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Calculation of beta-decay half-lives of proton-rich nuclei of intermediate mass

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We present the results of a calculation of the beta-decay half-lives of several proton-rich even-even nuclei of intermediate mass: ^{74}Sr, ^{76}Sr, ^{78}Zr, ^{80}Zr, ^{84}Mo, ^{86}Mo, ^{88}Ru, ^{90}Ru, ^{92}Pd, and ^{96}Cd. The calculation is based upon the random phase approximation with the quasiparticle formalism and takes into account the residual particle-particle interaction.

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Ever since Takahashi *et al.* [1] calculated in 1973 estimates for the beta-decay half-lives of virtually all beta-unstable nuclei, there has been a large effort to improve these estimates for neutron-rich nuclei because of applications in *r*-process theory and in the fate of fission products [2]. The theoretical effort to improve the estimates of half-lives of proton-rich nuclei has not been commensurate, although Hirsch *et al.* [3] and Muto *et al.* [4] presented calculations for light nuclei ($Z \leq 30$). (See, however, also Refs. [5–7].) In this Brief Report we present the results of an effort to improve the estimates for half-lives of several proton-rich nuclei of intermediate mass. These half-lives play a role in *rp*-process theory [8], that is, the process in which protons are quickly added onto C, N, O, and other "metals" with intervening fast positron decays resulting in heavy proton-rich nuclei. This process occurs in certain astrophysical contexts in which the temperature is greater than about 10^8 K. In particular, this process is predicted to occur in massive stars with degenerate neutron cores (if they exist) [9], and information about the longer-lived (> 1 s) beta-unstable nuclei would allow one to predict the nuclear abundances on the surfaces of these stars [10]. For this reason we undertook the calculation of half-lives of some proton-rich even-even nuclei of intermediate mass.

We are interested in even-even nuclei which have 0^+ ground states, so that the calculation is relatively simple. The positron-decay half-life $t_{1/2}$ is given by the following formula:

$$\frac{1}{t_{1/2}} = \sum_m \frac{B(\mathrm{GT})_m g_A^2}{6160\ \mathrm{s}} f(\Delta E_m, Z) , \quad (1)$$

where m labels the accessible 1^+ states in the daughter nucleus, $B(\mathrm{GT})_m$ is the Gamow-Teller β^+ strength (equivalent to $|\langle m|\sigma\tau^+|i\rangle|^2$ in this case), g_A is the axial-vector-current coupling constant (which we set to 1.25), and $f(\Delta E_m, Z)$ is the Fermi function (including Coulomb and relativistic corrections), which describes the size of phase space.

We obtain energy levels of the daughter nucleus and evaluate B(GT) using the random phase approximation based on the quasiparticle formalism (QRPA). (The generalization of the QRPA to charge-changing modes is due to Halbleib and Sorensen [11]. Particle-particle interactions were first included in the QRPA by Cha [12].) The formalism is described in detail in Vogel and Zirnbauer [13] and in Engel *et al.* [14]. In these papers the authors use the δ force as the residual interaction and describe the following four parameters: α_0, α_1 (the particle-hole interaction constants in the $S=0$ and $S=1$ channels, respectively), α_0', and α_1' (the particle-particle interaction constants). Although these constants are theoretically related, the authors present an argument that they can be treated independently in this calculation. Using the values given in Ref. [14], we set $g_{\mathrm{pair}} = -270$ MeV fm^3 when we solve the BCS equations, and we set $\alpha_0 = -890$ MeV fm^3 and $\alpha_1 = -1010$ MeV fm^3 for the RPA portion of our calculations. Because we are looking at positron decay of proton-rich nuclei, our results do not depend on α_0' in the RPA calculations. Our results do, however, depend strongly on the value of α_1', so we must take care to choose it carefully.

We divide the nuclei into two categories, those with $74 \leq A \leq 80$ and those with $80 < A \leq 96$. For the heavier nuclei in our study, we calibrated α_1' using the known decay half-lives of ^{88}Mo, ^{90}Mo, ^{92}Ru, and ^{94}Pd. In order to calculate these half-lives, we identified the lowest-lying 1^+ state in the daughter nucleus with the ground state given by the QRPA calculation. (This determines the values of ΔE_m, used in the phase space integrals.) Our calculation is for positron decay only, i.e., no electron capture. In three of the calibration nuclei positron decay dominates over electron capture; however, 75% of the decay of ^{90}Mo is due to electron capture. In that case we, therefore, use the proper partial decay rate. In our calculation, almost all ($\gtrsim 90\%$) of the predicted decays occur into the lowest-lying 1^+ state. Figure 1 shows the $\log_{10}$ of the ratio of calculated positron-decay half-life to experimental half-life versus α_1'. From this figure we see that α_1' may be anywhere within a window from -324 to -333 MeV fm^3 and yield values of half-lives correct to within a factor of 3. A value of $\alpha_1' = -329$ MeV fm^3 yields a least χ^2_{red} equal to 0.22, where

$$\chi^2_{\mathrm{red}} = \left[\frac{1}{3} \sum [\log_{10}(T_{\mathrm{calc}}/T_{\mathrm{exp}})]^2 \right]^{1/2} .$$

Thus we predict that our results in Table I are accurate to about a factor of $10^{0.22} = 1.7$. By comparison, the χ^2_{red}

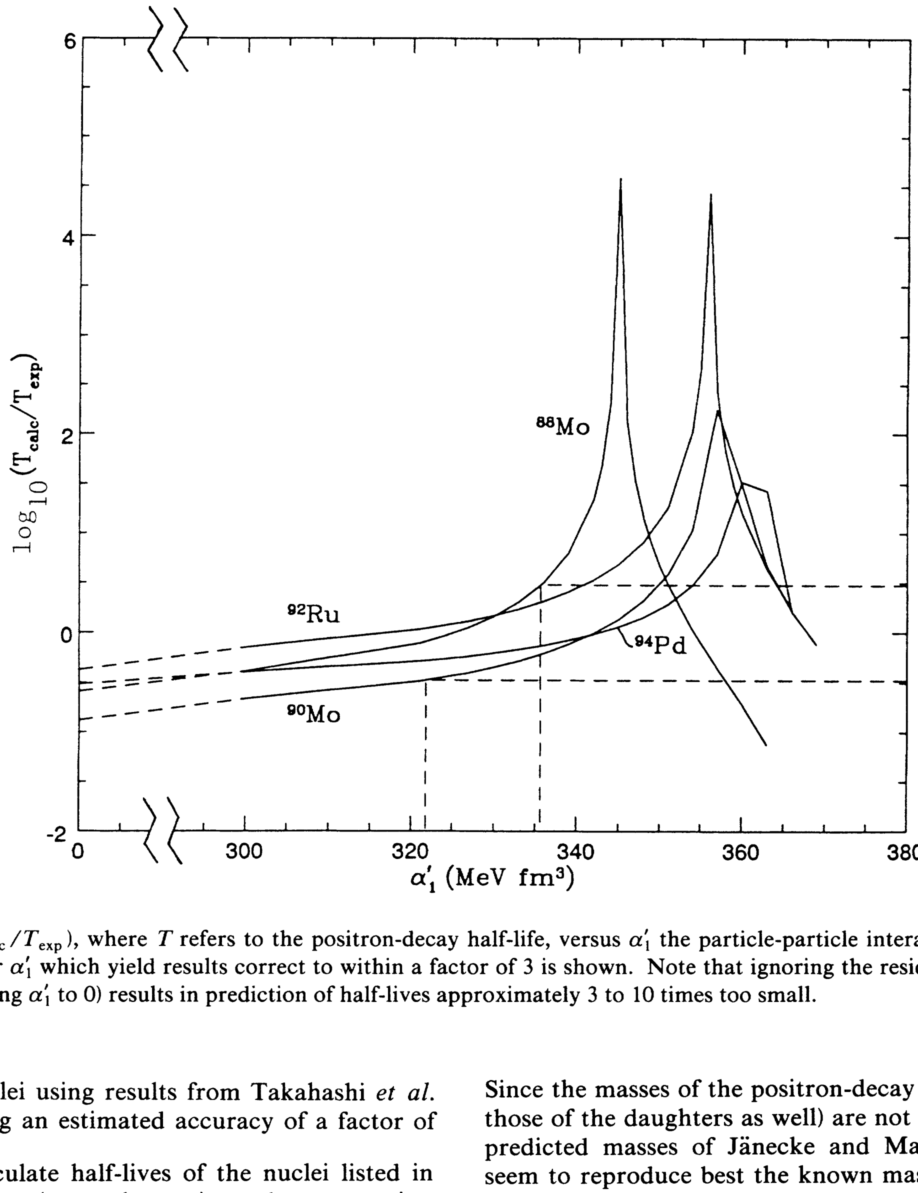

FIG. 1. $\log_{10}(T_{calc}/T_{exp})$, where T refers to the positron-decay half-life, versus α_1' the particle-particle interaction strength. The window of values for α_1' which yield results correct to within a factor of 3 is shown. Note that ignoring the residual particle-particle interaction (i.e., setting α_1' to 0) results in prediction of half-lives approximately 3 to 10 times too small.

for these four nuclei using results from Takahashi *et al.* [1] is 0.59, yielding an estimated accuracy of a factor of $10^{0.59}=4$.

In order to calculate half-lives of the nuclei listed in Table I, we need to know the positron-decay energies.

TABLE I. Predicted beta-decay half-lives.

Nucleus	$\Delta E_{m=0}$[a] (MeV)	Half-life[b] (s)	Takahashi *et al.* [1] half-life (s)
^{74}Sr	9.6	0.5	0.03
^{76}Sr	4.5	8.	3.
^{78}Zr	10.5	0.06	0.03
^{80}Zr	5.0	7.	3.
^{84}Mo	5.2	6.	0.8
^{86}Mo	3.9	90.	16.
^{88}Ru	5.8	1.2	0.8
^{90}Ru	4.7	16.	5.
^{92}Pd	6.8	0.9	0.4
^{96}Cd	8.0	0.6	0.3

[a] This is the maximum total energy of the positron for a transition to the lowest 1^+ daughter state.

[b] The estimated accuracy is a factor of 2. See the explanation in the text.

Since the masses of the positron-decay parents (and often those of the daughters as well) are not known, we use the predicted masses of Jänecke and Masson [15]. (These seem to reproduce best the known masses of proton-rich nuclei.) We set $\Delta E_{m=0}$, that is, the maximum total energy of the positron, to the difference of parent and daughter masses less 0.2 MeV. The 0.2 MeV represents a typical value for the energy difference between the ground state and the lowest-lying 1^+ state of the daughter nucleus. (For these decays, however, $\Delta E_{m=0}$ is large enough that the correction is trivial.) The results are shown in Table I. As stated in the previous paragraph, these values are accurate to within a factor of about 2. Electron capture is negligible in these nuclei, contributing less than 3% because of the large decay energies involved. (See Ref. [16].)

Similarly we use the known half-lives of ^{70}Se, ^{72}Kr, ^{74}Kr, and ^{80}Sr to calibrate α_1' and calculate half-lives for several nuclei with $A \leq 80$. In this case we obtain $\alpha_1' = -327$ MeV fm^3 for the best fit, yielding a least χ^2_{red} equal to 0.32. The results are also shown in Table I. We estimate that the results are accurate to within a factor of about $10^{0.32}=2$, and again electron capture is negligible.

Also shown in Table I are the predicted half-lives of

Takahashi *et al.* It is encouraging that our results are consistent with theirs, which are calculated by a different method; most of the difference is due to different Q values (i.e., $\Delta E_{m=0}$), especially in the case of ^{74}Sr.

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