Chapter 6

USE OF LONG-PERIOD SURFACE WAVES FOR DETERMINATION OF ELASTIC AND PETROLOGICAL PROPERTIES OF ICE MASSES

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Introduction

Elastic wave propagation has proved to be a powerful tool in the study of the mechanical properties and thicknesses of ice masses. The anisotropy, heterogeneity, and departure from perfect elasticity that plague conventional static tests can all be studied in detail by seismic techniques that have been developed for use both in the field and in the laboratory. Two types of elastic waves can be transmitted by an unbounded, isotropic, elastic media: the dilational and distortional. The velocities of these two waves, along with the density, completely describe the elastic behavior of an extended elastic body. In an inhomogeneous, anisotropic, and imperfectly elastic solid body, these basic wave types are modified. Bounded media will transmit, in addition, guided waves that can be used to give further information on elastic properties. The combined use of body wave and guided wave data permit a detailed description to be made of the mechanical properties of a bounded body, such as a sea ice sheet or a glacier.

The same battery of elastic waves can be applied in the laboratory, where the seismic, or ultrasonic, method becomes a sensitive analytical tool for the determination of composition and structure.

Theory

The basic theory of elastic wave propagation in floating ice sheets has been developed by Ewing and Crary (1), Press and Ewing (2), and Satô (3), under the assumptions of isotropy, homogeneity, and unattenuated propagation. With slight modification these theories can be used to interpret most of the features observed on seismograms taken on sea ice. By taking into account the departure of sea ice from the ideal model assumed, virtually the entire seismogram is interpretable. Ewing and Crary took into account the presence of gravitational forces that make it possible to apply their results to the low-frequency waves generated by moving vehicles and taxiing aircraft. Anderson (4) has developed the complete theory of elastic wave propagation in floating ice sheets, including the effect of anisotropy and space-time attenuation.

Surface waves are a form of guided wave where the guiding is controlled by the free surface. Each wave averages the properties along its path and the longer wavelengths sample deeper. Very long wavelengths sample the entire body under consideration. These waves on floating ice sheets correspond to the flexural wave. Studies on floating sea ice sheets in fact led to the discovery that long-period waves can be sensitive to microscopic details. The orientation of crystals less than a millimeter in width affects the properties of waves many tens of meters long. The theory of crystal orientation, i. e., anisotropy, on elastic wave propagation has been developed and successfully applied to the study of floating sea ice sheets. The theory has been extended to layered solid bodies such as the earth or glaciers and ice shelves. It has been applied to the earth, where it has been determined that the upper mantle displays an anisotropy of some 10%. This is attributed to the orientation of olivine crystals. The same technique can be applied to glaciers to determine the degree of crystal orientation and how it changes with depth and position on the glacier.

Figure 1 illustrates in a schematic fashion the dispersion curves of the lowest modes of elastic wave propagation in sea ice. These curves can be used to predict the sequence of arrivals from either an explosion source in the air, ice, or water, from a vehicle moving over the ice, or from ocean waves moving through an icefield. The weight of the lines is an approximate indication of the amount of energy that will appear in the seismic record; these modes with phase velocities greater than the sound velocity in air will be slightly attenuated owing to radiation of energy into the air; similarly, those modes with phase velocities greater than the speed of sound in water will be highly attenuated owing to radiation into the water. The higher modes are all strongly attenuated because of radiation into the water but must be invoked to explain some of the arrivals at small distances, such as the early arriving compressional wave. The routinely observed compressional plate waves, vertically polarized shear (SV) waves, flexural waves, air-coupled-flexural waves, and vehicle-coupled waves are all included in the above display.

Since it is the inhomogeneities, primarily the brine and air pockets, that control the properties of sea ice, it is possible to

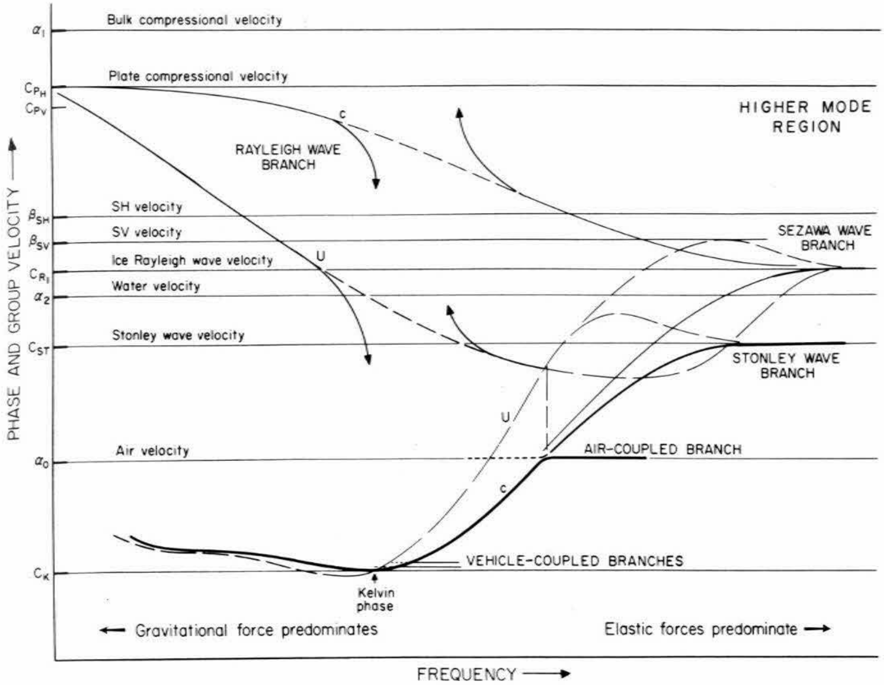

Fig. 1. Dispersion curves of the lowest modes of elastic wave propagation in sea ice.

correlate the plastic properties with the strength, and ultimately, the bearing capacity of a sea ice sheet. Using the results of a previous theoretical study (5), the author has been able to correlate elastic properties with brine content from field measurements and is presently undertaking a similar study in the laboratory, where more control is possible. The present indications are that the brine has more than just a volume effect; it also reduces elastic restoring forces by allowing interplatelet slippage.

The study of the anisotropy of sea ice is not only of theoretical interest but gives information about the location of the neutral surface, a critical parameter in bearing capacity calculations. Sea ice has both macroscopic anisotropy, controlled by the thermal gradient, and microscopic anisotropy, controlled by the crystal orientation. The second effect takes place over distances of millimeters, and the first over distances of meters, but by combining flexural wave and plate wave data, even with wavelengths of the order of hundreds of meters, it is possible to obtain a measure of the directional properties. Even on cold sea ice, this anisotropy amounts to 15%, the major part of which can be attributed to the crystal orientation. This is the reason why air-coupled flexural wave data in the past have underestimated the thickness of sea ice; anisotropy must be taken into account when an accurate interpretation of the air-coupled frequency is desired.

Ewing and Crary (1) give the following expression for long flexural waves in a floating ice sheet overlying incompressible water:

$$c^2 = \frac{(g/k) + Dk^3}{\coth(kH) + (\rho/\rho_1)\, kh} \tag{1}$$

where c is phase velocity, g is gravitational acceleration, k is wave number, D is flexural rigidity, h is thickness of ice, H is depth of water, and ρ/ρ_1 density ratio of ice to water.

For very long wavelengths, this becomes

$$c^2 = [(g/k) + Dk^3] \tanh kH \tag{2}$$

which is similar to the well-known equation giving the velocity of water waves when gravity and capillarity are both taken into account:

$$C_0 = [(g/k) + T'k] \tanh kH \tag{3}$$

The effect of the ice sheet is to increase the surface forces and to change the dispersion. Long waves see the ice only as a film on the surface. Gravity controls the long wavelengths, giving normal dispersion, while the flexural rigidity of the ice controls the short wavelengths, giving anomalous dispersion. This dual behavior gives rise to a phase velocity minimum, as shown in Fig. 1, and is one of the few places in nature where such an effect is observed. The existence of this minimum, in turn, introduces a powerful new method for determining the static deflection, and consequently, the static elastic properties of a sea ice sheet. A load placed on a floating ice sheet will cause the ice to deflect in a dish-shaped pattern, the mathematical description of which is the kei function. The displacement under the load and the "wavelength" of the deflection pattern, for a perfectly elastic plate, are controlled by the elastic properties. Until recently, the field technique has been to "shoot elevation" in the region of the load with a surveyor's level. This has been unsatisfactory because of the small deflections involved, the limited number and accuracy of the observations, the necessity for locating the level outside of the deflection region (strictly, an impossibility), and the plastic sagging of the ice that is taking phase simultaneously with the measurements. The presence of plastic sagging makes a truly static measure of ice elasticity impossible. The existence of a minimum in the phase velocity curve, however, means that a load moving sufficiently slower than this minimum will drag its static, elastic deflection pattern along with it. Thus it is possible to separate effectively the elastic and plastic deflection by moving the load slowly past a deflectometer, and then parking over the deflectometer. A load moving faster than the minimum phase velocity will be preceded by a high frequency "head" wave and followed by a lower frequency "rear" wave. This behavior can also be explained by the existence of a minimum phase velocity which requires two group (energy) velocities to be associated with each phase velocity. The frequencies of the "head" and "rear" waves are controlled by the elastic properties, and their relative amplitudes by the slope of the dispersion curves and the damping in the ice-water system. The same deflectometer can be used to measure these waves and introduces still another way to determine the elasticity of the sea ice. Instrumentation has been improved so that in a recent field trip to Thule, Greenland, it was possible to overlap the frequency ranges covered by the deflectometer and seismic techniques; this makes it possible to increase our confidence in extrapolating seismic results to bearing capacity calculations.

Summary

The general theory of elastic wave propagation in heterogeneous, anisotropic, floating or nonfloating ice bodies has been developed and applied. Since the theory includes gravitational forces, it applies to a wide range of wavelengths and brings ice-modified ocean waves and vehicle-coupled flexural waves into a consistent theory with high-frequency seismic waves. The inclusion of anisotropy into the theory makes it possible to use seismic instruments as a "petrographic microscope," making use of such phenomena as double refraction to determine crystal orientation over large volumes. This technique has been used on sea ice that is markedly anisotropic because of the crystal orientation. It can be applied to the study of glaciers to determine the degree of crystal orientation of large masses as a function of position and depth. This gives indirect evidence on the nature of the flow process and the kinds of stresses involved in glacier flow.

The form of the dispersion curve in the region where both gravitational and elastic forces are approximately equal in magnitude suggests a method for obtaining the "static" elastic constants by a nonstatic method, thus by-passing complications of creep and static (d-c) instrumentation; an instrument is described that can be used for "static" and dynamic load tests on floating ice sheets.

The theoretical basis of part of this study has been outlined by Anderson (6).

Acknowledgment

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