

# Distribution Networks and Electrically Controllable Couplers for Integrated Optics

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The power distribution as a function of propagation distance in a network of coupled optical waveguides is determined for several interesting cases. An electrically controllable coupler is proposed and analyzed in detail. High efficiency coupling and decoupling between two optical guides can be accomplished with the use of an electrooptically generated dynamic channel, of finite length, located in between the two guides.

## I. Introduction

Recently, a number of researchers<sup>1,2</sup> have reported the development of thin film and channel waveguide optical couplers for use in the emerging field of integrated optics. Applications of these couplers in optical networks, modulators, and multiplexers/demultiplexers, would be drastically increased if the coupling is dynamically controllable by an electric signal. Electrooptic substrates can be used to control the coupling coefficient between two waveguides, but such a scheme would be inefficient because of the upper limitation on the change of the refractive index of existing materials that could be achieved with reasonable voltage. In this communication, we study the power distributions as a function of the distance from the input plane in a network of  $N$  parallel guides. Then we discuss a number of functions that could be achieved using coupled optical waveguides, and we will study in detail a scheme for an electrically controllable coupler.

## II. Symmetric and Nonsymmetric Optical Networks

Let us consider  $N$  identical optical waveguides with  $K$  being the coupling coefficient between two neighboring guides. The field  $E_n$  in the  $n$ th guide is determined by the system of equations:

$$\begin{aligned} (dE_n)/(dz) &= -iKE_{n+1} - iKE_{n-1} \text{ for } 2 \leq n \leq N-1, \\ (dE_1)/(dz) &= -iKE_2, \\ (dE_N)/(dz) &= -iKE_{N-1}, \end{aligned}$$

where we have neglected the direct coupling between

nonneighboring guides. If the input light is fed into the  $m$ th guide, the normalized initial condition is

$$E_n(0) = \begin{cases} 1 & \text{for } n = m, \\ 0 & \text{for } n \neq m. \end{cases}$$

The above system of equations can be written in a matrix form as follows:

$$\begin{aligned} (d/(d\xi))E &= \mathbf{M} \cdot E \\ E(0) &= c, \end{aligned} \quad (1)$$

where  $\xi = Kz$  and

$$E = \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{pmatrix} \quad c = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \leftarrow m\text{th element}$$

$$\mathbf{M} = -i \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 1 & 0 & 1 & \dots & 0 \\ 0 & 1 & 0 & \ddots & 0 \\ & & \ddots & \ddots & 1 \\ 0 & & & & 1 & 0 \end{pmatrix}$$

Equation (1) is a well known differential equation<sup>3</sup> that can be solved by determining the eigenvalues and eigenvectors of the matrix  $\mathbf{M}$ .

For  $N \rightarrow \infty$ , the solution of Eq. (1) is the well known Bessel functions:

$$E_n = (-i)^{|n-m|} J_{|n-m|}(2\xi).$$

For  $N$  finite, the solution can be determined in a straightforward manner with a digital computer. In Fig. 1 we present the power  $P_n = E_n E_n^*$  for a number of configurations ( $N = 2, 3$ , and  $5$ ) with different input conditions. These various configurations can be used to perform a number of functions in optical networks. Some of the possible applications are shown in Fig. 2 and discussed below.

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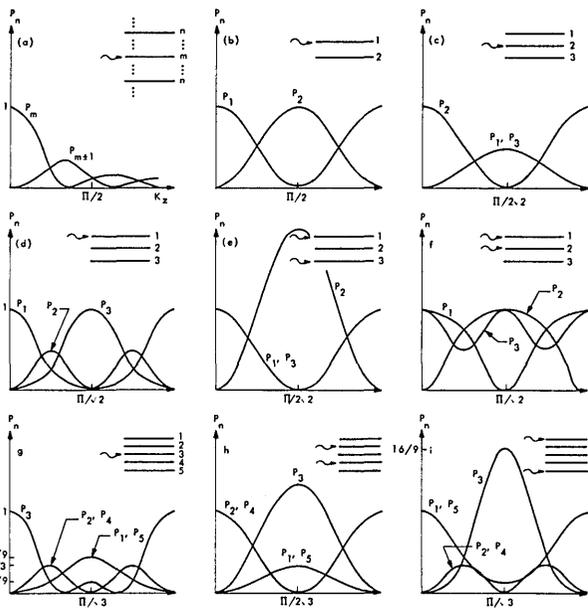


Fig. 1. Power distribution in coupled optical networks.  $P_n$  is the power in the  $n$ th guide as a function of the propagation distance  $z$ .  $K$  is the coupling constant between neighboring guides. Nonneighboring guides are assumed to be uncoupled. For the case where there is more than one input, these inputs are assumed to be in phase.

Figure 2(a) shows an energy transfer function configuration. Complete transfer occurs if the coupling length is an odd integer of  $\pi/2K$  [see Fig. 1(b)]. Figure 2(b) shows an energy divider configuration. The input energy is equally and completely divided between the two outputs if the coupling length is  $\pi/2K\sqrt{2}$  [see Fig. 1(c)]. An elementary logic system is shown in Fig. 2(c). The two inputs  $A$  and  $B$  are in phase, and the output is given by the truth table. Figures 2(d) and 2(e) correspond to energy transfer from two inputs to two outputs, where the useful information is the amplitude or the frequency of the signal (assuming that the two frequencies  $\omega_1$  and  $\omega_2$  are not very different so that  $K$  is approximately the same for both). In Figs. 2(f) and 2(g) we present a possible configuration for an electrically controllable coupler or switch that is discussed in the next section.

### III. Electrically Controllable Coupler

The basic configuration for an electrically controllable coupler is shown in Figs. 2(f) and 2(h). The two permanent channel guides are imbedded at the surface of an electrooptic substrate. They can be formed by proton bombardment,<sup>4</sup> ion implantation, diffusion, or other techniques. The two guides are located such that the direct coupling is very weak. In the region between the two guides, a third channel of finite length is dynamically generated by applying a voltage to the two electrodes shown in the figure. The resulting electric field generates a local change in the refractive index. The feasibility of such an electrooptically generated channel guide was

recently reported by Channin.<sup>5</sup> This controllable channel plays the role of a bridge between the two permanent guides. The electrodes should be located such that the cross section of the dynamic guide is similar to the cross section of the two permanent guides.

The power distribution as a function of the propagation distance in a three guides system, where the energy is fed in the first guide, is shown in Fig. 1(d) and is given by [from Eq. (1)]:

$$P_1(z) = \frac{1}{4}[\cos(K\sqrt{2}z) + 1]^2,$$

$$P_2(z) = \frac{1}{2}[\sin(K\sqrt{2}z)]^2,$$

$$P_3(z) = \frac{1}{4}[\cos(K\sqrt{2}z) - 1]^2,$$

where we assumed that the direct coupling coefficient  $K'$  between the two permanent guides is  $\ll K$ . Complete energy transfer between the two permanent guides occurs if the controllable channel has a length  $L = \pi/K\sqrt{2}$ .

In the absence of the bridge channel, the power distribution in the two permanent guides is

$$P_1'(z) = \cos^2(K'z),$$

$$P_2'(z) = \sin^2(K'z),$$

and the energy transferred over the length  $L$  is

$$\Delta P = \sin^2[(\pi/\sqrt{2})(K'/K)].$$

Therefore the dynamic efficiency of the coupler can be defined as

$$\eta = 1 - \Delta P = \cos^2[(\pi/\sqrt{2})(K'/K)].$$

The analytic expression of the coupling coefficients was derived by Marcatilli<sup>6</sup> as

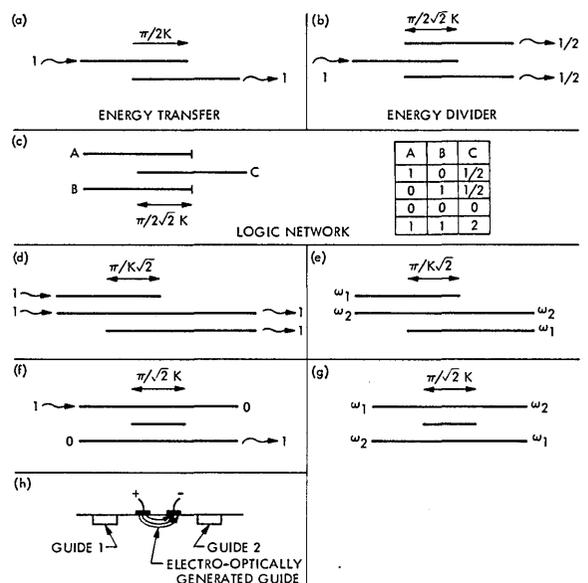


Fig. 2. Different configurations of optical network that could be used in energy transfer, energy distribution, controlled switching (see text).

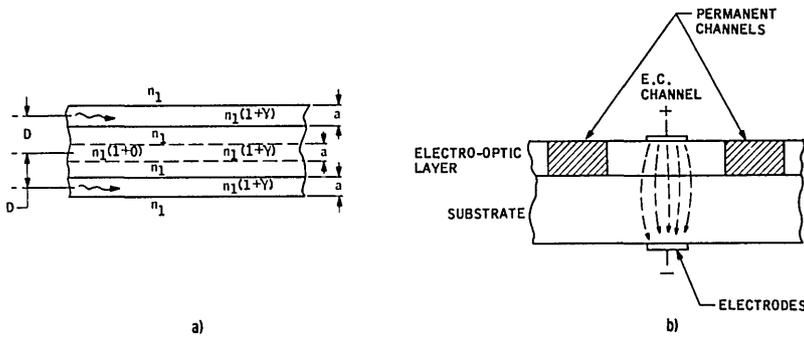


Fig. 3. (a) Simplified (ECC) for thin film guides.  $\gamma$  is the percentage change of the index of refraction in the permanent guides.  $\gamma$  is also taken as the percentage change due to the electrooptic effect. (b) Another possible configuration for an ECC.

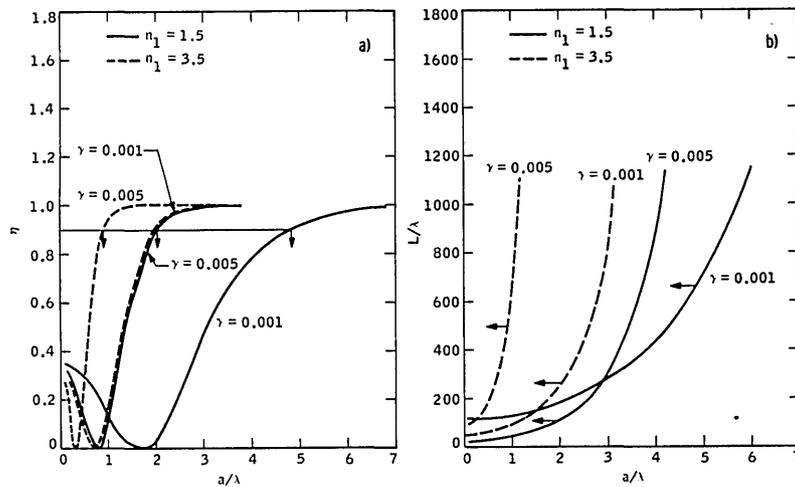


Fig. 4. Dynamic efficiency and effective length of the ECC shown in Fig. 3(a), as a function of  $a/\lambda$ , for different values of  $n_1$  and  $\gamma$ . The value of  $D/a$  is taken equal to 1.5.

$$K = [(2s^2\delta)/(s^2 + \delta^2)] \{ \exp[-\delta(D - a)] / (\kappa a) \}$$

and

$$K' = [(2s^2\delta)/(s^2 + \delta^2)] \{ \exp[-\delta(2D - a)] / (\kappa a) \},$$

where  $a$  is the width of each channel,  $D$  is the distance between the center lines,  $\kappa$  and  $s$  are, respectively, the propagation constants along and perpendicular to the propagation direction in the coupler guides. The above expressions were derived for well-confined modes, but they may be used as a good approximation in the general use. Using the above expressions of  $K$  and  $K'$  we can express  $L$  and  $\eta$  as

$$L = [\pi/(2\sqrt{2})] [(s^2 + \delta^2)/(s^2\delta)] \kappa a \exp[\delta(D - a)],$$

$$\eta = \cos^2[(\pi/\sqrt{2}) \exp(-\delta D)].$$

We carried out a numerical study of the simplified coupler scheme shown in Fig. 3(a), where the guides

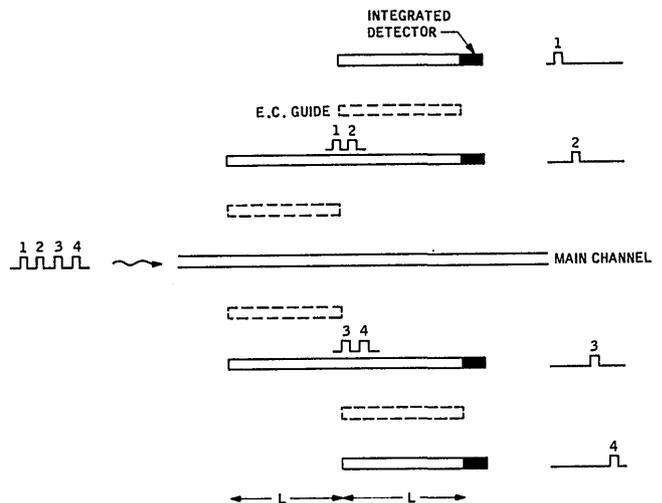


Fig. 5. Possible configuration for a 4-channel optical demultiplexer. The dashed guides are electrically controllable.

are thin film layers.  $n_1$  is the index of refraction of the substrate, and  $\gamma$  is the percentage increase generated when a voltage is applied.  $\gamma$  is also taken as the percentage index increase in the permanent guides. In Fig. 4 we plotted  $\eta$  and  $L/\lambda$  as a function of  $a/\lambda$  for a fixed value of  $D/a$ , where  $\lambda$  is the optical wavelength in vacuum. It is clear that high efficiency ( $\eta \geq 90\%$ ) is possible at high frequencies, and larger values of  $n_1$  or  $\gamma$  lead to a wider region of high efficiency. On the other hand, the value of  $L/\lambda$  increases with  $a/\lambda$ . To illustrate, let us choose  $n_1 = 3.5$ ,  $\gamma = 0.001$ , and  $\lambda = 1.15 \mu$ . For an efficiency of 90%,

$$a = 2.3 \mu, D = 3.45 \mu, \text{ and } L = 304 \mu.$$

If we want to increase the efficiency to 99%,

$$a = 3.4 \mu, D = 5.1 \mu, \text{ and } L = 920 \mu.$$

For shorter wavelengths, the above numbers are proportionally smaller. In Fig. 3(b) another scheme for an electrically controllable coupler is shown.

#### IV. Conclusion

The above results show that some simple function and efficient dynamic switching are possible over relatively short distances. The dynamic coupler may play a central role in complex optical networks and multiplexers/demultiplexers. A scheme for a 4-channel multiplexer/demultiplexer is shown in Fig. 5.

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