Scattering of Fast Electrons

In a private communication, Professor E. J. Williams has suggested that the electron scattering data recently reported in this journal[1] could be profitably compared with his latest computations on plural and multiple scattering. For comparison with cloud chamber observations the distribution in angle projected on a plane containing the initial direction of motion must be known. According to Professor Williams the projected plural and multiple scattering for fast electrons ($\beta=v/c\approx 1$) is Gaussian with an arithmetic mean projected angle given by:[2]

$$\alpha_m=k/W=(Ze^2N^{1/2}t^{1/2})(8.10+0.56\log_{10}\{Z^{4/3}Nt/Z_{\rm Pb}{}^{4/3}N_{\rm Pb}\})/W \quad (1)$$

$$=106/W \text{ degrees} \quad (1')$$

where t is the thickness, Z the atomic number, N the number of nuclei per cc of the scatterer and W is the kinetic plus mass energy of the electron. The numerical results, here and in what follows, are given for a lead scatterer of thickness $t=0.015$ cm as employed in the experiments and for W in Mev. In a form convenient for the treatment of the scattering of electrons of different energies we have:

$$P_M(W\alpha)d(W\alpha)=(2/\pi k)\exp(-(W\alpha/k)^2/\pi)d(W\alpha) \quad (2)$$

$$=0.006\exp(-(W\alpha/188)^2)d(W\alpha). \quad (2')$$

The first term in brackets in k appears squared in the expression for the projected single scattering[1] as follows:

$$P_S(W\alpha)d(W\alpha)=4\pi(Z^2e^4Nt)(W\alpha)^{-3}d(W\alpha) \quad (3)$$

$$=2840(W\alpha)^{-3}d(W\alpha) \quad (3')$$

while the second term is a statistical factor yielded by the calculation of plural and multiple scattering from single scattering. Professor Williams also points out that the transition from multiple to single scattering ($P_M=P_S$) occurs at $\sim 4\alpha_m$, this result depending only on the statistical factor discussed above. The single scattering contributes only about 4 percent to the mean scattering angle.

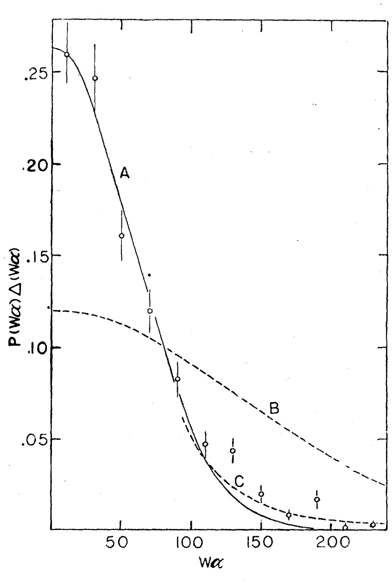

FIG. 1. Experimental and theoretical scattering of fast electrons in a lead lamina of thickness 0.015 cm. *A*, Empirical multiple scattering. *B*, Theoretical multiple scattering. *C*, Theoretical single scattering.

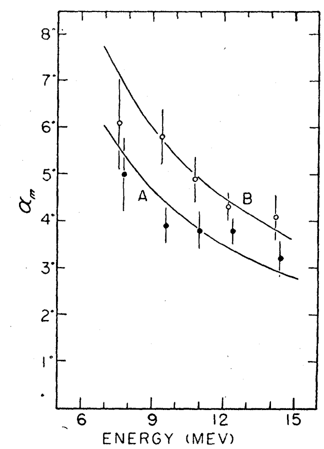

FIG. 2. Mean projected angles of scattering of fast electrons in a lead lamina of thickness 0.015 cm. *A*, For only electrons with $W\alpha\leq 120$ Mev degrees. *B*, For all electrons.

In Fig. 1 the fraction of 368 electrons (238 negatives, 130 positives) scattered by an 0.015 cm lead lamina into intervals $\Delta(W\alpha)=20$ Mev degrees is compared with equations (2′) (curve *B*) and (3′) (curve *C*). The electrons ranged in energy from 5 to 17 Mev with a mean total energy at approximately 11 Mev. A Gaussian curve (A) with $\alpha_m=45/W$ is found to fit the small angle scattering within the statistical fluctuations. The observed mean angle is thus smaller than the theoretical by a factor of 2.3 (compare (1′)). It is to be noted that any random error in measurement will tend to increase the observed mean angle. The large angle scattering is given qualitatively by the theoretical single scattering as pointed out in the first publication.[1] In Fig. 2 the observed mean angle for all electrons (open circles) and the observed mean angle for only those electrons with $W\alpha\leq 120$ Mev degrees (full circles) as functions of W are shown. The latter points fit the equation $42/W$ (curve *A*) whereas the former agree well with $55/W$ (curve *B*). The large angle scattering thus contributes approximately 25 percent of the mean angle of scattering. As this large angle scattering apparently is single, the single scattering is seen to become important at a critical angle smaller than that given by theory. These results indicate either that the statistical factor found from the plural and multiple scattering computations or the theoretical single scattering at small angles is too large. Professor Williams estimates his computational errors to be not more than three or four percent in the extreme and points out that the theoretical single scattering should hold especially for small angles as in this region the theory is independent of Dirac's treatment of spin. The preliminary experimental observations are now being supplemented by experiments with scatterers of different thickness in an attempt to discover the source of the discrepancy with theory.

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[1] Fowler and Oppenheimer, Phys. Rev. **54**, 320 (1938).

[2] Now in publication. We are greatly indebted to Professor Williams for his kind permission to use the results of his computations before publication.