

Bulk Modulus-Density Systematics¹

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The elastic constants of crystals show a general tendency to increase as the mean molar volume $\langle V \rangle = \langle M \rangle / \rho$ decreases. The role of other factors, such as cation radius, crystal field effects, and anion-cation coordination, can now be discussed with available elastic constant data. For a given coordination the parameter $\psi_0 = (\rho_0 / \langle M \rangle) \Phi_0^{-1/3}$ (where ρ_0 is the zero-pressure density, $\langle M \rangle$ is the mean atomic weight, and Φ_0 is the ratio of the zero-pressure bulk modulus to the density) decreases with increasing cation radius and with cell volume per oxygen atom unless a nonspherical transition element ion, such as Fe^{2+} , substitutes for a nontransition ion, such as Mg^{2+} . The calcium effect discovered by G. Simmons is a special case of the cation radius effect. The elastic ratio Φ_0 for complex oxides is approximately a molar average of the Φ_0 of the component simple oxides. For silicates it appears that an empirical table of $\Phi_0(\text{SiO}_2)$ can be constructed for various mineral groups. Tentatively, $\Phi_0(\text{SiO}_2)$ is roughly that of α quartz for the feldspars, β quartz for olivines and pyroxenes, coesite for garnets, and stishovite for spinels.

INTRODUCTION

The elastic properties of minerals depend on a variety of parameters including bonding type, crystal structure, cation-anion relationships, density, and atomic weight. For rocks and synthetic aggregates the presence of intercrystalline effects, cracks, and pores introduce additional complications. Because of the variety of phenomena affecting the elastic properties of solids, it is surprising that so many data can be summarized by one or two parameters.

Birch [1961], Anderson [1967a, b], Anderson and Kanamori [1968], Anderson and Nafe [1965], and Simmons [1964] have discussed relationships between elastic properties, density, and mean atomic weight. It is the purpose of this communication to discuss the effect of parameters other than the mean molar volume and, in particular, to present evidence that the cation radius affects the modulus-density relationship in a systematic way. Nonspherical transition elements must be treated separately and must form an important part of the present discussion. Simmons [1964] pointed out that calcium-rich rocks did not seem to fit into a

general scheme. Calcium has the largest ionic radius of any major rock-forming element, and one purpose of the present note is to present evidence that the cation radius affects the modulus-density relationship in a systematic way. Anderson and Kanamori [1968] introduced the parameter $\Psi_0 = (\rho_0 / \langle M \rangle) \Phi_0^{-1/3}$ and showed that its average value was about 0.048 for common rocks and rock-forming minerals. They also showed (their Table 4), however, that ψ_0 tended to increase above this value for the denser oxides with high mean atomic weights. The exponent of Φ_0 is also an average and is demonstrably different for different groups of compounds, but for present purposes it is convenient to keep this parameter fixed and to investigate the behavior of ψ_0 .

CATION RADIUS AND COORDINATION EFFECTS

Figure 1 is a plot of ψ_0 versus the radius of the largest cation and, in the insert, ψ_0 versus the cell volume per oxygen atom. The parameters on the lines are cation-anion coordination numbers, and the number in brackets in the compounds is the packing index of Fairbairn [1943], which is equal to $10 \times (V_{\text{ions}}/V_{\text{cell}})$. The solid lines join compounds having the same coordination; the dashed lines join end members of solid solution series involving Fe^{2+} substitution for Mg^{2+} .

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0.0498. Therefore, a value of $\psi_0 = 0.0492$ is recommended for interpreting lower mantle seismic data. The above range in ψ_0 would correspond to an uncertainty in $\rho_0/\langle M \rangle$ of 2.5%. By using this value of ψ_0 and by taking the high-pressure form (post spinel) of Mg_2SiO_4 to have a density appropriate for a mixture of $2\text{MgO} + \text{SiO}_2$ (stishovite), the predicted Φ_0 for this hypothetical phase is 59.2 (km/sec)².

Spinel of interest in the transition region have ψ_0 's ranging from about 0.0465 to 0.0475, the higher values corresponding to iron-rich spinels. Unfortunately, Mg_2SiO_4 spinel has not been measured. In lieu of other information, a value of 0.0465 is recommended for use in the spinel portion of the mantle. This value for ψ_0 gives a Φ_0 for Mg_2SiO_4 (spinel) of 54.1 (km/sec)². In a later section the corresponding Φ_0 is estimated to be $56 \pm 10\%$.

Ultrasonic measurements of rocks and minerals presumably from the upper mantle average about 0.0481, and, again, when making gross interpretations of upper mantle seismic data, this value is recommended.

Crustal rocks and common crustal minerals average about 0.0453 for ψ_0 . Values range from 0.0414 for anorthosite to 0.0477 for alkali feldspars.

EFFECT OF FeO , CaO , AND Al_2O_3

For geophysical purposes it is much more convenient to discuss systematics in terms of composition rather than cation radii or cell

volumes. Figure 2 gives the parameter ψ_0 for various rock and mineral systems plotted versus weight per cent FeO . The data are sparse and scattered, but there is a clear tendency of ψ_0 to increase with FeO content.

For spinels and olivines, $\psi_0 \simeq 0.0463 + 3.5 \times 10^{-5}f$, where f is weight per cent FeO . For the system MgO-FeO , $\psi_0 \simeq 0.048 + 5.8 \times 10^{-5}f$. Except for the magnesium end member, pyrope, garnets behave intermediate to the above two systems and roughly satisfy $\psi_0 \simeq 0.0482 + 5.7 \times 10^{-5}f$. There are inadequate data on pyroxenes, and about all that can be said is that pyroxenes containing approximately 16% FeO have a ψ_0 from 0.0465 to 0.0486.

CaO has the opposite effect of FeO ; an increase in CaO decreases ψ_0 . The following tentative formulas are derived from the limited amount of data in Figure 3:

MgO-CaO	$\psi_0 = 0.0498 - 1.3 \times 10^{-4}c$
pyroxene	$\psi_0 = 0.0482 - 1.9 \times 10^{-4}c$
olivine	$\psi_0 = 0.0462 - 1.6 \times 10^{-4}c$
feldspar	$\psi_0 = 0.0450 - 2.1 \times 10^{-4}c$

where c is the weight per cent CaO .

The effect of Al_2O_3 is even harder to pin down with available data. There is, however, a clear increase in both Al_2O_3 content and ψ_0 from pyrope to spinel to corundum, although the effect is small. In terms of mole per cent $\psi_0 =$

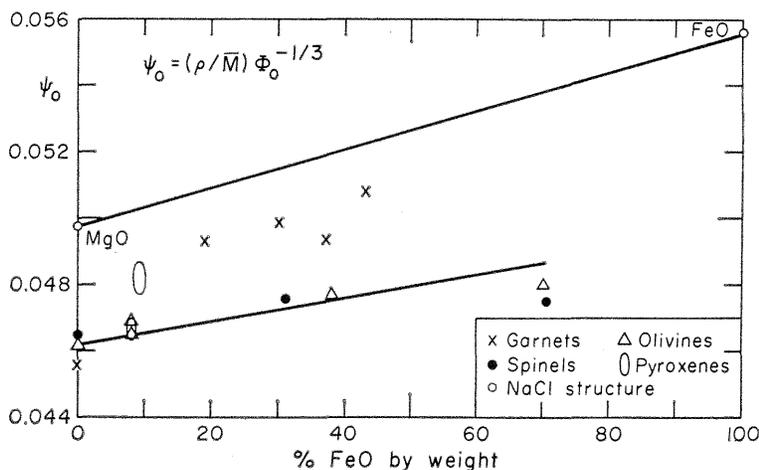
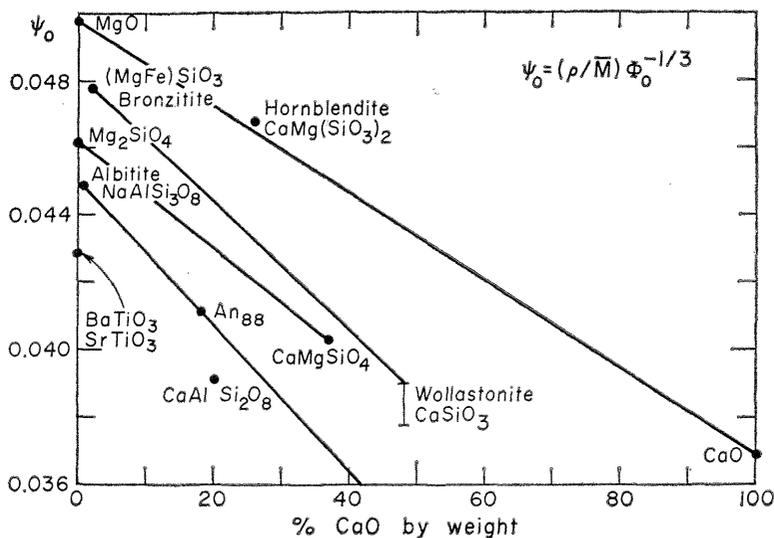


Fig. 2. Effect of iron substitution on ψ_0 .

Fig. 3. Effect of calcium substitution on ψ_0 .

0.0454 + 3.4 × 10⁻⁵a, where a is the mole per cent Al₂O₃.

ADDITIVITY OF Φ

In the course of this study it was noticed that the seismic parameter Φ_0 for oxide compounds was very nearly the same as the molecular weighted average of the Φ_0 's of the component oxides. For example, $2\Phi(\text{MgAl}_2\text{O}_4) = \Phi(\text{MgO}) + \Phi(\text{Al}_2\text{O}_3)$. This idea is tested in Table 2 where data on oxide compounds are compared with the Φ computed from the above consideration. The number of possible comparisons is very limited since both the oxide and its components must have been measured. This limits us primarily to compounds of Al₂O₃, MgO, CaO, TiO₂, and Fe₂O₃. Compounds involving FeO and NiO can be treated approximately, since crude X-ray data for the compression of these oxides are available. Unfortunately, stoichiometric FeO has not been measured, and there is reason to believe that the bulk modulus and Φ_0 of the nonstoichiometric FeO are less than appropriate for the ideal oxide. BaO and SrO can be estimated by extrapolating MgO and CaO data under the assumption that ψ_0 is linear with mean cell volume per oxygen atom. Table 1 gives the Φ_0 values used for the component oxides. The average error between the measured and computed Φ_0 's is 4%. There are errors in the meas-

urements of both the oxides and the oxide compounds, but it is not clear, of course, whether the true values would improve the agreement.

TABLE 1. Elastic Ratio of Simple Oxides

Substance	Formula	Φ_0 , (km/sec) ²	Method
α quartz	SiO ₂	14.2	u
β quartz	SiO ₂	22.3	u
Coesite	SiO ₂	45 ± 7	x
Stishovite	SiO ₂	85.4	s
Periclase	MgO	45.3	u
Wustite	FeO	26.8	x
Bromellite	BeO	73.2	u
Lime	CaO	32.2	u
—	BaO	12.9	ex
—	SrO	20.4	ex
Baddeleyite	ZrO ₂	14.4	u
Bunsenite	NiO	29.2	x
Manganosite	MnO	26.8	x
Hematite	Fe ₂ O ₃	39.3	u
Corundum	Al ₂ O ₃	63.5	u
Eskolaite	Cr ₂ O ₃	42.8	u

Notes. u = ultrasonic, x = X-ray compression; s = shock wave; ex = extrapolated.

Compiled from Anderson et al. [1968], Anderson and Liebermann [1966], Mao [1968], Drickamer et al. [1966], Anderson and Kanamori [1968], Rossi [1966], Simmons [1965], Simmons and England [1969], Ahrens et al. [1969], and personal communications from O. L. Anderson, G. Simmons, E. Schreiber, and T. Takahashi.

TABLE 2. Elastic Ratio of Oxide Compounds

Substance	Formula	Φ_0 (km/sec) ²		Error, %
		Calculated	Measured	
Spinel	MgAl ₂ O ₄	54.4	54.5	-0.2
'Spinel'	MgO·3.5Al ₂ O ₃	59.5	55.9	+6.4
Magnetite	Fe ₃ O ₄	33.2	36.0	-7.8
Ilmenite	FeTiO ₃	37.7	38.4	-1.8
(Al, Cr) ₂ O ₃	4.86 mole % Cr ₂ O ₃	62.5	61.7	+1.3
(Al, Cr) ₂ O ₃	15.1 mole % Cr ₂ O ₃	60.4	60.1	+0.5
(Al, Cr) ₂ O ₃	35.8 mole % Cr ₂ O ₃	56.1	55.9	+0.4
(Al, Cr) ₂ O ₃	59.2 mole % Cr ₂ O ₃	51.2	49.6	+3.2
(Al, Cr) ₂ O ₃	78.9 mole % Cr ₂ O ₃	47.1	43.2	+9.0
Chromite	MgCr ₂ O ₄	44.0	45.7	-3.3
Titanate	BaTiO ₃	30.7	27.0	+13.7
Titanate	SrTiO ₃	34.4	34.1	+0.9
Perovskite	CaTiO ₃	40.4*		
	MgSiO ₃ †	65.4*		
	FeSiO ₃ †	56.9*		
	Mg ₂ SiO ₄ †	58.6*		
	Fe ₂ SiO ₄ †	47.3*		
	Mg ₃ Al ₂ (SiO ₄) ₃ †	65.0*		
	Fe ₃ Al ₂ (SiO ₄) ₃ †	57.8*		

Note. Data taken from compilations given in note for Table 1 plus *McQueen et al.* [1964], *Rossi* [1966], *Lewis* [1966], and *Liu et al.* [1969].

* Predictions.

† Ultrahigh pressure phases, i.e. post spinel, etc.; SiO₂ taken as stishovite.

The agreement between the calculated and measured properties of compounds involving FeO would be greatly improved if stoichiometric FeO had a Φ_0 of about 30 (km/sec)². In any event, for the limited data available, it is certainly true that the Φ_0 for double oxides is bracketed by the Φ_0 's of the individual component oxides and is roughly a molecular average of the Φ_0 's of the component oxides. Assuming that the Φ_0 's are additive, the seismic parameter is calculated for hypothetical high-pressure phase of olivines, pyroxenes, and garnets by assuming that the SiO₂ component has a Φ_0 appropriate for stishovite, i.e. 85.4 (km/sec)². The Φ_0 of the lower mantle at standard conditions has been estimated from extrapolation of seismic data [*Anderson and Jordan*, 1969] to lie between 58.8 and 61.3 (km/sec)². For a predominately ferromagnesium mantle this estimation would imply 12 to 16 mole per cent FeO, substantially higher than thought appropriate for the upper mantle. Even more FeO is implied if the high-pressure forms of pyroxene and garnet are significant fractions of

the lower mantle. The estimated range of zero-pressure, low-temperature densities in the lower mantle is from 4.16 to 4.30 g/cm³. This value, for a predominately (Mg, Fe)₂SiO₄ mantle, implies a range of 10 to 15 mole per cent Fe₂SiO₄, in agreement with the above estimates. If the lower mantle were predominately garnet and/or pyroxene in their high-pressure phases, the seismic data indicates that they would be more iron rich than the olivine would be if the lower mantle were predominately olivine. Both pyroxenes and garnets are much richer in SiO₂ (0.50 and 0.43 mole fraction, respectively) than are olivines (0.33 mole fraction). The density and Φ_0 of the lower mantle are compatible with a range in SiO₂ of 39 to 46 mole per cent.

The approach used above for calculating the elastic ratio Φ_0 for oxides, even if valid, cannot be applied in a straightforward manner to silicates because of the many polymorphs of SiO₂. We can reverse the procedure, however, and use elastic constant data from silicates to estimate the Φ_0 of the SiO₂ component. Hopefully, the Φ_0 (SiO₂) so determined will show some con-

TABLE 3. Elastic Ratios of Silicates and Implied Elastic Ratio of SiO₂ Assuming Molar Additivity of Φ

Substance	Idealized Formula	Φ , (km/sec) ²	Φ_{SiO_2} , (km/sec) ²	Remarks
<i>Group 1</i>				
Forsterite	Mg ₂ SiO ₄	40	31	Verma [1960] Single crystal
Forsterite	Mg ₂ SiO ₄	40	29	~3% BaAl ₂ Si ₂ O ₈ , Soga and Anderson [1967]
Fayalite	Fe ₂ SiO ₄	38 ± 3	24 ± 10	*
Steatite	MgSiO ₃	70	20	~5% BaAl ₂ Si ₂ O ₈ Soga and Anderson [1967]
Wollastonite	CaSiO ₃	28	24	*
Anorthite	CaAl ₂ (SiO ₄) ₂	36	24	*
Diopside	CaMg(SiO ₃) ₂	35 ± 1	26 ± 2	*
Augite	Ca(Mg, Fe) (SiO ₃) ₂	29	19 +	FeO ignored*
Mullite	3Al ₂ O ₃ · 2SiO ₂	47	21	*
Zircon	ZrSiO ₄	43	29	*
<i>Group 2</i>				
Sillimanite	Al ₂ SiO ₅	59	55	*
Garnet	Mg ₃ Al ₂ (SiO ₄) ₃	55 ± 3	58 ± 11	Liu [1968]
Garnet	Fe ₃ Al ₂ (SiO ₄) ₃	42 ± 2	50 ± 5	Liu [1968]
Garnet	3(Mg, Fe)Al ₂ (SiO ₄) ₃	42	48 ± 2	2.43% FeO, Verma [1960]
Garnet	3(Mg, Fe, Ca)Al ₂ (SiO ₄) ₃	50 ± 1	58 ± 3	1.05% FeO, 0.3% CaO, Liu [1968]
Garnet	(MgFeMnCa) ₃ (Al Fe) ₂ (SiO ₄) ₃	43	50	Soga [1967]
Beryl	Be ₃ Al ₂ (SiO ₃) ₆	55 ± 2	44 ± 4	*
<i>Group 3</i>				
'Spinel'	Fe ₂ SiO ₄	44 ± 2	80 ± 10	Mao et al. [1969]
'Spinel'	(Mg _{0.1} Fe _{0.9}) ₂ SiO ₄	42 ± 2	70 ± 10	Mao et al. [1969]
'Spinel'	(Mg _{0.2} Fe _{0.8}) ₂ SiO ₄	45 ± 2	77 ± 10	Mao et al. [1969]
'Spinel'	Ni ₂ SiO ₄	40 ±	60 ±	Mao [1967]
'Spinel'	Mg ₂ SiO ₄ †	56 ± 6†		Spinel structure

* See notes on Tables 1 and 2.

† Predicted.

sistency in the various mineral groups so that they can be used empirically to estimate the Φ_0 of unmeasured minerals. The general philosophy is the same as constructing a table of empirical ionic radii that can be used to estimate cell volumes. The results are shown in Table 3. They divide naturally into three groups. In group 1 the $\Phi_0(\text{SiO}_2)$ averages about 25 (km/sec)². These minerals are all noncubic and have relatively open structures. The cell volumes per SiO₂ are mostly greater than 65 Å³, and the packing index is less than 6. For comparison, the Φ_0 's for α and β quartz are 14.2 and 22.3 (km/sec)², respectively. In group 2 the average $\Phi_0(\text{SiO}_2)$ is 52 (km/sec)². The $\Phi_0(\text{SiO}_2)$ for coesite is 50 ± 5 [Bassett and Barnett, 1969]. These are closer packed, mainly cubic minerals and except for sillimanite have cell volumes per

SiO₂ between 56 and 64 Å³ and packing indices of about 6.5 except for beryl. Group 3 includes only spinels, and the $\Phi_0(\text{SiO}_2)$ in this group is greater than 60 (km/sec)². The geophysically interesting iron and magnesium spinels average about 76 (km/sec)² with a fairly large uncertainty because of the uncertainty in $\Phi_0(\text{FeO})$. The Φ_0 's in group 3 are clearly larger, however, than for group 2. For comparison the Φ_0 for stishovite has been estimated to be 85.4 (km/sec)² [Ahrens et al., 1969]. From X-ray comparison data Liu et al. [1969] and Bassett and Barnett [1969] obtain Φ_0 for stishovite of 80 ± 5 and 70 ± 7 (km/sec)² (isothermal), respectively. The cell volume per SiO₂ in this group falls between 52 and 70, i.e. intermediate to the other two groups. The range is restricted to between 66 and 70 for spinels of geophysical

interest. The packing index for spinels is about 5.9 to 6.4.

Thus, the silicates can be differentiated into three groups on the basis of the implied $\Phi_0(\text{SiO}_2)$ and these $\Phi_0(\text{SiO}_2)$ are roughly comparable to $\Phi_0(\beta \text{ quartz})$ in group 1, $\Phi_0(\text{coesite})$ in group 2, and $\Phi_0(\text{stishovite})$ in group 3. The minerals in groups 2 and 3 are clearly more closely packed than those in group 1, but other arguments must be involved to explain why the Si-O bonding in spinels is apparently stronger than in the garnets. The presence of aluminum in the minerals of group 2 is probably significant.

On the basis of the range of $\Phi_0(\text{SiO}_2)$ for the spinel group the Φ_0 for Mg_2SiO_4 spinel has been predicted to be about $56 \pm 10\%$. This can be compared with the value 54 (km/sec)^2 used by Anderson [1967c], which was derived on other considerations. In Table 2 the Φ for the 'post spinel' form of Mg_2SiO_4 was estimated to be 58.6 (km/sec)^2 . This value is much smaller than the 70 (km/sec)^2 used by Anderson [1967c] in a straightforward application of the seismic equation of state. With the recommended value for ψ_0 in a previous section for high-pressure phases, the Φ_0 for Mg_2SiO_4 in the post spinel phase was found to be 59.2 (km/sec)^2 , in good agreement with the value found above. There is, therefore, presumably only a small increase in Φ as mantle minerals collapse from the spinel to the post spinel phase near 620 km.

It is clear that a great deal more ultrasonic and X-ray compression data on simple oxides and dense silicate minerals must be collected before the simple ideas expressed in this paper can be fully tested. For geophysical purposes we need the elastic constants of stoichiometric FeO , Na_2O , and K_2O . BaO and SrO are important for testing the systematics. Important minerals that need investigating are (Mg, Fe) SiO_3 pyroxenes, iron-rich olivines, and perovskites. Solid solution series involving Mg, Fe, Ca, and Al must be studied before we fully understand the compositional systematics. Polymorphism cannot be treated adequately with available data. It is important to measure the elastic constants on several polymorphs of the same material.

The alkali feldspars are even more open packed than those minerals discussed previously. Packing indices for feldspars are typically

5 and the cell volumes per oxygen atom are typically 21 to 33 \AA^3 compared with about 18 for olivine and pyroxene, 16 for garnets, and 16 to 18 for spinels. They have not been treated, owing to the lack of data on the alkali oxides. The low Φ_0 's measured on the alkali feldspars imply both that the alkali oxides, i.e., Na_2O , K_2O , must have low values for Φ_0 and that the appropriate Φ_0 for SiO_2 must also be low. For example, if $\Phi_0(\text{SiO}_2)$ for $\text{NaAlSi}_3\text{O}_8$ is 14.2 (km/sec)^2 , the value for $\alpha \text{ quartz}$, then $\Phi_0(\text{Na}_2\text{O})$ is 14 (km/sec)^2 . A correspondingly low value is implied for K_2O .

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REFERENCES

- Ahrens, T. J., D. L. Anderson, and A. E. Ringwood; Equation of shocked silicates and oxides, of states and crystal structures of high-pressure phases, *Rev. Geophys.*, 7, in press, 1969.
- Anderson, D. L., A seismic equation of state, *Geophys. J.*, 13, 9-30, 1967a.
- Anderson, D. L., Latest information from seismic observations, chapt. 12, *The Earth's Mantle*, edited by T. F. Gaskell, Academic Press, New York, 1967b.
- Anderson, D. L., Phase changes in the upper mantle, *Science*, 157, 1165-1173, 1967c.
- Anderson, D. L., and T. Jordan, The composition of the lower mantle, *Phys. Earth and Planetary Interiors*, in press, 1969.
- Anderson, D. L., and H. Kanamori, Shock wave equations of state for rocks and minerals, *J. Geophys. Res.*, 73, 6477-6502, 1968.
- Anderson, O. L., and R. C. Liebermann, Sound velocities in rocks and minerals, *VESIAC State-of-the-Art Rept.*, 7885-4-X, Institute of Science and Technology, University of Michigan, Ann Arbor, 1966.
- Anderson, O. L., and J. E. Nafe, The bulk modulus-volume relationship for oxide compounds and related geophysical problems, *J. Geophys. Res.*, 70, 3951-3963, 1965.
- Anderson, O. L., E. Schreiber, R. C. Liebermann, and N. Soga, Some elastic constant data on minerals relevant to geophysics, *Rev. Geophys.*, 6, 491-524, 1968.
- Bassett, W. A., and J. D. Barnett, Isothermal compression of stishovite and coesite to 100 kilobars by X-ray diffraction (abstract), *Trans. Am. Geophys. Union*, 50, 312, 1969.

- Birch, F., Composition of the earth's mantle, *Geophys. J.*, *4*, 295-311, 1961.
- Drickamer, H. G., R. W. Lynch, R. L. Clendenen, and E. A. Perez-Albuerne, X-ray diffraction studies of the lattice parameters of solids under very high pressure, *Solid State Phys.*, *19*, 135-228, 1966.
- Fairbairn, H. W., Packing in ionic minerals, *Bull. Geol. Soc. Am.*, *54*, 1305-1374, 1943.
- Lewis, M. F., Elastic constants of magnesium aluminate spinel, *J. Acoust. Soc. Am.*, *40*, 728-729, 1966.
- Liu, L., Isothermal compression of magnesium garnets up to 300 kilobars and its implication to the earth's mantle, M.S. thesis, Department of Geological Sciences, University of Rochester, N. Y., 1968.
- Liu, L. G., T. Takahashi, and W. A. Bassett, Compression of stishovite and magnesium ilmenite at 25°C (abstract), *Trans. Am. Geophys. Union*, *50*, 312, 1969.
- Mao, H.-K., The pressure dependence of the lattice parameters and volume of ferromagnesian spinels, and its implications to the earth's mantle, Ph.D. thesis, University of Rochester, N. Y., 1967.
- Mao, H.-K., T. Takahashi, W. A. Bassett, J. S. Weaver, and S. Akimoto, Effect of pressure and temperature on the molar volumes of wustite and of three (Fe·Mg)₂ SiO₄ spinel solid solutions, *J. Geophys. Res.*, *74*, 1061-1069, 1969.
- McQueen, R. G., J. N. Fritz, and S. P. Marsh, On the composition of the earth's interior, *J. Geophys. Res.*, *69*, 2947-2966, 1964.
- Rossi, L. C., The elastic properties of oxide solid solutions; The system Al₂O₃-Cr₂O₃, *Monthly Rept. 363*, S.U.N.Y. College of Ceramics, September 1966.
- Simmons, G., Velocity of compressional waves in various minerals at pressures up to 10 kilobars, *J. Geophys. Res.*, *69*, 1117-1121, 1964.
- Simmons, G., Single crystal elastic constants and calculated aggregate properties, *J. Grad. Res. Center*, *34*, Southern Methodist University, Dallas, Texas, 1965.
- Simmons, G., and A. W. England, Universal equations of state for oxides and silicates, in press, 1969.
- Soga, N., The elastic constants of garnet under pressure and temperature, *J. Geophys. Res.*, *72*, 4227-4234, 1967.
- Soga, N., and O. L. Anderson, High-temperature elasticity and expansivity of forsterite and steatite, *J. Am. Ceram. Soc.*, *50*, 239-242, 1967.
- Verma, R. K., Elasticity of some high-density crystals, *J. Geophys. Res.*, *65*, 757-766, 1960.

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