plowed ahead 24 m. during the Winter of 1959–60, pushing up a small moraine. This advance was nearly destroyed by increased melting during the Summer.

Department of Electrical Engineering, A. E. HARRISON
University of Washington,
Seattle, Washington
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SIR, *Growth rate of sea ice*

During the course of field work at Thule, Greenland, in conjunction with an Air Force Cambridge Research Center sea ice project, ice thicknesses were measured in a protected part of North Star Bay, an arm of Baffin Bay, over the greater part of two ice growth seasons. Measurements were made at several locations both on the sea ice and on pools that were artificially opened up in it daily for the first month after freeze-up of the 1956–57 season. Snow cover was slight (several centimeters or less) throughout the period of most intensive measurement and averaged less than 30 cm. in March for both years of record. All air temperatures were measured at least 6 m. above the ice surface. Ice thicknesses were measured in a small area on a single uniform sheet. There existed no complications such as rafting, above freezing temperatures, runoff, or large and variable amounts of snow. The resulting curve is particularly well documented in the initial parts where data have previously been sparse. Two important conclusions may immediately be drawn from the data:

1. The growth curve cannot be fitted by a single, simple power law such as results from the pure ice growth theories of Stefan [1] or Tamura,[2] which have been extensively applied to sea ice.

2. The relatively small scatter indicates that for a first approximation time and temperature may be compounded into a single parameter, namely, the exposure (degree-days or degree-hours of frost), and this is the most important parameter controlling the thickness of ice forming under a wide variety of conditions even on such a temperature-sensitive material as sea ice. This commonly accepted procedure needs statistical justification since a proper theory of sea ice growth requires knowledge of the actual thermal history, as well as such meteorological variables as humidity, wind velocity and radiation.

Although the principal purpose of this note is to make available the thickness data it might be useful to develop briefly the pertinent theory. The growth equation may be derived by equating the latent heat of ice formation to the heat removed from the ice to the air:

$$\int L\rho de = \int \frac{k\,(\theta_s - \theta_f)}{e}\,dt = \int \frac{h(\theta_a - \theta_s)}{e'}\,dt, \tag{1}$$

where L = latent heat, θ_a = air temperature,
ρ = ice density, θ_s = ice surface temperature,
e = ice thickness, θ_f = freezing point,
k = ice thermal conductivity, e' = effective boundary layer thickness,
t = time, h = transfer coefficient of boundary layer.

The effective transfer coefficient, K, of the ice and boundary layer system may be written:

$$K = \frac{e+e'}{ke'+he}(hk),$$

leading to the growth equation

$$\frac{k}{L\rho}\int_0^t \Theta dt = e\left(\frac{e}{2}+\frac{1}{h'}\right), \tag{2}$$

where $\Theta = \theta_a - \theta_f$, $\int_0^t \Theta dt$ = total exposure, and $h' = \frac{h}{2e'k}$ = contact coefficient,

which is in the form of empirical growth equations of Barnes [3] and Zubov,[4] and is identical to the often overlooked theoretical result of L. V. King.[3]

The contact coefficient, among other things, includes the effect of snow insulation, wind, radiation, humidity and evaporation or, generally, anything that makes the effective surface temperature of the ice different from the ambient air temperature. It is obviously difficult to evaluate theoretically.[3, 5] If the effects of wind, radiation and evaporation are ignored, h is simply the thermal conductivity of the snow layer and e' is its thickness. The other constants are easy to evaluate for a pure, homogeneous substance with constant properties and freezing point, such as fresh-water ice.

The continuous phase changes taking place in the interior of sea ice lead to heterogeneity and cause the effective physical properties to be time and temperature dependent. However, since conductivity and effective latent heat are approximately the same function of temperature, it might be expected that the use of average values to retain the simplicity of equation (2) would not be too dangerous. Any errors due to this approximation will be partially absorbed by the empirically determined contact coefficient.

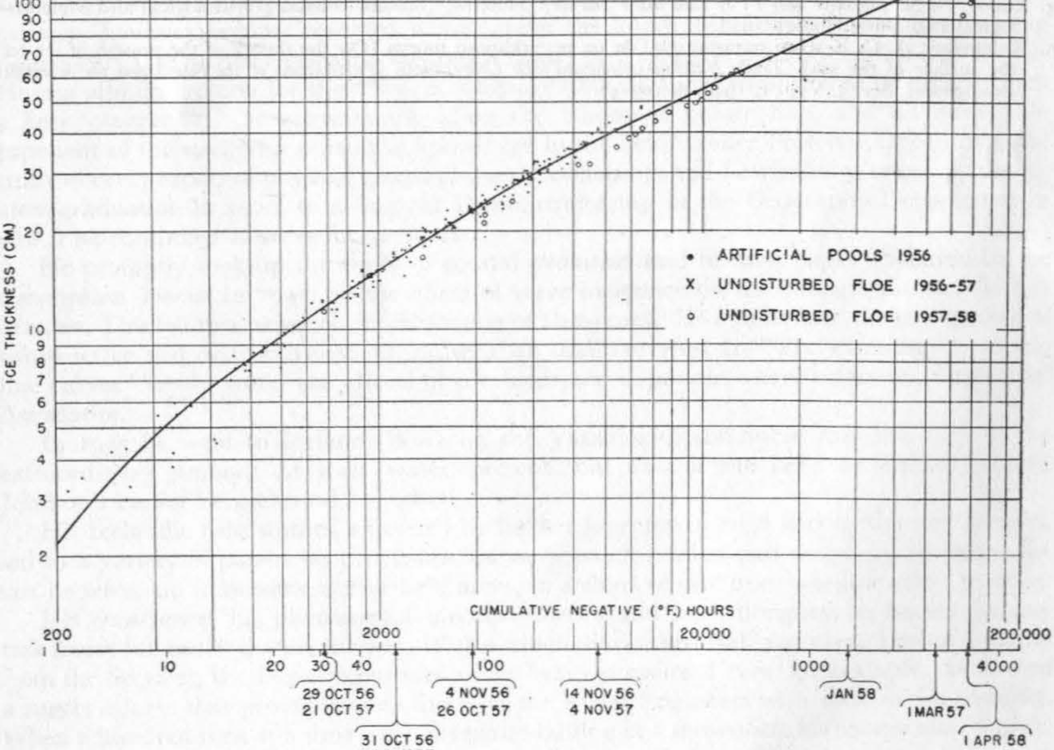

Fig. 1. Sea ice growth curve; Thule, Greenland

In practice, however, the coefficient multiplying the exposure is always lower than even extreme choices of thermal properties would warrant, indicating the complexity of the factors controlling sea ice growth. It is dangerous to attempt to determine thermal conductivity from sea ice growth curves.

An equation of the form of equation (2) can be fitted very well to observed growth curves, both for fresh-water ice and sea ice. The semi-empirical constant h' varies approximately from $0\cdot 04$ to $1\cdot 00$ cm.$^{-1}$, the smaller values applying to oceanic conditions. The solid line in Figure 1 is an eye fit to the 1956–57 data. An approximate ice growth equation for ice less than 80 cm. thick is given by

$$e^2+5\cdot 1\ e = 6\cdot 7\int \Theta dt$$

in units of cm. °C. days.

Arctic Institute of North America,
1530 P Street, N.W.,
Washington, 5, D.C.
11 July 1960

DON L. ANDERSON

Present address:
Division of Geological Sciences,
California Institute of Technology,
Pasadena,
California

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