslant 0. \quad (4)$$

Co-existence of Smoluchowski Distributions and Boltzmann Distributions in the Realm of Nebulae

When in a physical system, such as the coexistence of a liquid (i) and its vapor (e) in a closed vessel the number of impacts between the elementary particles is very great, equipartition of energy (constancy of temperature) is ultimately established and we have $\bar{E}_{ke}=\bar{E}_{ki}$. However the manifold of the field nebulae and the cluster nebulae in some fundamental aspects differs from the physical system just described because of the length of some of the mean free paths involved.

Terrestrial systems in which some of the mean free paths are so long that deviations from the Boltzmann Gibbs distribution law result were considered by Maxwell[10] and later on particularly by Smoluchowski[11] and Knudsen.[12] In the theory of the large scale distribution of matter considerations of the types described by these authors play a decisive role. They explain, as we shall see, some of the important characteristics of nebulae and of clusters of nebulae. They also make it clear that the lack of equipartition of energy (constancy of temperature) among the large scale condensations of matter does not necessarily indicate that the universe is not in a thermodynamically stable state.

In the realm of the nebulae we deal with mean free paths in the large clusters which are comparable with the diameters of the clusters while for the field nebulae the mean free paths are much larger. This means that in the central parts of stationary large clusters we may expect a Boltzmann distribution of matter and of its kinetic energy, while in "extra cluster" space Smoluchowski distributions will have to be introduced. This suggests that a "cosmic temperature jump" exists between the inside and the outside of clusters. Strictly speaking all nebulae, the field nebulae included, are more or less closely associated with one cluster of nebulae or another.[13] The existence of a region around a cluster in which a relatively sharp transition from a higher average kinetic energy to a lower average kinetic energy takes place justifies the introduction of intracluster nebulae and extra-cluster nebulae and lends a precise meaning to the long used distinction between field nebulae and cluster nebulae.

In Fig. 1 the distribution[3] of the average kinetic energy per unit mass is shown as a function of the distance r from the center of a large cluster.

The distance $r=r_c$ from the center at which the "cosmic temperature jump" occurs is given roughly by the relation

$$\bar{E}_{ki}-\bar{E}_{ke}\cong\Phi(r_c+\lambda(r_c))-\Phi(r_c), \quad (5)$$

which expresses the fact that for $r=r_c$ the mean free path of the nebulae becomes so large that running from r_c to $r_c+\lambda$ the nebula may transform a considerable fraction of its excess kinetic energy into potential energy.

For $r_c>0$ the cluster is of the Boltzmann compact type while for $r_c=0$ the cluster is of the Smoluchowski open type. In passing we mention that the Coma cluster of nebulae is of the first type showing a distribution of mass and of kinetic energies which would be expected from Boltzmann's principle,[1] while the system of the field nebulae including many open groups represent a distribution of the Smoluchowski type. The two masses M_i and M_e previously introduced correspond roughly to the total masses of the two types of groupings in a given very large volume.

An estimate of these total masses M_i and M_e as well as of the kinetic energies $\bar{E}_{ki}$ and $\bar{E}_{ke}$ may

* The possibility that the expression (4) is negative cannot be dismissed *a priori*. If negative, it would mean that the total kinetic energy of all the nebulae is not sufficient to disrupt all of the clusters. This, however, would make necessary values for the masses of nebulae much larger than we can tolerate at the present time without running into contradiction with a number of other phenomena previously discussed (see reference 1), such as the characteristics of the velocity dispersion in clusters and the fact that no gravitational lens effects among the nebulae have as yet been found.

[10] C. Maxwell, *Scientific Papers* (Cambridge Univeristy Press, 1890), Vol. II, p. 703.

[11] M. v. Smoluchowski, Wied. Ann. **64**, 101 (1898).

[12] M. Knudsen, Ann. d. Physik **34**, 593 (1911).

[13] F. Zwicky, Pub. Astr. Soc. Pac. **50**, 218 (1938).

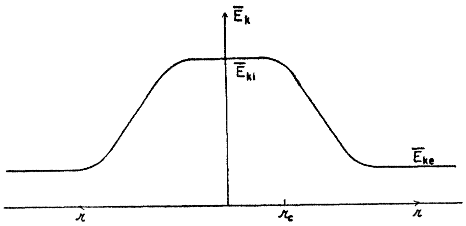

FIG. 1. Average kinetic energy of nebulae per unit mass as a function of the distance from the center of a large cluster of nebulae.

be obtained from an application of the virial theorem and the Boltzmann principle.

The Virial Theorem

Large clusters of nebulae represent stationary assemblies in the sense that the number of nebulae which in a given time escape from a cluster through its boundary ($r=r_c$) on the average is equal to the number of nebulae captured by the cluster. The virial theorem states[1] that for any mechanical system whose particles interact according to the inverse square law of attraction

$$\tfrac{1}{2}\langle d^2\theta/dt^2\rangle_{\text{Av}}=(2\bar{E}_{ki}+\bar{E}_{pi})M_i, \tag{6}$$

where M_i and θ are the total mass and the polar moment of inertia of the system. For a stationary cluster the left side of (6) is zero and therefore

$$\bar{E}_{pi}=-2\bar{E}_{ki}. \tag{7}$$

Substituting (7) in (4) we obtain

$$M_e\bar{E}_{ke}-M_i\bar{E}_{ki}=(M_i+M_e)\bar{E}_{ko}\geqslant 0. \tag{8}$$

From this and the fact that certainly $\bar{E}_{ki}$ is greater than $\bar{E}_{ke}$ we conclude that

$$M_e/M_i\geqslant \bar{E}_{ki}/\bar{E}_{ke}\geqslant 1. \tag{9}$$

This relation expresses the important fact that clustering among nebulae as such has its natural limit. At the same time as clusters grow more numerous and become more compact $\bar{E}_{ki}$, which is equal to $-\bar{E}_{pi}/2$, increases correspondingly. The resulting increase of M_e therefore automatically depletes the population of the clusters in favor of the field nebulae. We have here an excellent illustration of the universal principle that *every process which, generally speaking, involves a contraction automatically is associated with another process which involves expansion,* and vice versa. The contraction of the stellar cores in novae and in supernovae with the subsequent expulsion of the outer layers of the stars involved provides another illustration while the examples of *explosions* caused by molecular and nuclear *contractions* are too numerous to be mentioned specifically.

Although more observations are badly needed, relation (9) can be subjected to a qualitative test with the data on hand. While the average velocity for cluster nebulae[3] is of the order of $v_i=500$ km/sec. the average velocity of the field nebulae is of the order $v_e=250$ km/sec.[1,3] and consequently

$$\bar{E}_{ki}\cong 4\bar{E}_{ke}, \tag{10}$$

from which it follows that

$$M_e\geqslant 4M_i, \tag{11}$$

a relation which is in accordance with the approximately known relative abundance of field nebulae and of cluster nebulae.

While from the considerations given so far the relation (11) can be only derived if (10) is taken as an observational fact we may drive the theory one step further which leads to another justification of (11). This additional step is furnished through an application of Boltzmann's principle and in turn justifies our initial assumption of the stationary character of the distribution of the nebulae and of their kinetic energies.

The Boltzmann Principle

For a gravitational isothermal gas sphere which has reached a statistically stable state and in which the mean free paths are small compared with the total dimensions of the sphere the radial distribution of matter was determined by Emden.[14] We shall call this the Emden distribution. Vice versa if we know from observation that a spherically symmetrical cluster of objects satisfies the Emden distribution we may conclude that this distribution is in accordance with Boltzmann's principle and that between the objects the ordinary Newton inverse square law of attraction is operative. From an analysis of the distribution of nebulae in the Coma cluster it was shown previously[1] that the conditions just mentioned are satisfied and that therefore

[14] R. Emden, *Gaskugelm* (Teubner, Leipzig, 1907).

the interaction between the nebulae is governed by Newton's law. The application of the Boltzmann principle to the whole realm of nebulae on the other hand is not quite correct since in a great part of this realm the mean free paths are too long and the number of encounters too small, a fact which leads us to the recognition of a cosmic "temperature jump" between the cluster nebulae and the field nebulae. We may, however, still assume that the Boltzmann principle leads to qualitatively correct results for the relative population of the clusters and of the general field of nebulae because of the fact that the relative abundance of the field nebulae and of the cluster nebulae is determined by the relative volumes V_e and V_i occupied by the two types of nebulae and by their average kinetic energies. Qualitatively we may therefore write*

$$M_i/M_e = \exp\left[-\bar{E}_{pi}/kT\right] V_i/V_e, \quad (12)$$

where

$$kT \cong 2\bar{E}_{ke}/3, \quad (13)$$

since for $\bar{E}_{ke}=0$ the clusters cannot be disrupted by any encounters with field nebulae, which in this case would disappear entirely. Furthermore from (7) and (9)

$$\bar{E}_{pi} = -2\bar{E}_{ki} = -2\gamma \bar{E}_{ke} M_e/M_i, \quad (14)$$

where $0 \leq \gamma \leq 1$ and therefore

$$M_i/M_e = \left[\exp\left(3\gamma M_e/M_i\right)\right] V_i/V_e \quad (15)$$

or

$$\left[\exp\left(-3M_e/M_i\right)\right] V_e/V_i \leq M_e/M_i \leq V_e/V_i. \quad (16)$$

While the upper limit for M_e/M_i corresponds to the case that $\bar{E}_{ko}$ is very large and practically no clustering takes place, the lower limit represents the maximum possible clustering ($\bar{E}_{ko}=0$) and the smallest ratio M_e/M_i.

From the fact that matter is not uniformly smeared out over space but exhibits a great tendency to agglomeration we conclude that $\bar{E}_{ko}$ is small compared with $\bar{E}_{ki}$ or even $\bar{E}_{ke}$. For purposes of discussion we shall set $\bar{E}_{ko}=0$ although the exact value of $\bar{E}_{ko}$ can probably be deduced only when more is known about the nature of the redshift on the one hand and perhaps the problem of relativistic cosmology on the other. With $\bar{E}_{ko}=0$ we have

$$V_e/V_i = xe^{3x},$$

where

$$x = M_e/M_i. \quad (17)$$

Strictly speaking this is an equation for x since $\bar{E}_{pi}$ can be expressed in terms of M_i and V_i which in turn leads to a relation between V_e/V_i and M_e/M_i. This relation, however, is so complicated that for purposes of a clear illustration of the physical principles discussed it will be more advantageous to compare (17) directly with the observational data. In this comparison we are also interested in the values of $\bar{\rho}_i/\bar{\rho}_e$ where $\bar{\rho}_i$ and $\bar{\rho}_e$ are the average density of matter inside and outside of the large clusters. It is

$$\bar{\rho}_i/\bar{\rho}_e = M_i V_e/M_e V_i = e^{3x}. \quad (18)$$

Table I gives some of the values of x, V_e/V_i and $\bar{\rho}_i/\bar{\rho}_e$ in the range in which we shall be interested.

Observationally the field nebulae are several times as numerous as the cluster nebulae, so that, in order of magnitude $x=3$ and consequently $V_e/V_i \sim 20000$ which also is in approximate agreement with the observations. It will be of great interest to determine accurate values of V_e/V_i through a more general survey of clusters of nebulae and to compare the values of M_e/M_i and $\bar{\rho}_i/\bar{\rho}_e$ resulting from Table I with other independent determinations of these quantities. At the present not enough observations are available to make an accurate comparison although we can state that qualitatively the data on hand check the validity of our considerations.

It may be asked why, instead of analyzing step by step the distribution in space of nebulae and of their velocities through the application of various fundamental theorems we have not approached the problem in the stereotype manner of statistical mechanics, which is to determine directly the most probable distribution in phase space of our assembly of nebulae. Such an approach which would be the only rigorous one, however, meets with two difficulties which have not yet been overcome. These difficulties are

(a) The number of encounters among nebulae in a great part of the universe known to us is not

* A strict application of Boltzmann's principle involves the introduction of partition sums with the potential energies of the individual nebulae entering the exponentials so that a segregation of nebulae of different mass will result.

large enough to make the distribution of nebulae random in a statistical sense. The problem of the coexistence of Boltzmann and *Smoluchkowski distributions* therefore remains to be investigated in more detail.

(b) The assembly of nebulae is a *cooperative assembly* in the sense that the potential energy is represented through *conditionally convergent sums*, the values of which depend on the shapes of the configurations (clusters and clouds) involved. This fact leads to peculiar deviations from the laws of thermodynamics as formulated for non-cooperative assemblies for which ordinary equations of state exist. For cooperative assemblies the laws of thermodynamics need reformulation.

(β) On the Theory of the Frequency Distribution of Nebulae

The distribution function $n(M)$ for nebulae of different absolute magnitudes M now generally adopted is explicitly given by (2). In Section B_β we advanced some reasons which suggest that this distribution function cannot be correct. With the help of the physical principles which we discussed in the previous section C_α and which enabled us to derive some of the major characteristics of the clustering of nebulae we now can proceed to derive a more quantitative estimate on just how far the function (2) is in error.

We first list some of the reasons why the intergalactic space between the nebulae incorporated in the distribution function (2) must be populated by very considerable amounts of matter in the form of intergalactic gases and dust, individual stars and groups of stars or nebulae which are fainter and smaller than those which obey the distribution function (2).

TABLE I.

$x=M_e/M_i$	$V_e/V_i=xe^{3x}$	$\bar{\rho}_i/\rho_e=e^{3x}$
1	20	20
1.5	135	90
2	807	403
2.5	4520	1808
3	2.43×10^4	8103
3.5	1.27×10^5	3.63×10^4
4	6.52×10^5	1.63×10^5
4.5	3.28×10^6	7.29×10^5
5	1.64×10^7	3.27×10^6
5.5	8.03×10^7	1.46×10^7
6	3.94×10^8	6.56×10^7
6.5	1.91×10^9	2.94×10^8

(1) The kinetic energy of translation of the field nebulae per unit mass is

$$\bar{E}_{ke}=\langle v_e^2\rangle_{\text{Av}}/2. \tag{19}$$

The available observational data indicate that[3]

$$(\langle v_e^2\rangle_{\text{Av}})^{\frac{1}{2}}\cong 250 \text{ km/sec.}, \tag{20}$$

while for the nebulae in large clusters

$$\bar{E}_{ki}=\langle v_i^2\rangle_{\text{Av}}/2, \tag{21}$$

where

$$(\langle v_i^2\rangle_{\text{Av}})^{\frac{1}{2}}\cong 500 \text{ km/sec.} \tag{22}$$

The average potential energy $\bar{\epsilon}_{pi}$ per unit mass of the stars in the nebulae themselves, by the virial theorem is

$$\bar{\epsilon}_{pi}=-2\bar{\epsilon}_{ki}, \tag{23}$$

where

$$\bar{\epsilon}_{ki}=\langle w_i^2\rangle_{\text{Av}}/2 \tag{24}$$

and w_i rarely exceed 100 km/sec. Therefore

$$|\bar{\epsilon}_{pi}|<\bar{E}_{ke}. \tag{25}$$

Encounters between nebulae consequently are quite capable of *disrupting* nebulae to a large degree and producing fragments or stellar systems which are smaller than the original objects involved in the encounters. Because of the occurrence of encounters the frequency distribution of small and of large nebulae must be similar to the frequency distribution of atoms and smaller and larger molecules in a gas for which the average translational energy $3kT/2$ is larger than the dissociation energy of the molecules. Because of collisions which frequently result in dissociation the smaller units must exist in preponderant numbers. This effect is enhanced because of the much greater *a priori* probability (larger volume available) of the smaller units.

(2) In the nebulae themselves, as contraction proceeds, $\bar{\epsilon}_{ki}$ increases at the same time with $|\bar{\epsilon}_{pi}|$. During this process a certain amount of kinetic energy is liberated from the system in order to maintain internal equilibrium in accordance with the virial theorem (24). This kinetic energy will be liberated through the expulsion of stars or groups of stars into intergalactic space and we again have a phenomenon which illustrates the principle that contraction on the one hand and expansion on the other are necessarily related effects. The net result is that intergalactic

space will be populated by stars and groups of stars.

(3) In passing we mention further phenomena which involve contractions such as the collapse of stars in novae and in supernovae during which processes gas clouds are ejected into interstellar and intergalactic space.[7] Light pressure on the interstellar gas and dust clouds acts to populate intergalactic space with the same objects.

In order to obtain a *minimum estimate* of the stellar population of intergalactic space we disregard the effects (1) and (3). To the effect (2) a similar analysis then applies which we have given for the relation between cluster nebulae and field nebulae. We can thus again use a relation analogous to (15) to estimate the ratio between the total mass m_i of the large nebulae incorporated in the distribution function (2) and the mass m_e of the intergalactic objects not represented by this function. We have

$$m_e/m_i \cong [\exp(-3m_e/m_i)]\Omega_e/\Omega_i, \qquad (26)$$

where Ω_e and Ω_i are the partial volumes occupied, respectively, by the nebulae incorporated in the distribution function (2) and by the "intergalactic" space between these nebulae. Since approximately

$$\Omega_e/\Omega_i = 10^6, \qquad (27)$$

we read from Table I that

$$m_e/m_i \cong 4. \qquad (28)$$

Because of the neglect of the effects (1) and (3) the ratio m_e/m_i will be greater than estimated by (28). The fact, for instance, that the nebulae possess already very considerable kinetic energies which are a result of the clustering of nebulae rather than due to the release of potential energy in the formation of the nebulae themselves will make the effective kT in the Boltzmann factor of relation (12) greater than indicated by the formulae of the type (13) and (14), with the result that m_e/m_i becomes greater than given by these formulae.

Since no good reason seems to exist why any particular agglomeration of stars in interstellar space should be excluded, we conclude that *individual stars, multiple stars, open and compact star clusters and stellar systems of increasing population will be found in numbers presumably decreasing in frequency as the stellar content of the systems in question increases.* This important conclusion from our theoretical considerations is so radically at variance with the now generally adopted luminosity function (2) that a serious effort seems justified to decide between the two alternatives through further systematic observations. Some suggestions regarding such observations will be discussed in Section *D.*

D. THE SEARCH FOR INTRINSICALLY FAINT NEBULAE

(α) Discovery of New Faint Stellar Systems in the Local Group

The reasons given in the preceding for the probability of the existence of a far greater number of faint nebulae than was hitherto suspected seemed powerful enough to warrant a new search for such nebulae. Because of its great speed and large field the 18-inch Schmidt telescope on Palomar Mountain is well suited for this task. During the extensive search for supernovae in the period from September, 1936, until date quite a number of faint and relatively extended objects was noted. Without the possession of some distance criterion however, it would have been necessary to photograph these nebulae indiscriminately with one of the large reflectors under good seeing conditions in order to ascertain their distance and intrinsic nature. Such a procedure obviously would have been tedious and inefficient. The problem therefore was to develop, if possible, some criteria which would enable us to search for nearby nebulae with some greater chance of success. Although we cannot hope with the Schmidt telescope to resolve in faint nebulae, even if in the local group, a sufficient number of stars to determine their character, the three following qualitative criteria may sometime be of considerable help.

(1) Nebulae of small stellar content N which are of the extended "Smoluchowski" type may often be expected not to possess any kind of central condensation and to show relatively great *irregularities in surface brightness* ($\sim N^{\frac{1}{2}}$) giving them a patchy granulated appearance. Repeated long exposures of the same fields were therefore taken in order to see whether in some of the faint nebulae any such intrinsic granulation could be

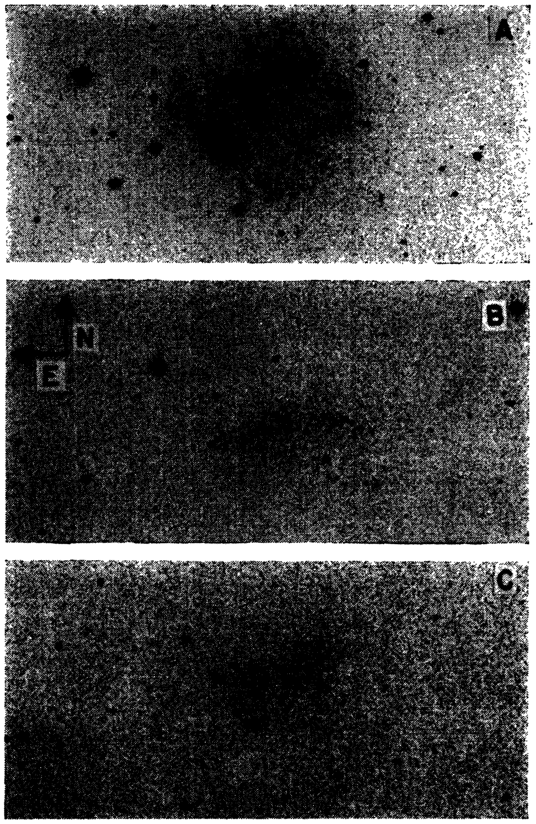

FIG. 2. (B) This nebula (1940: R.A. 9^h 55.9^m, Decl. $+31°$ $2'$) which is located in the constellation of Leo at a distance of about 1.3 million light years is the faintest stellar system now known. Its absolute brightness, according to Baade is about 600,000 times that of the sun. (A) This nebula (1940: R.A. 10^h 8.1^m; Decl. $-4°$ $24'$) in the constellation of Sextans is about one million light years distant and is probably the second faintest system known at the present time. Its brightness is about 1.5 million times that of the sun. The irregularities in surface brightness in both the Sextans system and the Leo system appear much less pronounced when photographs are made in red light only. Both stellar systems belong to the so-called local group of nebulae. (C) This nebula (1941: R.A. 9^h 36^m, Decl. $+71°$ $27'$) is located in Ursa Major. Its average surface brightness which is exceedingly low and its structure suggest that it is a system similar to IC 1613 which nebula is a member of the local group. If we put the Ursa Major system tentatively at a distance of about one million parsecs (=3.26 million light years) and incorporate it in the group of nebulae associated with the bright systems Messier 81 and Messier 82, its diameter becomes about 4000 light years and its brightness about four million times that of the sun making it very similar to IC 1613. All of the three systems shown were discovered with the 18-inch Schmidt telescope on Palomar Mountain. The fact that even with this unusually fast telescope it is difficult to make such nebulae as the Ursa Major system stand out clearly against the various lights of the night sky indicates that a considerable number of quite nearby dwarf nebulae may thus far have escaped discovery. In the reproductions shown the contrasts have been very strongly enhanced in order to make the nebulae stand out clearly.

found which is coarse enough to be distinguished from the accidental granulation of the photographic grain.

(2) For faint nebulae which show bright stars in numbers increasing with decreasing brightness in about the same way as IC 1613 or Messier 33 the granulation effect may be expected to be more pronounced on "blue" films than on panchromatic films shielded by a red filter.

(3) In cases like the Fornax system[5] where neither one of the criteria (1) or (2) is of any help, the system may happen to be near enough to be conspicuous through its relatively large angular diameter.

With the help of the principles just mentioned six objects were singled out for further investigation with the 100-inch telescope. Among these six objects two nebulae located in Leo and in Sextans, respectively, turned out to be new extremely faint nebulae in the local group. Reproductions of 100-inch photographs of these two nebulae may be found in the article by E. Hubble in the *Scientific Monthly** of November, 1940. These show clearly the absence of a central condensation and the existence of great irregularities in surface brightness. Since the search for dwarf nebulae of the type described is most effectively conducted with medium size telescopes we here reproduce in Fig. 2 photographs of three of these nebulae which were obtained by Dr. N. U. Mayall with the 36-inch Crossley reflector at the Lick Observatory. For permission to publish these photographs thanks are due to Dr. W. H. Wright, director of the Lick Observatory, and to Dr. Mayall.

In addition to the fact that the Leo system and the Sextans system are the faintest extragalactic systems known so far, their location in the sky is such as to be of value for the investigation of the relative motion of the Milky Way, an investigation which in the past was hampered by the fact that the members of the local group of nebulae known previously show a lopsided distribution favoring a cap around the south galactic pole.

It should be emphasized that the criteria (1), (2), and (3) often may help in detecting nearby

* Some inaccuracies in this article regarding the discovery of the new systems in Leo and Sextans were corrected by Hubble in a note to Sci. Monthly **52**, 486 (1941).

nebulae of the Smoluchowski type, but that they are still considerably selective and that, worst of all, they do not in any way facilitate the establishing of a representative collection of intrinsically small nebulae of the elliptical and globular types.

(β) The Luminosity Function of the Local Group

If we limit the local group to nebulae of distance moduli $\mu = m - M \leqslant 23$ (distance 1.3×10^6 L.Y.) the fourteen objects now known in this group may be arranged in the following ranges of absolute luminosity M.

The rectangles in Fig. 3 with a total area equal to unity graphically represent the frequency distribution of absolute luminosities in the local group.

It is seen that the faint nebulae are much more prominent than would be expected from the luminosity function $n(M)$ given by (2). While for the unobscured areas in the sky presumably no more nebulae brighter than about $M = -14$ and of distance modulus $\mu \leqslant 23$ exist than are listed in Table I it is very unlikely that we now know all of the nebulae fainter than say $M = -13$. Extrapolating from the progress which has been made during the past few years in shifting the mean value M_0 towards smaller luminosities it is very probable that this trend will continue through further discoveries.

It should here be mentioned that the globular cluster NGC 2419, the absolute magnitude of which is $M = -8.3$, lies at a distance of 200,000 light years from the center of the Milky Way.[15]

TABLE II.*

Range in Absolute magnitude	Systems of the local group of nebulae
-8.25 to -9.10	Leo system
-9.10 to -9.95	None
-9.95 to -10.80	Wolf-Lundmark nebula, Sextans and Sculptor systems
-10.80 to -11.65	NGC 205, NGC 6822, IC 1613
-11.65 to -12.50	Fornax system
-12.50 to -13.35	Messier 32
-13.35 to -14.20	None
-14.20 to -15.05	Messier 33
-15.05 to -15.90	Small Magellanic cloud
-15.90 to -16.75	Large Magellanic cloud
-16.75 to -17.60	Messier 31, galaxy

* I am indebted to Dr. W. Baade of the Mt. Wilson Observatory for some of the data given in this table.

[15] W. Baade, Astrophys. J. **82**, 396 (1935).

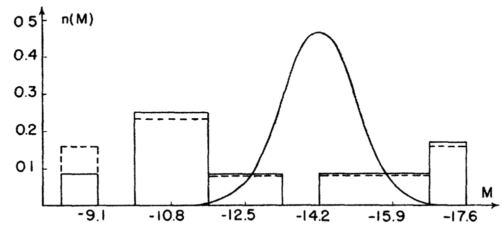

FIG. 3. Luminosity function of the local group of nebulae. The curve represents the luminosity function (2) of the text. The drawn out rectangles represent the frequency distribution in absolute magnitudes of the "local" nebulae listed in Table II. If the globular cluster NGC 2419 is admitted as an independent system a frequency distribution results which is indicated by the broken lines. It is probable that many more "local" nebulae will be found which fall into the range $M > -14.2$.

Although its radial velocity of $+9$ km/sec. is not in itself larger than the velocity of escape from the Milky Way its total velocity might well be larger. But even if this is not the case it may be justified to consider it as a companion of the Milky Way which must independently be incorporated in the luminosity function of the nebulae.*

It must also be remembered that the systematic search for faint nebulae has only just begun and that the results achieved so far justify the expectation that many additional faint nebulae will be found. It will, however, be very difficult to establish the complete luminosity function by an investigation of the local group alone, although such telescopes as the 48-inch Schmidt telescope now in construction will no doubt be of great help in achieving a solution of this problem.

In regard to the question whether or not the local group is a representative sample collection of nebulae it follows from our previous considerations that this group as well as any other group or cluster rather favors the massive and more luminous nebulae because of the segregation resulting from Boltzmann's principle. *In a large sample collection of nebulae including both clusters and field nebulae the number of faint nebulae should be relatively larger than that found in the local*

* A more exact theory will have to take into account the fact that the stellar systems themselves are not rigid units but are made up of various types of subsystems, the frequency distribution of which is in some definite way related to the luminosity function of nebulae. In such a theory it may appear convenient and justified to consider as statistically independent systems all of the globular clusters of the Milky Way system, of the Andromeda nebula and of other stellar systems.

group. The ultimate result may therefore well be in accord with the predictions of our theory. That is, it may be found that in contradistinction with the luminosity function (2) the relative number of nebulae increases as their intrinsic brightness decreases. Some additional attempts which are now being made to establish the correctness of this expectation may be briefly sketched.

(γ) The Luminosity Function in the Virgo Cluster of Nebulae and its Surrounding Regions

The Virgo cluster lies at a distance of about seven million light years. Its six hundred known constituent nebulae have a mean apparent magnitude $+12.7$ which is equivalent to a mean absolute magnitude of about $M=-14.2$ and seem to be adequately represented by the luminosity function (2). Because of the segregation effect which results from Boltzmann's principle it must be suspected that the more luminous nebulae are represented in unduly large numbers if only the cluster itself is investigated. The following procedure therefore suggests itself. Determine the radial distribution in nebular density to a distance of say three times the apparent diameter of the Virgo cluster of nebulae in the range of apparent magnitude $+9$ to $+10$, $+10$ to $+11$, etc. With the 18-inch Schmidt telescope it will be possible to explore these ranges to a limit of about $+15$ to $+16$. It should then be expected that distribution curves of the type shown in Fig. 4 will be found.

If these expectations can be verified through the observations which are now being conducted at Palomar it should be possible to establish a fairly complete luminosity function in the range from the brightest nebulae ($M=-18$) to those of intrinsic brightness $M=-10.5$ (luminosity equal to about one million suns). In addition curves of the type shown in Fig. 4 should enable us to determine the relative masses of nebulae.

(δ) The Luminosity Function in Distant Clusters of Nebulae

In the case of a distant cluster the following method suggests itself in order to obtain a third independent check of our theory. First, count the individual nebulae discernible with a large telescope, determine their luminosities and calculate the total integrated brightness of these nebulae. Then run microphotometer tracings well across the cluster and determine the total brightness of the cluster. The difference between the two luminosities represents the contribution of the faint not individually discernible nebulae in the cluster, which, compared to the integrated brightness of the discernible nebulae should give an estimate of the relative importance of the fainter branch of the luminosity function to the brighter branch. In order for this procedure to be successful the cluster in question should lie in a region of the sky in which the intervening number of bright stars is small. Such clusters are rare but a new cluster in Pisces (Right ascension $23^h\ 7.5^m$, Decl. $+7°\ 13'$ (1940)) which promises to meet all of the necessary requirements has recently been found with the 18-inch Schmidt telescope on Palomar Mountain.

Finally it should be mentioned that the search for supernovae conducted during the past few years has led to some interesting results concerning the stellar population of intergalactic space. It was found that supernovae occur far outside the nebulae suggesting that stars are present in these regions although they cannot be detected under normal circumstances. Further refinements of the observational technique will no doubt enable us in the future to detect objects of fainter surface brightness than was hitherto possible and to obtain in this way additional information about the amount of matter dispersed throughout intergalactic space.

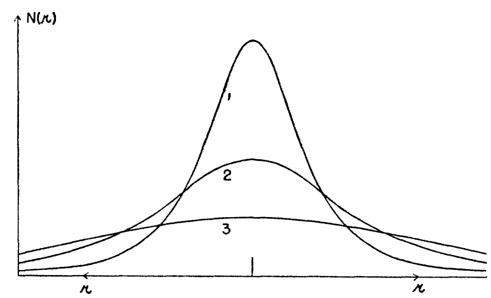

FIG. 4. Radial distribution of nebulae of different mass to be expected in a stationary cluster of nebulae. The curves 1, 2 and 3 qualitatively represent the numbers $N(r)$ per unit volume of nebulae of different mass $\mathfrak{M}_1>\mathfrak{M}_2>\mathfrak{M}_3$.

E. BRIEF REVIEW OF SOME OF THE EVIDENCE NOW AVAILABLE WHICH FAVORS THE ASSUMPTION OF A STATIONARY RATHER THAN AN EXPANDING UNIVERSE

In the preceding discussion it is shown that on the assumption of a stationary universe the luminosity function (2) for nebulae cannot be correct and that the jump in the velocity dispersion from cluster nebulae to field nebulae can quantitatively be accounted for on the basis of the coexistence of a Boltzmann distribution for the cluster nebulae and a Smoluchowski distribution for the field nebulae. On the assumption of an expanding universe our general conclusions concerning the luminosity function of nebulae still remain qualitatively valid although the necessary proof has not here been given. On the other hand it was shown previously[16] that the magnitude and even the sign for the jump in the velocity dispersion from the clusters to the general field is in contradiction with the hypothesis of an expanding universe. It is therefore of interest to mention briefly some additional evidence which favors the hypothesis of a stationary universe.

(a) The observed large scale distribution of matter in space and in velocities exhibits the degree of uniformity which permits the assumption of a stationary universe. The uniformity in the morphological types of nebulae throughout the observable regions of space also is in accord with this assumption.[8] These aspects of uniformity have been discussed at length by Hubble.[4] On the quantitative characteristics of the large scale distribution of matter the following evidence is available.

(b) As is to be expected in a stationary universe a large proportion of the clusters of nebulae exhibit quantitatively correct characteristics of stationary assemblies. It was shown for the first time with the help of the 18-inch Schmidt telescope that the clusters of nebulae are far more extended than was previously thought[1, 17] and that many of the clusters possess spherical symmetry. Among the clusters of this type, arranged in order of decreasing membership are the clusters in Coma[1] (at least 2000 nebulae), Perseus, Hydra,[17] Cancer, Pegasus[17] and Fornax (R. A. $3^h\ 34^m$, Decl. $-36°\ 0'$, about 100 nebulae) as well as a number of small condensations in the Pisces cloud.[18]

(c) The observed radial distributions of the numbers of nebulae in clusters are such that with the help of two reduction factors affecting the density and the distances from the center of the clusters all of the known distribution curves of globular clusters can quantitatively be reduced to *one standard curve* as will be shown in another place. This curve is practically identical[1] with the distribution curve of surface brightness observed in globular nebulae and both curves are represented quantitatively by the density curve derived theoretically for a bounded isothermal gravitational (Emden) gas sphere. Incidentally, these results furnish the first proof that Newton's law is valid as a first approximation in describing the interactions among objects separated by distances of the order of one million light years. Previously, from observations on double stars, the validity of Newton's law had been established only for the interactions of objects separated by distances of less than one light year.

The observed radial distribution in clusters of nebulae has not been accounted for on the hypothesis of an expanding universe. If in an originally contracted universe all of the present clusters had existed in a much more condensed state their present radial distribution curves should be determined by the original velocity distribution of the nebulae which would result in radial distributions in clusters quite different from the actually observed Boltzmann-Emden distributions.

From the remarkable fact that even clusters which contain only about one hundred nebulae exhibit the Emden distribution we may conclude that the exchange of momentum and energy among the nebulae of such a cluster has been effective and the time available long enough to establish a statistically stationary assembly and that we are therefore justified, as was done in this paper, to analyze the problem of the luminosity function of nebulae on the assumption of a stationary universe.

[16] F. Zwicky, Proc. Nat. Acad. Sci. **25**, 604 (1939).
[17] F. Zwicky, Proc. Nat. Acad. Sci. **27**, 264 (1941) and J. B. Edson and F. Zwicky, Proc. Nat. Acad. Sci. **27**, 366 (1941).
[18] F. Zwicky, Proc. Nat. Acad. Sci. **23**, 251 (1937); and Clyde W. Tombaugh, Pub. Astr. Soc. Pac. **49**, 259 (1937).

(d) The velocity distribution in clusters of nebulae such as the Virgo cluster for which enough observations are available is independent of the distance from the center.[19] (For information on a great number of unpublished observations I am indebted to Mr. Humason and to Dr. Hubble.) Equipartition of energy among nebulae of presumably about the same mass, in order of magnitude, is therefore established to distances from the center of a cluster where the effects of encounters become too weak and the characteristics of the velocity distribution go over into those of Smoluchowski distributions. As was shown in this paper the average jump in velocity dispersion from the interior of the large clusters to the general field can be quantitatively related to the ratio of the total mass of all cluster nebulae and of all field nebulae in a very large volume of space [Eqs. (10) and (11)]. Furthermore, according to data kindly supplied me by Dr. Hubble the velocity dispersion in clusters depends on the central density of the cluster. These observations check a theoretical correlation between the total radius, the velocity dispersion and the average central density (or the total mass) of a cluster which may be derived on the assumption that a globular cluster is a statistically stationary assembly.

(e) The time of formation of a stationary cluster of nebulae according to an estimate previously made[16] is of the order of 10^{18} years. Since clusters of nebulae in their present extents could not have existed in a greatly contracted universe[16] we are here confronted with a contradiction to the hypothesis of an expanding universe which allows only a few billion years for the clusters to be formed.

(f) In a stationary universe the formation of clusters of nebulae should, according to Boltzmann's principle be accompanied by a segregation of nebulae of different mass. Evidence for this effect has now also been secured and will be presented in a forthcoming publication.

[19] S. Smith, Astrophys. J. **83**, 23 (1936).

(g) A number of the structural and kinematic features of nebulae such as the density and velocity distributions in globular and elliptical nebulae furnish most convincing evidence[8] that these nebulae have reached statistically stationary states and that the procedure employed in the derivation of the fundamental relations (26) and (28) concerning the luminosity function of nebulae can thus be directly justified. For the quantitative discussion of the morphology of nebulae and the relaxation times involved we refer to previous publications.[8]

(h) The observed second-order effects in the redshift of light from nebulae so far favor the assumption of a stationary universe. Hubble[4] states as the conclusion of his investigation on the number of nebulae depending on apparent magnitude, that "Careful examination of possible sources of uncertainties suggests that the observations can probably be accounted for if redshifts are not velocity shifts. If redshifts are velocity shifts, then some vital factor must have been neglected in the investigation." The second-order effects involved must however be rediscussed once a better luminosity function is established and also more information is available on the effects of "nebulae as gravitational lenses."

In conclusion this relative appraisal may be made. The hypothesis of the expanding universe compensates for the instability of a uniform distribution of matter through an all over expansion and explains the redshift as due to real velocities. On the other hand this hypothesis is in contradiction with several features of the large scale distribution of matter which can satisfactorily be accounted for on the hypothesis of a stationary universe. Expansion on this theory is assumed to be unnecessary because of dynamic stabilization of the universe through the formation of large scale condensations. The redshift in this case must find an independent explanation, for instance as a gravitational drag of lights as I have proposed it some time ago.[20]

[20] F. Zwicky, Proc. Nat. Acad. Sci. **15**, 773 (1929) and Phys. Rev. **34**, 1623 (1929).

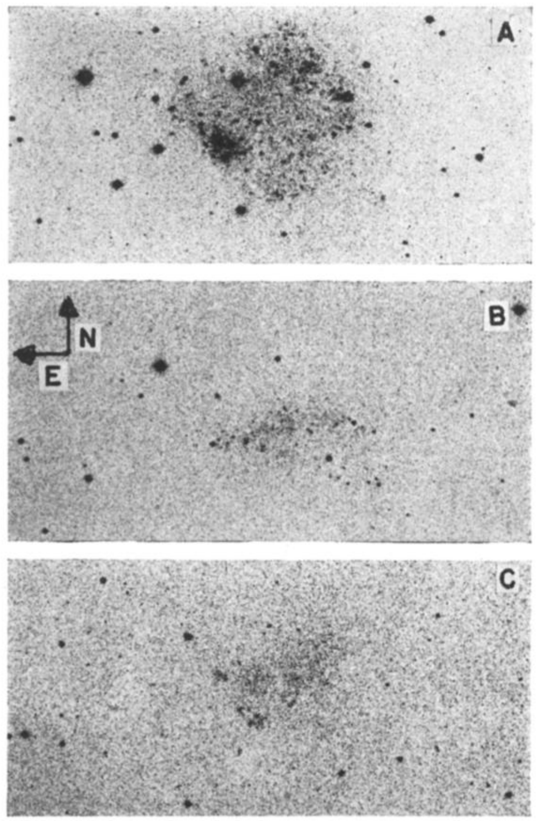

FIG. 2. (B) This nebula (1940: R.A. 9^h 55.9^m, Decl. $+31°$ $2'$) which is located in the constellation of Leo at a distance of about 1.3 million light years is the faintest stellar system now known. Its absolute brightness, according to Baade is about 600,000 times that of the sun. (A) This nebula (1940: R.A. 10^h 8.1^m; Decl. $-4°$ $24'$) in the constellation of Sextans is about one million light years distant and is probably the second faintest system known at the present time. Its brightness is about 1.5 million times that of the sun. The irregularities in surface brightness in both the Sextans system and the Leo system appear much less pronounced when photographs are made in red light only. Both stellar systems belong to the so-called local group of nebulae. (C) This nebula (1941: R.A. 9^h 36^m, Decl. $+71°$ $27'$) is located in Ursa Major. Its average surface brightness which is exceedingly low and its structure suggest that it is a system similar to IC 1613 which nebula is a member of the local group. If we put the Ursa Major system tentatively at a distance of about one million parsecs (=3.26 million light years) and incorporate it in the group of nebulae associated with the bright systems Messier 81 and Messier 82, its diameter becomes about 4000 light years and its brightness about four million times that of the sun making it very similar to IC 1613. All of the three systems shown were discovered with the 18-inch Schmidt telescope on Palomar Mountain. The fact that even with this unusually fast telescope it is difficult to make such nebulae as the Ursa Major system stand out clearly against the various lights of the night sky indicates that a considerable number of quite nearby dwarf nebulae may thus far have escaped discovery. In the reproductions shown the contrasts have been very strongly enhanced in order to make the nebulae stand out clearly.