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Influence of early plate tectonics on the thermal evolution and magnetic field of Mars

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Abstract. Recent magnetic studies of Mars suggest that (1) it possessed a periodically reversing magnetic field for the first $\sim$ 500 Myr of its existence and (2) plate tectonics may have been operating during this time. On Earth the geodynamo is thought to occur because of convection in the outer core. This paper estimates the amount of heat the Martian core can conduct in the absence of convection. It uses parameterized, variable-viscosity thermal evolution models to show that the core heat flux increases if the planet's surface heat flux is increased above the value required to eliminate instantaneous radiogenic heat production. Conversely, a sudden reduction in surface heat flux causes the mantle to heat up and the core heat flux to become negative. Thus, if plate tectonics, or some other process causing high surface heat flux, was occurring on early Mars, it is likely to have caused convection in the core and hence generated a magnetic field. Conversely, a reduction in surface heat flux would probably have caused the core to stop convecting and shut off the magnetic field. There is thus an important link between surface processes and core magnetism, which may also be relevant to planets such as Earth and Venus.

1. Introduction

The thermal evolution of terrestrial planet interiors is thought to be primarily dictated by mantle convection. In the "standard" or simplest picture of this evolution, a monotonic cooling of the planet takes place throughout geologic time. This steady mantle cooling follows inexorably from the following four assumptions:

1. Convective heat flux is a strong positive function of mantle temperature. Typically, convection scaling laws predict that a decrease of mean mantle temperature by 100 K leads to a decrease of heat flow of around a factor of 2. This strong dependence arises from the very strong dependence of mantle viscosity on temperature.

2. The "equilibrium" mantle heat flow decreases throughout geologic time. Here, equilibrium is defined as the heat flow predicted by the instantaneous heat production of radioactive elements. The decline in this heat flow arises from the half-lives of the relevant isotopes ^{40}K,^{235}U,^{238}U, and ^{232}Th.

3. Accretional heating is large, so that the planet began with a mantle temperature that was at least as large as that required to sustain the equilibrium heat flow at that time.

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4. The convective circulation pattern is simple, is unchanging, and extends throughout the mantle.

This simple picture has several consequences. First, it predicts more volcanism in the past than later (at least to the extent that one can associate the propensity for volcanism with the mean mantle temperature). Second, it provides an estimate for the relative importance of secular cooling and radioactivity in the total heat output. Third, it offers a possible explanation for the presence and vigor of core convection by predicting the rate at which the core cools. (The core is assumed to have no radiogenic heat sources.) One can then attempt to understand the absence or presence and persistence of dynamo-driven magnetic fields [e.g., *Stevenson et al.*, 1983].

The simple picture also has some obvious shortcomings. First, it is based on convection scaling laws that do not explicitly account for melting. These scaling laws are probably incorrect, at least at early times, when the melting was probably significant. Second, it assumes a particularly simple and unchanging form of the convection, unimpeded by phase transitions or compositional layering. Third, and most important, it does not appear to agree very well with some observations. On Earth, estimates of total heat production [*Sun and McDonough*, 1989] are roughly a factor of 2 smaller than the observed heat flow [*Sclater et al.*, 1980], a larger discrepancy than simple whole mantle convection models permit [*McKenzie and Richter*, 1981]. Possibly, there is layering (either chemical or due to phase transitions) which may have varied through geologic time. On Venus, estimates of current thermal lithospheric thickness suggest a far lower current heat flow than for the similar mass Earth and probably lower than the equilibrium value [*Solomatov and Moresi*, 1996; *Nimmo and McKenzie*, 1998], suggesting that the interior may actually be heating up. This could arise if Venus is passing through a transition in convective behavior, either a secular transition from plate tectonics to a single plate planet regime or an episodic behavior [*Turcotte*, 1993; *Solomatov and Moresi*, 1996; *Nimmo and McKenzie*, 1997].

New data for Mars also present a challenge to simple models. There is striking evidence for an early Martian dynamo [*Acuña et al.*, 1999], consistent with simple thermal evolution models [e.g., *Stevenson et al.*, 1983; *Schubert and Spohn*, 1990; *Weizman et al.*, 1996], but those models predict that the field would likely have persisted for of order one or two billion years, or even more. In these conventional monotonically cooling models, the existence or absence of a dynamo depends not only on the cooling rate but the presence or absence of an inner core. This depends, in turn, on the abundance of sulfur in the core. Typically, these models require that the sulfur content be larger than some critical value, since otherwise an inner core will form and the dynamo will persist, perhaps through to and beyond the present day. These kinds of models are reviewed by *Schubert et al.* [1992] and *Spohn et al.* [1998].

Although magnetic anomalies within the southern highlands are large ($\sim$ 1000 nT), neither the Hellas nor the Argyre impact basins shows a magnetic field greater than at most 100 nT at $\sim$100 km altitude. This observation suggests that the Martian dynamo was already extinct when these basins formed, probably at $\sim$4 Gyr B.P. [*Strom et al.*, 1992], in contrast to the predictions of the models outlined above.

In this paper we examine one particular modification to simple thermal evolution models: we suppose that Mars had an early plate tectonic episode, replaced later by a single plate or "stagnant lid" regime. *Sleep* [1994] proposed a plate tectonic episode for Mars. Part of the attractiveness of this idea is the striking differences between the younger, smoother, thinner-crust northern hemisphere and the older, rougher, thicker-crust southern hemisphere. Recent Mars Global Surveyor (MGS) altimetry and gravity data reaffirm the differences between north and south [*Smith et al.*, 1999]. The claimed connections between surface morphology and tectonic regime are controversial and permissive (i.e., there is no strong argument either for or against a plate tectonic epoch). *Connerney et al.* [1999] propose a plate-spreading explanation for the southern hemisphere magnetic lineations (necessarily a disjoint proposal from that of Sleep). One problem for this proposal is that it probably requires plate spreading so slow that the hypothesized plate tectonics has very little ability to eliminate heat. This can be seen as follows: if a given magnetic stripe requires uniform magnetization of a crustal thickness of d, then the time to make a stripe is necessarily at least d^2/κ, where κ is the thermal diffusivity. This follows from the expectation that if the field reversed more rapidly than this timescale, then the magnetic intensity would be greatly reduced; the observed intensities are in fact very large. For a stripe width of order 200 km it follows that the plate-spreading velocity can be at most $\sim$ 0.5 cm/yr$(30 \text{ km}/d)^2$. At the early epoch in question, Mars has heat flows like the current Earth and thus needs plate motions of order 10 cm/yr to eliminate heat. Of course, it might be that the magnetization is confined to a thinner layer. Another possibility is that the lineations may be a consequence of plate tectonics but not a local spreading center. We conclude that arguments for an early plate tectonic epoch are not compelling but neither can they be readily dismissed.

There are many possible implications of a plate tectonic epoch on Mars, but for our consideration, one is paramount: plate tectonics is potentially a far more efficient eliminator of heat than the stagnant lid regime. The main reason for this is that plate tectonics recycles the entire lithosphere, including material that is up to a thousand degrees colder than the mantle. By contrast, the material mobilized and recycled in a stagnant lid regime is only at most a few hundred degrees colder than mean mantle. Of course, plate tectonics may also have impediments special to that mechanism and not so readily quantified because they are not conventional fluid mechanics. These involve the mechanics of faults and collision zones and may influence the scaling laws for this kind of convection [*Conrad and Hager*, 1999]. Nevertheless, Earth shows us that plate tectonics can manage to behave somewhat like constant viscosity convection, and it is the implications of this hypothesis that we wish to examine in more detail.

Evidently, a planet that changes from plate tectonics to no plate tectonics will be changing from an efficient to an inefficient mode of heat expulsion. It accommodates this transition by heating up internally, since that lowers the viscosity and remobilizes the convection. A necessary (but not sufficient) condition for core convection is core cooling, and the core cannot continue to cool if the overlying mantle is heating up. Accordingly, core convection may turn off and the dynamo may cease to operate. We examine here the hypothesis that the cessation of the Martian dynamo is intimately linked to the cessation of an early plate tectonic regime. We are not advancing any explanation for the cessation of plate tectonics at $\sim$500 Ma, nor are we necessarily saying that plate tectonics ever happened at all. Our main conclusion is that if plate tectonics did happen early on, it would have made the generation of a magnetic field more likely. It is the proposed link between surface processes and dynamo generation which is the important point. We note also that in the models suggested here, the link between the presence of an inner core and the persistence of a dynamo to later times no longer need exist. Even if Mars has an inner core, it will not generate a dynamo if the core is not cooling significantly at later times, as the new models proposed below suggest. Unfortunately, there is no experiment or observation in the near future that is likely to tell us whether Mars has an inner core.

2. Martian Core

Neither the nature nor the exact composition of the Martian core is well known. Assuming a mantle Mg number of 0.75, the most likely core radius from moment of inertia data is from 1300 to 1700 km [*Folkner et al.,* 1997]. Compositional models based on SNC meteorites and gravity data suggest that the core may contain from 3.5 to 33.8 wt% sulfur [*Longhi et al.,* 1992], which lowers the melting point by several hundred kelvins [*Boehler,* 1996]. Although there is no direct evidence for whether the Martian core is at least partially liquid, the sulfur content and previous thermal evolution models [*Stevenson et al.,* 1983; *Schubert and Spohn,* 1990] strongly suggest that this is the case.

The heat flux out of the core of a terrestrial planet is due to secular cooling, plus any contribution from settling out of a light element or solidification of an inner core. Assuming that the temperature gradient in the core is adiabatic, the heat flux F_{cond} which can be transported across the core by conduction is given by

$$F_{\text{cond}} = \frac{k_c \alpha_c g T_c}{C_{\text{pc}}}, \tag{1}$$

where k_c, α_c, and C_{pc} are the thermal conductivity, expansivity, and heat capacity of the core, respectively, g is the acceleration due to gravity, and T_c is the core temperature. On Earth the heat flux out of the core is thought to exceed this conductive value; the resultant convection, which may also be due to inner-core solidification [*Braginsky,* 1963; *Stacey and Loper,* 1984; *Labrosse et al.,* 1997], drives the terrestrial geodynamo.

The pressure at the core-mantle boundary (CMB) on Mars is ~25 GPa, about one fifth of that at the terrestrial CMB. The acceleration due to gravity at this point is 3 ± 0.3 m s^{-2} [*Longhi et al.,* 1992]. The solidus temperature at this pressure is ~2300 K for FeO [*Boehler et al.,* 1995], whereas that for Fe with 14.5 wt% S is ~1800 K [*Longhi et al.,* 1992] and for the Fe-FeS eutectic is ~1600 K [*Boehler,* 1996]. The density of liquid iron at 25 GPa is ~8300 kg m^{-3}, on the basis of an equation of state using an isentrope centered at atmospheric pressure and 1811 K [*Anderson & Ahrens,* 1994]. That of liquid FeS is 7100 ± 500 kg m^{-3} [*Longhi et al.,* 1992]. The heat capacity of iron is not greatly pressure dependent; values obtained for the terrestrial core of 820-860 Jkg^{-1} K^{-1} [*Anderson,* 1995] are probably similar to those for Mars. Measurements of the thermal expansivity in solid iron at 25 GPa give values of $2 - 3 \times 10^{-5}$ K^{-1} [*Boehler et al.,* 1990]. The value for liquid iron at the terrestrial CMB is around $1.5 - 2 \times 10^{-5}$ K^{-1} [*Anderson,* 1995].

Anderson [1998] used measurements of Fe-Si electrical conductivity to infer thermal conductivies of Fe-S. For Fe-14.5 % S, his method gives a k_c of 43 – 88 Wm^{-1} K^{-1} for Martian CMB conditions. That for pure Fe is ~200 Wm^{-1} K^{-1}; estimates for the value at the terrestrial CMB range from 28.6 to 60 Wm^{-1} K^{-1} [*Anderson,* 1998]. The value for liquid iron or an iron alloy is probably 10% lower than that for the solid [*Anderson,* 1998].

Clearly, the largest uncertainties in calculating the conductive core heat flux using (1) are in T_c and k_c, both of which depend mainly on the amount of sulfur present in the core. Most models arrive at a value of ~15 wt% for this value [*Longhi et al.,* 1992; *Wanke and Dreibus,* 1988; *Treiman et al.,* 1986]. Taking T_c to be 1800 K and k_c to be 43 Wm^{-1} K^{-1}, the expected adiabatic heat flux out of the Martian core is 4.8-9.4 mW m^{-2}. For a k_c of 88 Wm^{-1} K^{-1} the value increases to 9.9-19.1 mW m^{-2}. Thus, unless the amount of sulfur present in the Martian core is significantly less than current estimates, the maximum conductive heat flux the core can support is in the range 5-19 mW m^{-2}.

3. Theory

If we restrict ourselves to viscous rheologies, then theory and experiment agree that the only convective regime available for a terrestrial planet is the asymptotic stagnant lid regime. In this regime the outermost regions are not participating in the flow because they are enormously more viscous than the deep interior. As a result, the outer shell in which conduction dominates can be subdivided into two parts. The deeper, usually thinner portion consists of the material that has no more than about 3×10^3 times the mantle viscosity at the top of the adiabatic region; fluid dynamically speaking, this is the true upper boundary layer of the system. The temperature drop across this boundary layer at the onset of convection is thus about $8/\gamma$, where

$$\gamma = \frac{-d \ln \eta}{dT_i} \tag{2}$$

evaluated at the interior temperature T_i. The shallower, usually thicker outer conductive portion is immobile and has no fluid dynamical role except to provide a rigid boundary condition for the deeper fluid. The stagnant lid heat flow F_{sl} is well approximated in these circumstances by

$$F_{\text{sl}} = \frac{k}{a} \left(\frac{\rho g \alpha}{\kappa \eta_i} \right)^{1/3} \gamma^{-4/3}, \tag{3}$$

where k is the thermal conductivity, κ is the thermal diffusivity, a(~ 2 [*Reese et al.,* 1999a]) is a dimensionless constant, ρ is the mantle density, g is the gravitational acceleration, α is the coefficient of thermal expansion, and η_i is the mantle viscosity at the top of the adiabatic interior. This relationship holds for both spherical [*Reese et al.*, 1999b] and Cartesian [*Moresi and Solomatov*, 1995] geometries. As expected, this result has no dependence on the planetary surface temperature. It has an implicitly strong dependence on mantle temperature through the temperature dependence of the viscosity. It also has no dependence on the depth of the convective layer, which is a well-known property of the simplest version of boundary layer analysis (and is only approximately correct). Since heat transport is primarily conductive in the boundary layer, it also follows that the upper boundary layer thickness δ_u is precisely that required to carry F_{sl} through a temperature drop of $8/\gamma$:

$$\delta_u = \frac{8k}{\gamma F_{sl}}. \tag{4}$$

Although this result is based in part on a global energy argument and is well calibrated by recent numerical experiments, it is largely in agreement with a much older "local" analysis, which states that the boundary layer has a fixed "local" Rayleigh number close to the value needed for instability (i.e., of order 500.) This is explained in the appendix. This local analysis helps to explain and justify the approach developed below for the transition from plate tectonics to stagnant lid, and it also motivates one possible treatment of the lower boundary layer.

Except for minor uncertainties in the value of a, the result for F_{sl} should apply to both internal-heating and bottom-heated systems. However, we also need a prescription for the bottom boundary layer (the thin layer immediately above the core-mantle boundary). Existing numerical and theoretical efforts do not appear to provide a simple result for this, and there is unlikely to be a simple result (i.e., it surely depends on the ratio of internal to bottom heating as well as on the local material properties.) The inapplicability of results contained in Cartesian geometries is a particular concern.

Irrespective of geometry, the core heat flux F_c is given by

$$F_c = k(T_c - T_m)/\delta_b, \tag{5}$$

where T_c is the core temperature, T_m is the mantle temperature, and δ_b is the boundary layer thickness at the bottom. Two limiting cases can be envisaged. In one, it is supposed that the large-scale circulation is presumed to be dominated by the top boundary layer (where the total heat flow is much larger) and dictates the time that material spends at the CMB. In other words, the boundary layer thickness at the bottom is the same as at the top, since this will mean that the time spent in the boundary layer is the same at the top and bottom. Obviously, this can be at best a crude approximation, since it ignores complexities of geometry and viscosity variation with depth. This assumption implies that, neglecting the complications of spherical geometry, the ratio of heat flow from the core to heat flow at the planet surface is the same as the ratio of $(T_c - T_m)$ to $8/\gamma$; the latter is around 300-400K for $\gamma \sim 0.02 - 0.03$.

An alternative viewpoint, probably more realistic, proposes that the bottom boundary layer has a local Rayleigh number that is the same as that of the top boundary layer. This implies that

$$\frac{\rho g \alpha (8/\gamma)\delta_u^3}{e^4 \eta_i \kappa} = \frac{\rho g \alpha (T_c - T_m)\delta_b^3}{e^{-0.5\gamma(T_c - T_m)}\kappa \eta_i} = Ra_c, \tag{6}$$

where Ra_c is the critical Rayleigh number for the onset of convection, from which one finds that

$$\frac{\delta_b}{\delta_u} = 0.5(\gamma[T_c - T_m])^{-1/3} e^{-\gamma(T_c - T_m)/6}. \tag{7}$$

For example, if the core heat flux is one third the surface heat flux, then the bottom boundary layer is about one third the thickness of the top boundary layer, and the temperature drop is accordingly almost an order of magnitude smaller. Recent experiments suggest that the temperature drop in the bottom boundary layer is of order $1/\gamma$ [*Trompert and Hansen*, 1998], a similar result. In our models, termination of core convection and dynamo generation is fastest and most striking if the core-mantle temperature difference is small. On the other hand, if we acknowledge, as we should, a probable increase of viscosity with depth by a factor of A, then the boundary layer thickness and temperature drop should be increased by a factor of order $A^{1/3}$, and this could easily be a factor of 2 or 3. The net effect may well be comparable top and bottom boundary layer thicknesses, but there is evidently a major uncertainty in this estimate.

The plate tectonic regime is fundamentally different because it mobilizes the entire surface layer rather than a thin sublayer. Since the early days of plate tectonics it has been noticed that simple constant viscosity convection does a quite good job of reproducing plate tectonics on Earth in the sense of providing reasonable heat flows and convective velocities provided one chooses a mantle viscosity appropriate to the mean mantle. The constant viscosity law for the plate tectonic heat flow is then

$$F_{pt} = \frac{k}{e}\left(\frac{\rho g \alpha}{\kappa \eta}\right)^{1/3} (T_m - T_s)^{4/3}, \tag{8}$$

where T_s is the planetary surface temperature, $e \sim 2$ [*Solomatov*, 1995]. In the case of Earth and perhaps Mars the viscosity η is not the value η_i referred to in the stagnant lid analysis, since the latter is actually the viscosity at the shallow level corresponding to the base of the thermal boundary layer. Post glacial rebound studies, geoid studies, and experimental work on rheology all support a substantial increase of viscosity with depth (even for the reduced pressure range pertaining to Mars.) This formula predicts the observed heat flow for current Earth if $\eta \sim 10^{21} - 10^{22}$ Pas, whereas reasonable estimates for η_i based on the assumption that the lowermost pressure Earth mantle is at the solidus are nearer 10^{20} Pas. The formula for $F_{\rm pt}$ may also mildly overestimate the heat flow because plate tectonics is surely different from constant viscosity convection (in aspect ratio of the convective cells and the possible role of friction at collisional boundaries, for example.) It follows that

$$\frac{F_{\rm pt}}{F_{\rm sl}} = [\gamma(T_m - T_s)]^{4/3} \left(\frac{\eta_i}{\eta}\right)^{1/3} \sim 100 \left(\frac{\eta_i}{\eta}\right)^{1/3}. \quad (9)$$

There is a large uncertainty in this estimate but little doubt about the much higher efficacy of plate tectonics, even for $\eta_i/\eta \sim 0.01$. The bottom boundary layer might conceivably be as thick as the top boundary layer, which in this case is $k(T_m - T_s)/F_{\rm pt}$ and thus much thicker than in the stagnant lid regime. It might instead be the much thinner value (with correspondingly smaller temperature drop) corresponding to the local analysis, the prediction of which is independent of whether one is in the plate tectonic or stagnant lid regime. The latter would seem more probable and is adopted below.

To consider the effect of a cessation of plate tectonics, suppose that, at some instant in time $t = t_{\rm pte}$, the plate tectonic transport ceases. Since, as we have seen, plate tectonics is much more efficient than stagnant lid convection, it follows that the local Rayleigh number for the sublayer of the lithosphere is subcritical. Specifically, it must be smaller than critical by $(F_{\rm sl}/F_{\rm pt})^3$, where $F_{\rm pt}$ is the predicted heat flow for the mantle temperature at that time. Since the actual temperature profile cannot have breaks in slope, the system must relax conductively until the state is reached where the actual heat flow can be carried by the stagnant lid convection at that time. In other words, during the time $t_{\rm pte} << t << t_{\rm slb}$, where $t_{\rm slb}$ is the beginning of stagnant lid convection, there is no mantle convection except for the possibility of a small amount carried by plumes from the core. The total "thermal lithosphere" thickness (it should not be thought of as a boundary layer any more, since most of it is fluid dynamically immobile) $L(t)$ will therefore increase as it cools from the top by conduction. During this period the heat flux is given by

$$F_l = k(T_m - T_s)/L(t). \quad (10)$$

Consider a time interval dt during which the average mantle temperature has increased by dT_m and the thermal lithospheric thickness has increased by dL. By the first law of thermodynamics, the heat output of the planet in time dt is then $\rho C_p[(T_m - T_s)(L/2 + dL) - (T_m + dT_m - T_s)(L + dL)/2 + T_m(L + dL)]$. The term $T_m(L + dL)$ arises from assuming that the radiogenic heat occurs uniformly everywhere, including the lithosphere. It follows that in a quasisteady state approximation (which is not the exact solution to the thermal diffusion equation but adequate for our purpose)

$$\frac{\kappa(T_m - T_s)}{L(t)} = 0.5(T_m + T_s)\frac{dL}{dt} + 0.5\frac{dT_m}{dt}L. \quad (11)$$

If we ignore T_s relative to T_m and $dT_m = T_m/\tau$ (an adequate approximation for radioactive heating on a timescale of a few million years), then the solution is

$$L^2(t) = L^2(t_{\rm pte})e^{-2\theta/\tau} + 2\kappa\tau(1 - e^{-2\theta/\tau}), \quad (12)$$

where $\theta = t - t_{\rm pte}$, the time elapsed since $t_{\rm pte}$. In the limit where this elapsed time is small compared to τ, L^2 has increased by $4\kappa\theta$.

The critical boundary layer thickness $L_{\rm crit}$ at which stagnant lid convection will initiate is given by *Solomatov* [1995]:

$$L_{\rm crit} = 2.75 \left(\frac{\eta_c\kappa}{\rho g \alpha}\right)^{1/3} \gamma^{4/3}(T_c - T_s), \quad (13)$$

where η_c is the mantle viscosity at the CMB. Stagnant lid convection is assumed to initiate at the point where $L(t)$ exceeds $L_{\rm crit}$.

Approximating the core to be isothermal, and neglecting internal heat generation in the core, the rate of change of core temperature is given by

$$\frac{dT_c}{dt} = -\frac{A_c F_c}{h_c}, \quad (14)$$

where A_c is the core surface area and h_c is its heat capacity. Similarly, the rate at which the mantle temperature changes is given by

$$\frac{dT_m}{dt} = -\frac{A_m F_m}{h_m} + \frac{A_c F_c}{h_m} + \frac{H_{int}}{h_m}, \quad (15)$$

where F_m is the heat flux out of the surface, H_{int} is the mantle heat generation rate, A_m is the planetary surface area, and h_m is the heat capacity of the mantle.

4. Model

The model consisted of a spherical planet of radius r_m containing a core of radius r_c. The mantle was assumed to convect in a single layer, and core solidification and crustal generation were not included in the calculations. The mantle possessed radiogenic heat sources with abundances assumed to be the same as those of the primitive terrestrial mantle of *Sun and McDonough* [1989] (see Table 1). The heat flux out of the core was F_c, and that out of the top of the mantle was F_m. F_c was calculated using (5) and (6) with a critical Rayleigh number of 500. F_m was calculated using (3) with a value of a of 2.0. After each time step the core and mantle temperatures were updated according to (14) and (15). The interior viscosity was updated using (2), and the cycle then repeated. The time step used was 1 Ma.

The initial temperatures T_c and T_m were usually assumed to be equal. Different models were characterized by different initial temperatures and reference viscos-

Table 1. Parameter Values for Mars Thermal Evolution Model

Property	Symbol	Units	Value	Reference
Planetary radius	r_m	km	3400	-
Core radius	r_c	km	1450	see text
Surface acceleration	g	m s^{-2}	3.7	-
Mantle density	ρ	kg m^{-3}	3400	-
Core density	ρ_c	kg m^{-3}	9000	*Longhi et al.* [1992]
Mantle heat capacity	C_p	Jkg^{-1} K^{-1}	1200	-
Core heat capacity	C_{pc}	Jkg^{-1} K^{-1}	840	*Anderson* [1995]
Mantle conductivity	k	Wm^{-1} K^{-1}	3.2	-
Core conductivity	k_c	Wm^{-1} K^{-1}	43-88	see text
Mantle expansivity	α	$°C^{-1}$	4×10^{-5}	-
Core expansivity	α_c	$°C^{-1}$	2×10^{-5}	*Anderson* [1995]
Mantle diffusivity	κ	m^2 s^{-1}	8×10^{-7}	-
Surface temperature	T_s	°C	-80	-
Reference viscosity at 1300°C	η_0	Pas	10^{20}	-
K mantle abundance	-	ppm	250	*Sun and McDonough* [1989]
Th mantle abundance	-	ppm	0.085	*Sun and McDonough* [1989]
U mantle abundance	-	ppm	0.021	*Sun and McDonough* [1989]

ity values η_0. In some models the surface heat flux was assumed initially to be governed by plate tectonics (equation (8)), using a value of $e = 2.0$. To account for the higher effective viscosity during plate tectonic episodes (see section 3), the plate tectonic viscosity η was assumed to equal $10\eta_i$ when determining the heat flux in (8). After a time t_{pte}, plate tectonics was assumed to have ceased; the thickness of the conductively cooling lithosphere was determined by using (11) until stagnant lid convection began. In other models, stagnant lid convection (equation (3)) was assumed to be operating throughout.

The core heat flux was generally assumed to be governed by (5). However, in some models the core temperature T_c dropped below that of the mantle T_m. In such cases, conduction of heat from the mantle into the core was found by solving the spherical heat diffusion problem using a finite difference model discretized into 80 nodes with a time step of 10^5 a. The initial temperature profile was assumed to be adiabatic. The total heat flux from the mantle into the core was found by radial integration of the change in the core temperature profile with time.

5. Results

Figure 1a shows the result of a typical model of the thermal evolution of Mars over 5 Gyr, starting from $T_c = T_m$=2000 K and using the parameters defined in Table 1. The system starts very close to equilibrium, so the heat fluxes demonstrate simple secular decline after an initial transient. The core heat flux never exceeds 15 mW m^{-2}. A lower starting temperature would cause the mantle to heat up initially more rapidly than the core, producing a negative core heat flux, but after 3 Gyr the situation would be almost identical.

Figure 1b shows the same model, starting from the same temperature but this time calculating the heat flux for the first 0.5 Gyr according to (8). The mantle temperature initially drops rapidly because the surface heat flux is much greater than in Figure 1a (see (9)). This temperature drop is accomplished within a few hundred megayears, which means that the system is not very sensitive to the initial conditions (see below). The drop in mantle temperature causes a higher value of core heat flux than in Figure 1a, 24.6 mW m^{-2} at 0.5 Gyr. After 0.5 Gyr, plate tectonics ceases, and the mantle heat flux decreases as the upper boundary layer thickens according to (11). At t = 1.4 Ga the heat flux out of the core becomes negative because of the increasing mantle temperature, and at t = 1.7 Ga, stagnant lid convection initiates, causing the mantle heat flux to increase again. After 5 Gyr the heat fluxes and temperatures are almost indistinguishable from Figure 1a.

Figure 1c shows the same model as in Figure 1a, but this time starting with a core 100 K hotter than the mantle. Because of this large core-mantle temperature difference, the initial heat flux out of the core is also large. However, the time constant over which the temperatures adjust to quasiequilibrium is short, a few hundred megayears, so after 500 Myr the heat flux out of the core is only 2.5 mW m^{-2}.

Figure 2 shows the effect of the mantle viscosity parameter γ, starting temperature, and reference viscosity on the heat flux out of the core at the end of the 0.5 Gyr plate tectonic episode. A higher reference viscosity causes a lower core heat flux because the mantle heat transport is less efficient at higher viscosities (equation (3)). Higher initial temperatures cause a higher core heat flux because the mantle starts further out of equilibrium and the viscosity is lower. The behavior as a function of mantle viscosity parameter is more complicated. The temperature at which the stagnant lid heat flux (equation (3)) balances the internal heat generation decreases as γ increases, so for a particular starting temperature, higher values of γ are likely to be associated with greater disequilibrium and thus higher core heat flux. However, because the time constant of the man-

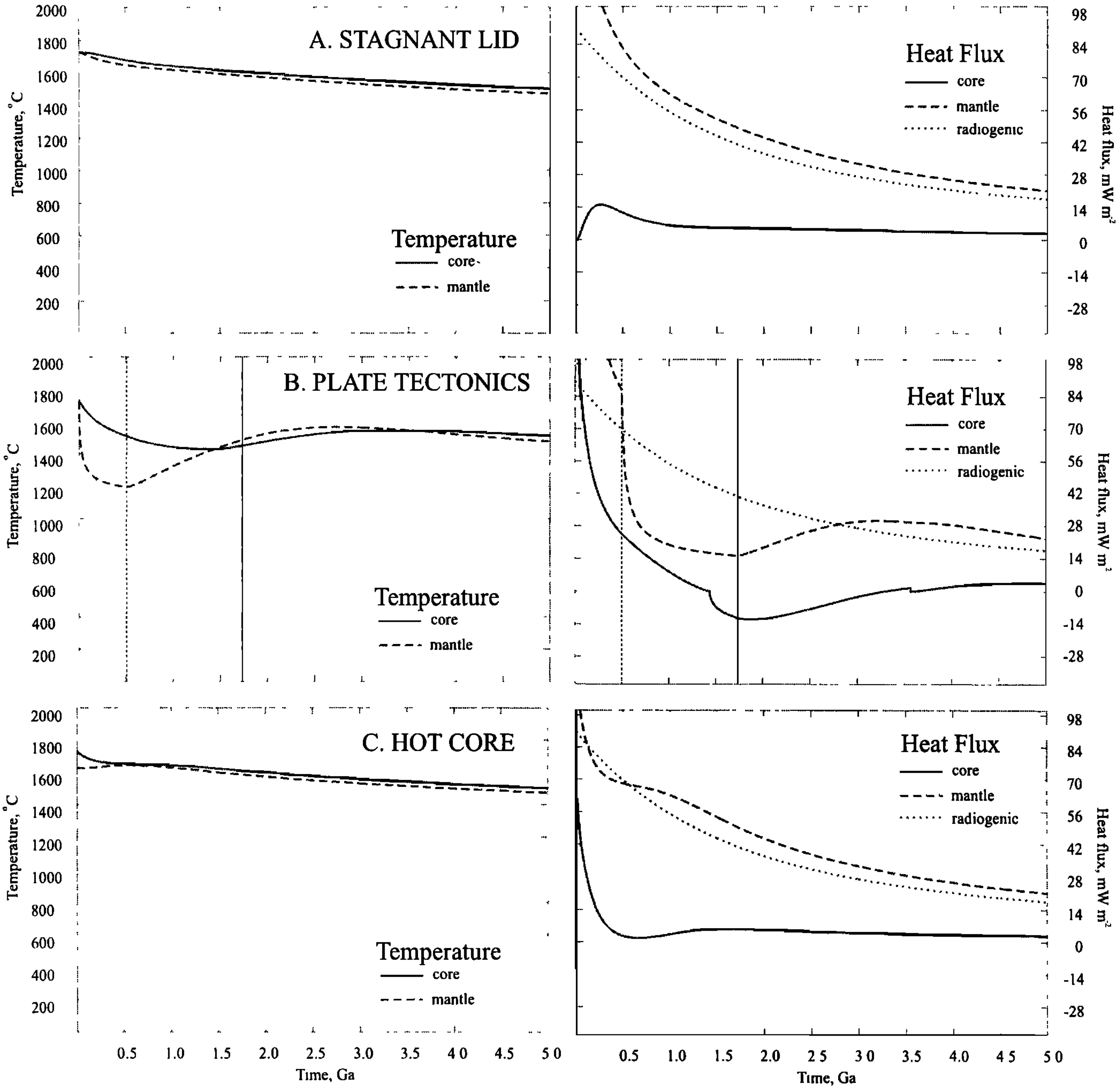

Figure 1. Models of thermal evolution of Mars, using the parameters given in Table 1 and a viscosity parameter $\gamma = 0.023$. In each case the left panel is the evolution of core and mantle temperatures (in °C) with time, and the right panel is the evolution of heat fluxes across the core, across the top boundary layer, and due to radioactive elements, with time. For Figures 1a and 1b the initial conditions were that the core and mantle were at the same temperature. (a) Starting temperature 2000 K. (b) As for Figure 1a, but the surface heat flux is specified by equation (8) for the first 500 Myr. The dotted vertical line indicates the time at which plate tectonics ceases, and the solid vertical line indicates the time at which stagnant lid convection initiates. Between these two times, the lithosphere is thickening conductively according to equation (11). (c) As for Figure 1a, but the initial mantle temperature is 1900 K.

tle decreases with increasing γ, the mantle approaches equilibrium more rapidly, decreasing the core heat flux. There is thus a trade-off between these two effects. Because the mantle time constant increases with increasing viscosity, at lower temperatures (higher viscosities) the second effect predominates (see Figure 2). Figure 2 also shows the core flux after 0.5 Gyr for cases where stagnant lid convection operated throughout. The core heat flux decreases as γ decreases because the equilibrium mantle temperature increases.

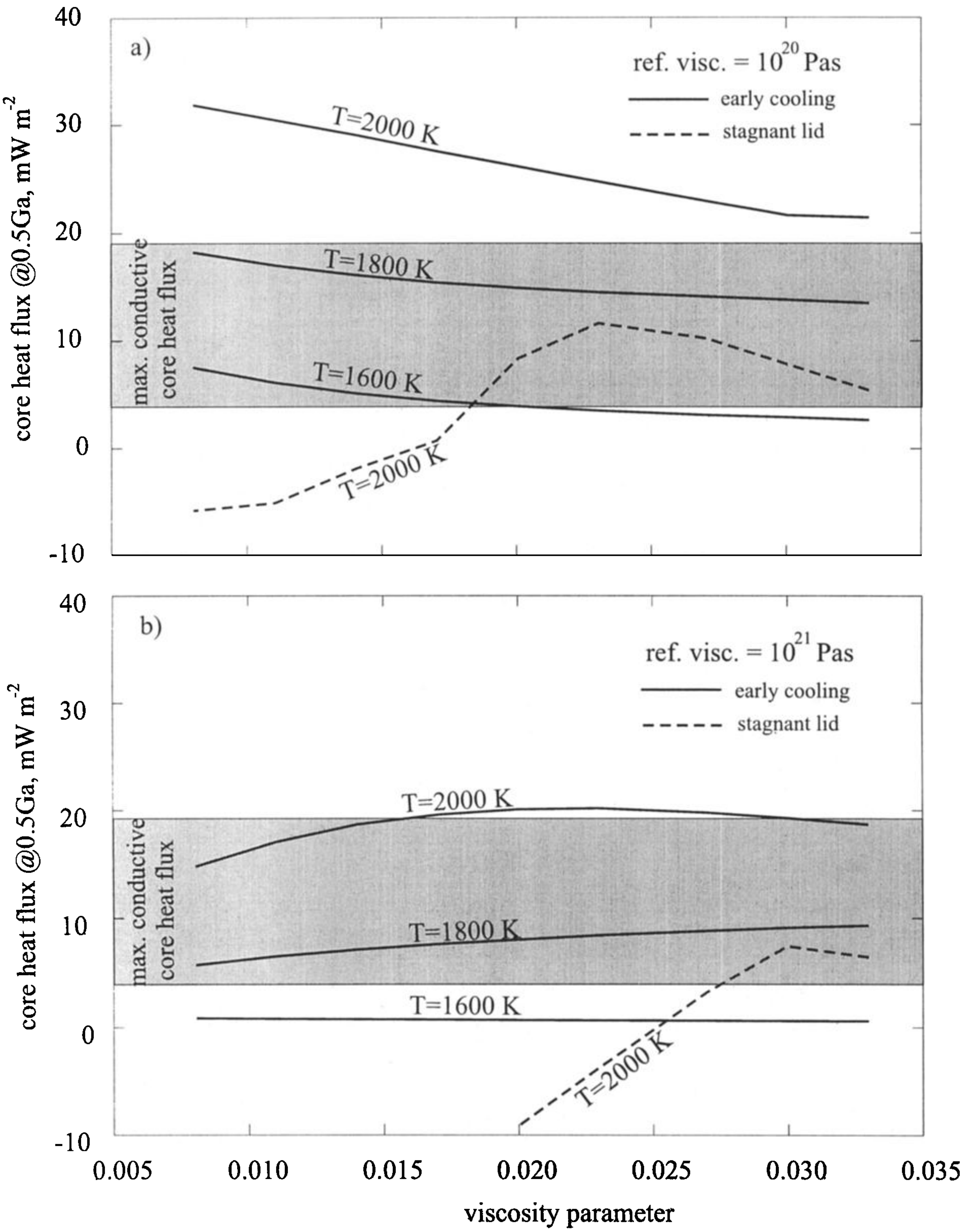

Figure 2. (a) Variation in core heat flux at 500 Ma with viscosity parameter (γ), initial temperature T (in K) for reference viscosity of 10^{20} Pas. The lines marked "stagnant" have no plate tectonics (see Figures 1a and 1c); others have a 500 Myr plate tectonic episode (see Figure 1b). Stagnant lid models with starting temperatures of 1800 K and less always have core heat fluxes less than -10 mW m^{-2}. (b) As for Figure 2a but with a reference viscosity of 10^{21} Pas.

The core heat flux is not very sensitive to the initial conditions: varying the starting temperature by 200 K causes a change of about 10 mW m^{-2}. Nor are the results particularly sensitive to the duration of the plate tectonic event: for the same conditions as Figure 1b but with a plate tectonic event ending at 1.0 Ga, the core heat flux at t_{pte} is 13 mW m^{-2}.

6. Discussion and Conclusions

The most important result of this study is that an increase in early surface heat flux over the value due to stagnant lid convection causes the heat flux out of the core to increase significantly. For a starting temperature of 2000 K the core heat flux at the end of 0.5 Gyr of plate tectonics is from 15 to 30 mW m^{-2} (see Figure 2), whereas in the stagnant lid regime it is <12 mW m^{-2}. Since the maximum conductive core heat flux is 5-19 mW m^{-2} (see section 2), during the stagnant lid regime conduction is probably sufficient to transport all the heat flux from the core. However, if there is enhanced heat flux at the surface, the core is likely to be convecting. Because of the uncertainties in the core conductive heat flux, it is possible that core convection might also occur during secular cooling. However, Figure 1 shows that this is most likely to occur after a delay of several hundred megayears, and continue for several gigayears, which is not consistent with the observations at Mars. By contrast, a cessation of increased surface heat flux leads to a low or negative core heat flux within <1 Gyr. Therefore the generation of a Martian magnetic field is most likely when there is a period of enhanced surface heat flux, such as an early episode of plate tectonics.

It is important to examine how robust this conclusion is. The easiest way of increasing the conductive heat flux the core can support would be by increasing the conductivity, presumably by reducing the sulfur content. The calculations in section 2 show that a pure iron core could sustain a conductive heat flux of up to 41 mW m^{-2}. However, unless the model values of ~15 wt% sulfur are completely incorrect, this heat flux value is probably a factor of 2 too large.

The main variables affecting the heat flux out of the core are the value of the enhanced surface heat flux, the initial conditions, and the mantle parameter γ.

The assumption that an early episode of plate tectonics can be represented by (8) is unlikely to be correct in detail, but there is currently no justification for using a more complicated model. If the viscosity multiplier in the plate tectonic regime is reduced to 1 (see section 4), the core heat flux typically increases by a few mW m^{-2}. Simply fixing the surface heat flux to 120 mW m^{-2} for the first 0.5 Gyr gives similar results to the present study. Any situation in which the rate of heat loss exceeds the heat generation rate will produce a strong temperature contrast at the core-mantle boundary and be likely to generate core convection.

As Figure 2 shows, changing the starting temperature has rather little effect on the outcome, because the time constant of the mantle is short relative to 0.5 Ga. It is generally accepted that accretional energies are large enough to at least partially melt planetary interiors during formation, so initial mantle temperatures lower than ~1600 K are implausible. At the base of the Martian mantle the pressure is ~25 GPa which gives a liquidus temperature for peridotite of about 2650 K [*McKenzie and Bickle,* 1988], corresponding to a mantle potential temperature of ~2050 K. It is highly unlikely that the Martian mantle can have remained above this temperature for any significant period of time. We do not include the effects of melting on convective dynamics in our parameterization, though these may be important, because it is unclear how to do so.

Another way of generating a large core-mantle temperature difference, and hence core convection, is by starting with a core hotter than the mantle (see Figure 1c). A packet of core material initially at 1600 K moving from the top to the base of the mantle will undergo an adiabatic increase in temperature of ~250 K if the transport time is rapid compared to the thermal diffusion time. However, theoretical work [*Stevenson,* 1990] suggests that iron and silicate equilibration may take place at the centimeter scale, implying that core and mantle should start at the same temperature. Siderophile abundances in SNC meteorites also support complete equilibration [*Schubert et al.,* 1992]. Moreover, it is not clear that even large initial temperature differences can sustain core convection for the length of time required, because of the short time constant of the mantle.

It might be thought that the abundance of radiogenic elements in the mantle would affect the core heat flux. However, the heat flux out of the core depends mainly on the existence of a core-mantle temperature difference, which is governed by the balance between heat production and heat loss. A model with the same starting conditions as Figure 1b but with only 50% of the radiogenic elements present has a core heat flux after 500 Myr of 24.7 mW m^{-2}, essentially identical to the Figure 1b result.

The reference viscosity value does have a significant effect on the results. However, the range of 10^{20} – 10^{21} Pas at 1300°C used in this study is similar to estimates for the terrestrial upper mantle and is probably appropriate to Mars. One possibility is that if the mantle of Mars is very dry, the viscosity may be higher, as appears to be the case on Venus. Higher viscosities would reduce the core heat flux. However, the mantle water content of Mars is currently not sufficiently well known to be able to model this effect.

The parameter γ may be related to measurable rheological constants by assuming that the viscosity is governed by an Arrhenius relationship. It can be shown that [*Reese et al.,* 1999b]

$$\gamma = E/RT_i^2, \tag{16}$$

where E is the activation energy of the mantle material, R is the gas constant, and T_i is the temperature. The value of E for dry olivine undergoing diffusion creep is 300 kJ mol^{-1} [*Karato and Wu,* 1993], so with typical values of T_i of ~1800 K, γ is ~0.01. Viscosity increases as a function of depth; for mid mantle pressures of 12 GPa and an activation volume of 2×10^{-5}m^3 mol^{-1} [*Karato and Wu,* 1993], the activation energy would increase to 540 kJ mol^{-1}, and γ would be 0.02. For γ less than ~0.005, the assumption of stagnant lid convection no longer holds.

If there are additional sources of heat within the core, there exists the possibility that the core may convect even during secular cooling. Assuming a latent heat of fusion of 540 kJ kg^{-1}, the rate of freezing required to produce an extra 10 mW m^{-2} of heat is 4.9×10^5 kg s^{-1}, which is equivalent to the entire core solidifying in 8 Gyr. This process might therefore be capable of sustaining core convection; in the absence of any information about the current state of the Martian core it is difficult to conclude whether core solidification is an important effect or not. *Stevenson et al.* [1983] and *Schubert and Spohn* [1990] both concluded that if the core contains 15 wt% or more sulfur, core solidification is unlikely to have occurred. In any case, a robust feature of the models is that the end of the plate tectonic regime causes a temporary increase in core temperature (see Figure 2b), which would certainly stop core solidification.

On Earth it is thought that some convective plumes arise from the core-mantle boundary region. An interval in which the core is cooler than the mantle will not generate such plumes. Figure 1b suggests that, for models with an initial episode of plate tectonics, this style of plume activity might have occurred for the first 1-2 Gyr but not for a few gigayears thereafter. In contrast, models without plate tectonics (Figure 1a) should show plume activity throughout the geological record. Whether such differences would be detectable in the record of Martian volcanism [*Greeley and Schneid,* 1991] is unclear.

In summary, the conclusion that a period of early enhanced surface heat flux leads to an increase in core heat flux by a factor of 2 or more is robust. In particular, it does not depend on the details of the radiogenic heat abundance, surface heat loss mechanism, or rheological parameters. Unless the core contains very little sulfur (which is in conflict with most models of Mars' composition), this increased heat flux is likely to have caused core convection, whereas during secular cooling the core heat flux could have been supported by conduction alone. A period of core convection early in Mars' history is probably required to have created the ancient magnetic stripes on the planet's surface. The apparent cessation of the Martian dynamo prior to the creation of the large impact basins might be linked to a cessation in plate tectonic activity. If correct, this model has implications for the generation of magnetic fields on Venus and Earth. On the latter planets the mantle behavior may be complicated by layering, which increases the response time to surface perturbations [*McKenzie and Richter,* 1981]. However, the ongoing plate tectonics on Earth is likely to cause the core heat flux to be higher than if the surface layer were not moving. Similarly, on Venus the apparent cessation of resurfacing ~500 Myr ago [*Phillips et al.,* 1992] would have reduced the core heat flux and may explain that planet's present lack of a magnetic field.

Appendix

Howard [1964] proposed that one could understand the behavior of convection by analyzing the local stability of the thermal boundary layers. In the context of a fluid with temperature-dependent viscosity, this could be interpreted as follows: for a specified mantle temperature T_m and fixed heat flow, choose the thickness of the upper boundary layer so that the local Rayleigh number is maximized and so that its extremum value is the critical value Ra_c for convection, say 500. The "local" Rayleigh number is evaluated at the viscosity halfway through the boundary layer, as suggested by the work of *Booker* [1976]; see also *Booker and Stengel* [1978]. This approach was proposed by *Reynolds and Cassen* [1979] and has been used in Galilean satellite problems [e.g., *Kirk and Stevenson,* 1987].

For the top boundary layer,

$$Ra(\delta) = \frac{\rho g \alpha \beta \delta^4}{\kappa \eta (T_m - \beta\delta/2)}, \tag{A1}$$

where $\eta(T)$ is the viscosity and β is the temperature gradient, so $k\beta$ is the heat flow F_{sl}. The maximum value of $Ra(\delta)$ can be found by differentiation. This extremum exists because the viscosity is a very strong function of temperature. One finds that to a good approximation, an Arrhenius viscosity law leads to an extremum at $\beta\delta = \beta\delta_0 = 8/\gamma$. Thus $\rho g\alpha(8/\gamma)\delta_0^3/50\kappa\eta \sim Ra_c$, and solving for the heat flow gives

$$F_{sl} = k\left(\frac{80}{Ra_c}\right)^{1/3}\left(\frac{\rho g \alpha}{\eta_i \kappa}\right)^{1/3}\gamma^{-4/3}. \tag{A2}$$

The approach used by *Solomatov* [1995] is superior but yields similar results. He also finds that the boundary layer temperature drop for the initiation of stagnant lid convection is $8/\gamma$ and his heat flow would be the same as found by the local approach if $\mathrm{Ra_c} \sim 80\mathrm{a}^3 \sim$ 600.

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