The Interiors of the Terrestrial Planets[1]

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Abstract. Conclusions regarding the internal constitution of the terrestrial planets are dependent on the assumption as to the nature of the earth's core. It has previously been supposed that if the terrestrial planets, Earth, Venus, and Mars, are of similar composition the material of the core must represent a phase change, but if the core material is chemically distinct the planets must differ in over-all chemical composition. An equation of state for the mantle and core based on recent free oscillation and shock wave data is used in developing models of the terrestrial planets. It is demonstrated that Earth, Venus, and Mars can be satisfied with the hypothesis of chemical uniformity and a chemically distinct iron-rich core, provided that the external radius of Mars is about 3310 km. The radius of Mars could be as large as 3325 km and could differ only slightly from the gross composition of the earth, i.e. 2% less iron. Astronomical data indicate that Mars must be an almost homogeneous body, but compositional identity with the earth can be maintained by mixing mantle and core material.

Introduction. The variation in the mean densities of the terrestrial planets is usually taken as a compelling argument for the inhomogeneity of this part of the solar system. *Urey* [1952] ascribed the general decrease of densities from Mercury (through Earth, Venus, and Mars) to the moon to a variation in the proportions of the silicate and metallic phases within the different planets. The moon and Mercury are special cases, and on the basis of their mean densities alone must certainly differ in composition from the remaining terrestrial planets. *MacDonald* [1962] presented detailed calculations supporting the view that these planets differ markedly in composition, both in the abundances of heavy elements and radioactive elements, substantiating Urey's earlier conclusions. MacDonald stresses that 'any theory of the origin of the solar system must take into account chemical differences among the inner planets.'

Ramsey [1948], *Bullen* [1949, 1957], *Lyttleton* [1963], and *Levin* [1964] have maintained that the terrestrial planets have the same composition. However, their conclusion requires that their cores, if present, be composed of a high-density modification of silicates and not of iron-rich alloys differing in chemistry from the material of the mantle. The presence or absence of a core in their view is governed entirely by pressures in the deep interior.

It thus appears that opinion regarding the terrestrial planets is divided as follows:

1. If the earth's core is chemically distinct from the mantle—an iron-rich alloy for example—the terrestrial planets cannot all have the same composition. This has important bearing on the formation and history of these planets and the composition and homogeneity of the primordial dust cloud.

2. If the terrestrial planets all have the same over-all composition, their cores result from a pressure phenomenon and are not chemically distinct.

The problems associated with nonhomogeneity (differentiation) of the iron-silicon region of the solar system apparently are the major sources of support for the phase-change core hypothesis. Mars has been used as a critical test because of the difficulty of reconciling the hypotheses of chemical identity with the earth and a chemically distinct core for the earth. We will show that it is possible to construct models for Venus and Mars that are identical in composition to the earth without violating the hypothesis of a chemically distinct, i.e. iron-rich, core. In particular, we will investigate the following classes of planetary models:

1. The planets are all similar to the earth, having a chemically distinct mantle and core as

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the major internal subdivisions. The density in each subdivision is determined by pressure alone. The location of the core-mantle boundary is arbitrary and determines the over-all 'chemical composition,' i.e., mass ratio of core to total planetary mass. The astronomical data, mass and moment of inertia, determine the size of the core for actual planets satisfying these assumptions. The terrestrial planets, then, all have different chemical composition.

2. The planets are all identical in composition to the earth but differ in the distribution of heavy elements. Heavy material from the core is mixed into the mantle in order to satisfy the available moment-of-inertia data for the planets. The resulting planets all have cores of identical composition but differing in relative mass. The mantles of these planets have different compositions from the earth's mantle but the over-all composition of the planet is the same as that of the earth. The cores of the terrestrial planets can then be envisaged as being formed by iron draining downward from the mantle, and their size is an indication of the evolutionary state of the planet.

3. The planets are all identical in composition to the earth but differ in the distribution of the lighter compounds, i.e. the silicates. Mantle material is mixed with the core, thereby increasing the size of the core and decreasing its density. The mantles of the resulting planets are all identical in composition but the cores are not. The over-all compositions of the planets are identical. The cores of these planets can be considered to have formed by the rejection upward of the lighter silicates and oxides.

History of the controversy. Before 1948 it was widely held that the core of the earth consists largely of iron or nickel-iron. On various grounds this view was challenged by *Kuhn and Rittman* [1941], *Kronig et al.* [1946], *Ramsey* [1948], and *Bullen* [1949]. In spite of the persuasive arguments for an iron-rich core by *Birch* [1952, 1961] and *Wildt* [1961], there is apparently much sympathy for the view that the earth's core represents a phase change of mantle material [*Beloussov,* 1962; *Lyttleton,* 1963; *Bullen,* 1952, 1963, p. 248; *Levin,* 1964].

In 1937, *Jeffreys* [1937] and *Bullen* [1963, p. 247] demonstrated that under certain conditions, including a chemically distinct core, the over-all compositions of Earth, Mars, and Venus must be widely different. The mean densities of the planets were satisfied by starting with planetary models having the same ratio of core mass to total planetary mass as the earth and then reducing the size of the core until the desired mean densities were achieved. Each planet would then have a different composition, and the hypothesis of chemical uniformity—actually geometric uniformity—would be violated. Calculations of this type were also made by *Jobert* [1962] and *MacDonald* [1962].

On the other hand, *Ramsey* [1948] assumed that the terrestrial planets had the same primitive composition and that any theory fails which does not satisfy this condition. Lacking pertinent data or a requisite physical theory, Ramsey attached great significance to the astronomical test of his hypothesis. Using *Bullen's* [1940] earth density model A as a prototype, in which it is assumed that except at discontinuities the increase of density with depth is due entirely to adiabatic compression, he constructed a mass–mean density relation for terrestrial planets. Assuming that density depends on pressure alone, Ramsey concluded that the observed mean densities of the planets provided strong support of a pressure-induced transformation at the core-mantle boundary, notwithstanding the 10% discrepancy with Mars (Figure 1).

Bullen [1949, 1957] repeated these calculations with a newer earth density model B based on the hypothesis that the incompressibility and its gradient with pressure varies smoothly with pressure. Bullen achieved better agreement with Venus and Mars than Ramsey did and was apparently convinced that the earth's outer core was a phase change of the mantle and not a chemically distinct iron alloy. *Lyttleton* [1963] and *Levin* [1964] have reopened the question of a pressure-induced phase change at the core-mantle boundary and have concluded that the terrestrial planets are similar in composition to the earth. *Bullen* [1963, p. 249] points out that Lyttleton's theory of mountain building 'revives interest in the view that the outer core is a high density modification of the material of the mantle.'

Equations of state. For the construction of planetary models, equations of state for mantle and core material are needed. In previous studies the only constraints on the density distribution in the prototype earth model were the mass and

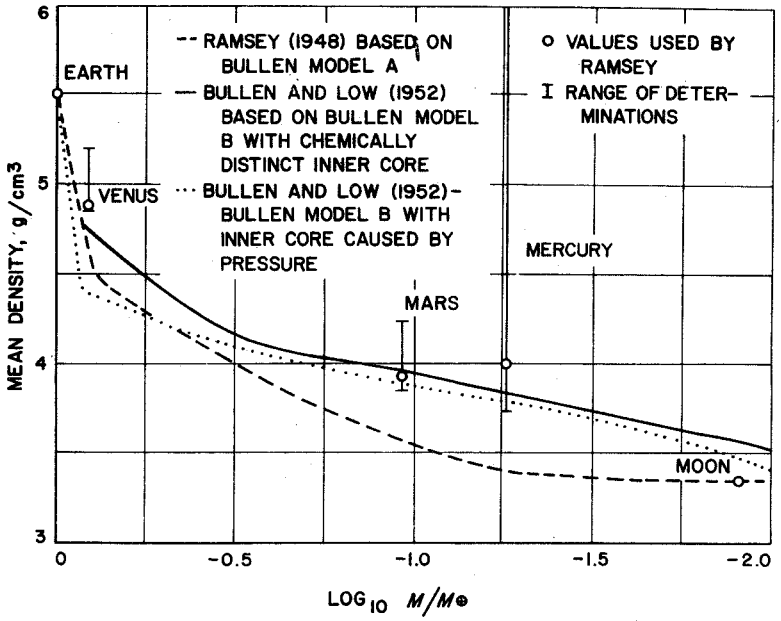

Fig 1. Mass-mean density curve for the terrestrial planets based on the hypothesis that the earth's outer core is a pressure-induced phase change.

moment of inertia. These two parameters are related to a density in the mantle and in the core, either the mean density or the density at the top of the zone. Integration downward in each zone is accomplished by invoking an equation of state of some type which relates density to pressure. Finite strain theory and the Adams-Williamson theory are examples of the equations of state that have been assumed. Adjustments in the two free parameters are made until the mass and moment of inertia are satisfied, but restrictive assumptions regarding homogeneity, thermal gradients, and hydrostaticity are implicit in these approaches. We will use the density distribution (Table 1) determined by *Anderson* [1964] from analyses of the periods of free oscillations of the earth, which is not restricted by these assumptions. The model, designated E8, has been modified slightly to be in accord with more recent data on the moment of inertia of the earth and on shock waves and to facilitate the extrapolations required for mixing core and mantle material. The mantle equation of state allows for phase changes in the upper mantle.

The equation of state assumed for the core is parallel to the O°K isotherm for iron computed from the shock wave Hugoniot [*Al'tshuler et al.*, 1958] and is consistent with a core composition of 84% iron and 16% mantle material, presumably silicates and oxides. The equation of state for core material is extrapolated to lower pressures by assuming an ideal mix of iron and mantle material. In Figure 2 the E8 equation of state is compared with those derived from the familiar Bullen A and Bullen B density models.

For completeness, a low-density crust and an inner core were added to the earth model. The change from low-density surficial material to mantle material and the change to an inner core were assumed to be pressure controlled, although inclusion of a crust and inner core is not critical for any of our principal conclusions.

With the equations of state for the two major regions of the earth, over a pressure range appropriate for the terrestrial planets, we will design two-zone planets under the following assumptions:

1. The boundary of the two zones is a compositional discontinuity and is the only such discontinuity in the planet. Phase changes in the mantle are assumed to be functions of pressure alone and are automatically taken into account in the E8 equation of state for the mantle.

2. The density in each zone is controlled by pressure alone and can be determined by a suitable linear combination of the two equations of

state. Implicit in this calculation is the assumption that temperature is the same function of pressure as in the earth or, alternatively, that the effect of temperature gradients different from those in the earth are negligible.

A coefficient of thermal expansion of $4 \times 10^{-5}/°K$ was used to correct the mean density of Venus to allow for its presumed high surface temperature of 600°K [*Mayer*, 1961], which implies that at equivalent pressures the temperature is higher than in the earth. This temperature correction is small, amounting to a 1.6% increase in the effective mean density for Venus.

Discussion. Figure 3 shows the results of constructing a suite of model planets for different ratios of the mass of chemically distinct cores to total planetary masses. The model planets were constructed by numerical integration of the familiar hydrostatic equilibrium equations together with the selected equations of state. Table 2 lists pertinent physical data for the inner planets.

TABLE 1. Equation of State

Pressure, megabars	Density, g/cm³	
0.000	2.740	Crust
0.009	2.740	
0.009	3.421	
0.053	3.421	
0.060	3.552	
0.101	3.552	
0.155	3.653	
0.158	3.875	
0.212	3.885	
0.251	3.996	
0.294	4.360	
0.339	4.491	
0.383	4.521	Mantle
0.475	4.663	
0.571	4.915	
0.668	4.945	
0.843	5.197	
1.050	5.288	
1.320	5.338	
1.347	5.551	
1.500	5.742	
2.000	6.300	
2.500	6.580	
3.000	6.910	
0.009	6.494	
0.053	6.618	
0.060	6.684	
0.155	7.010	
0.200	7.406	
0.300	7.810	Core
0.400	8.071	
0.600	8.592	
0.900	9.236	
1.350	9.908	
1.500	10.172	
2.000	10.888	
3.300	12.279	
3.300	12.936	Inner core
4.000	12.936	

The mean density of the moon can only be satisfied by decreasing the size of its core until it comprises about 8.5% of its total mass. For this computed model the ratio of its principal moment of inertia to the product of the mass and the square of the mean radius, C/MR^2, is 0.37. For a homogeneous sphere C/MR^2 is 0.4. Data on its axis of rotation and orbital motion lead to a value of 0.56 ± 0.12 for C/MR^2 [*Jeffreys*, 1961]. Jeffreys considers the disagreement with a homogeneous moon to be tolerable. A model with a chemically distinct core cannot be constructed for the moon which does not violate compositional identity with the earth and the observational data on its mean density and moment of inertia.

The curve for the same gross composition as the earth misses the temperature-corrected upper limit for the mean density of Venus (solid body radius of 6050 km and mean density of 5.25 g/cm³) by only 0.4%. If we decrease the ratio of core mass to planetary mass to 0.30, the computed curve intersects the mass of Venus at an external radius of 6100 km. The radius of the solid surface of Venus is not known because of uncertainty about the height of its atmosphere. However, recent determinations of the *optical diameter* of Venus have all yielded values around 6100 km [*de Vaucouleurs and Menzel*, 1960; *Martynov*, 1961, 1963; *Smith*, 1964], and it is safe to conclude that the solid body radius is somewhat less than the visual radius. The computed models for Venus which have cores containing 30 to 32% of the total planetary mass are thus entirely compatible with observational data on the mass and radius of Venus. *Urey* [1952] has already pointed out that Venus is so similar to the earth that it can be made to fit any theory that accounts for the earth.

The mass of Mars is satisfied by a model for which the core mass is 32.5% of the total mass and therefore has the same gross composition as the earth if its mean radius is 3309 km; this is in agreement with *Trumpler's* [1927] deter-

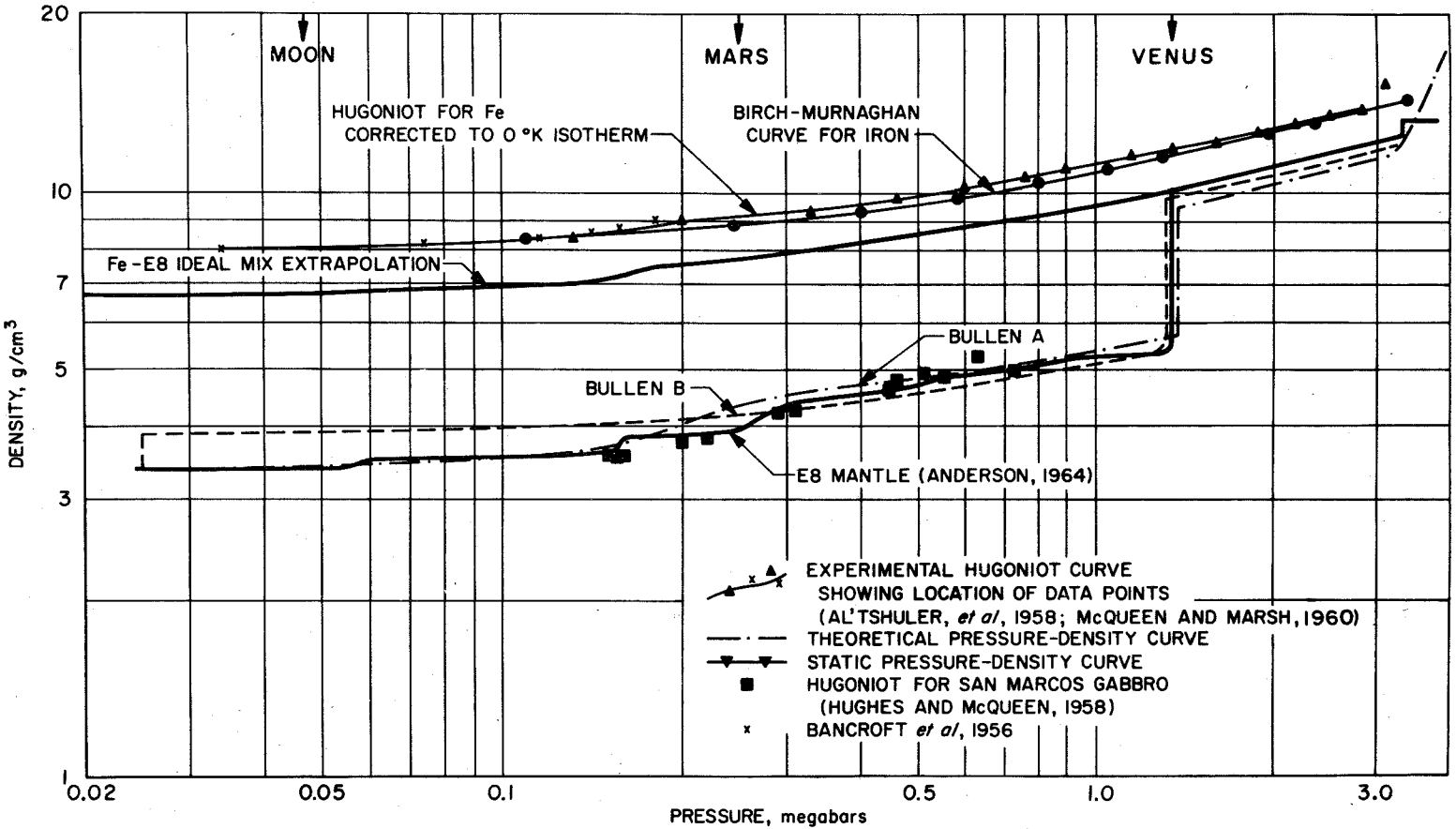

Fig. 2. Pressure-density relation used for mantle and core compared with Bullen A and Bullen B relations and experimental shock wave data. Minimum central pressures in the moon, Mars, and Venus are shown by top arrows.

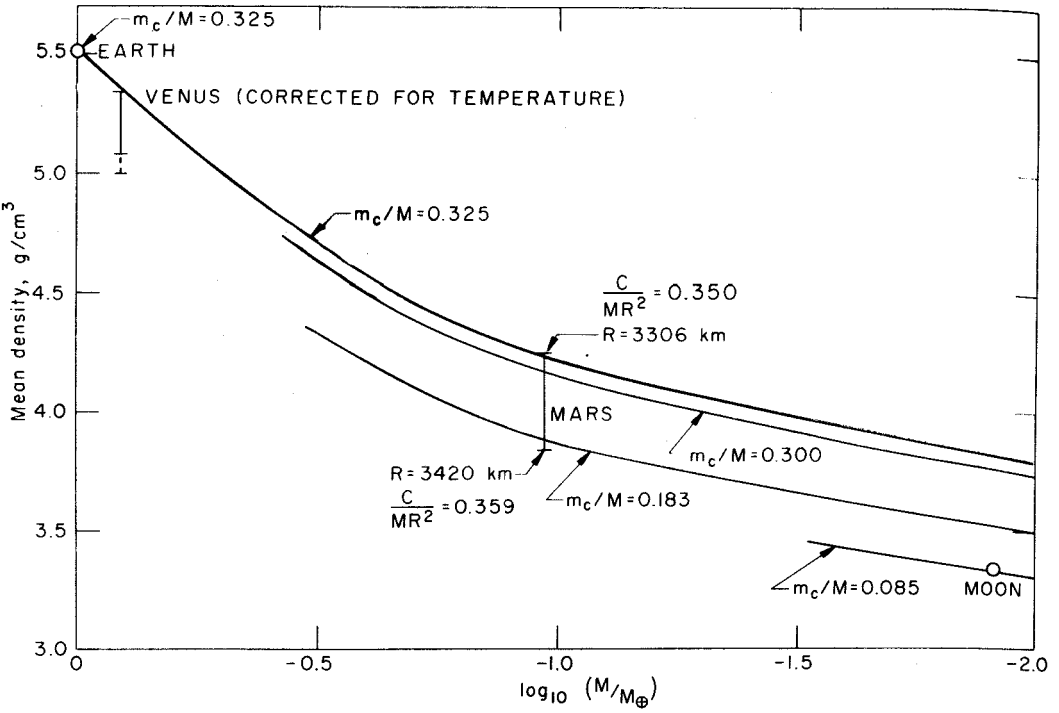

Fig. 3. Mass-mean density relation for different ratios of core mass to total mass. Curve for ratio of core mass to total mass of 0.325 is for a composition which is the same as that of the earth. The uncertainties of the radii of Venus and Mars are shown by the height of the error bars.

mination of 3310 ± 12 km. If an external radius much greater than 3309 km is adopted for Mars, the mass percentage of core material in the computed models must be decreased and compositional identity with the earth cannot be maintained (Figure 3).

$(C - A)/MR^2$ for Mars is known from analyses of the motion of Phobos [*Woolard,* 1944]. Assuming hydrostatic equilibrium in Mars, this value may be used with the well-known Clairaut-Radau relations to obtain a value for the hydrostatic ellipticity e and, in turn, an estimate of C/MR^2 for the planet. For a mean radius of 3309 km the hydrostatic ellipticity of Mars is 0.0052 ($e^{-1} = 192$). However, the actual ellipticity of the earth is greater than its hydrostatic ellipticity, which implies that stress differences are supported within its interior [*MacDonald,* 1962; *Jeffreys,* 1963]. MacDonald concludes that the range of acceptable values for the ellipticity of Mars lies in the range $182 < e^{-1} < 200$. The computed value of e^{-1} for our Mars model, which maintains the same composition as the earth, is 231.2 (Table 3), and this is outside of the presumed range of possible ellipticities.

It is possible to redistribute the material in the interior of a planet to change the moment of inertia and therefore the hydrostatic ellipticity without changing the over-all composition. We will construct a suite of models which have the same gross composition as the earth but which vary in the distribution of heavy elements by mixing core material into the mantle and mantle material into the core. Starting with the model which has the same ratio of core mass to planetary mass as the earth, we strip off different mass percentages of the core and ideally mix them throughout the mantle using the relation

$$\rho_m = \frac{M_1 + M_2}{M_1/\rho_1 + M_2/\rho_2}$$

where

ρ_1 = density of mantle material at a specific pressure.

ρ_2 = density of core material at the same pressure.
ρ_m = density of core-mantle mixture.
M_1 = mass of mantle.
M_2 = stripped mass of core.

In the construction of models which have mantle material mixed into the core the roles of M_1, M_2, ρ_1, and ρ_2 are interchanged.

Figure 4 shows the results of mixing 80% of the mass of the core into the mantle. The model which is identical in composition to the earth and has 80% of its core mass distributed throughout the mantle satisfies the mass of Mars for an external radius of 3306 km (ρ = 4.25 g/cm^3). This model has a moment of inertia factor of 0.3784 and a value of 205.0 for e^{-1}, which is outside the presumed allowable range of values for Mars. If an initial ratio of core mass to planetary mass of 0.3 is selected, the mass of Mars is satisfied at a radius of 3322 km (ρ = 4.185 g/cm^3) and gives 202.0 for e^{-1}.

TABLE 2. Physical Properties of Venus, Mars, and the Moon

	Source
Venus	
Mass	
$(4.8695 \pm 0.0006) \times 10^{27}$ g	Mariner 11 value
Radius	
(6050 ± 3) km	*Smith* [1964]
(6100 ± 34) km	*Martynov* [1961]
(6085 ± 10) km	*de Vaucouleurs* [1964]
Mars	
Mass	
$(0.6434 \pm 0.0006) \times 10^{27}$ g	*Brouwer and Clemence* [1961]
Radius*	
(3310 ± 12) km	*Trumpler* [1927]
(3423 ± 11) km	*Rabe* [1929]
(3396 ± 8) km	*Russell et al.* [1945]
(3360 ± 10) km	*Camichel* [1954]
(3362 ± 10) km	*de Vaucouleurs* [1964]
Moon	
Mass	
$(7.3446 \pm 0.0006) \times 10^{25}$ g	*Brouwer and Clemence* [1961]
Radius	
(1737.95 ± 0.05) km	*Russell et al.* [1945]

* Based on 1 AU = (149,598,845 ± 250) km [*Muhleman et al.*, 1962]. Mass of the earth adopted as 5.9760×10^{27} g, mass of the sun as 1.9868×10^{33} g.

TABLE 3. Parameters of Models for Mars

Distribution of Material	Radius, km	C/MR^2	η	e^{-1}
Undifferentiated	3309	.3449	.4568	231.2
80% core into mantle	3306	.3784	.1686	205.0
90% core into mantle	3305	.3828	.1332	201.7
80% mantle into core	3307	.3787	.1664	204.3
97% mantle into core	3309	.3836	.1289	200.3

Mixing 80% of the mantle into the core (Figure 5) gives a model for Mars having a moment of inertia factor of 0.3787 and 204.3 for e^{-1} (Table 3). Iterating to a solution for Mars that falls within the allowable range of ellipticities (and hence moment of inertia) we find that mixing 97% of the total mass of the mantle into the core gives a satisfactory solution (Table 3). Thus an internal density model for Mars can be constructed (Figure 6) which has the same gross composition as the earth, provided its mean radius is about 3309 km and its ellipticity is 0.005 (e^{-1} = 200).

A definitive determination for the mean radius of Mars would place a very strong constraint on discussions of its compositional similarity to the earth. If the mean radius of Mars should prove to be different from the favored *Trumpler* [1927] measurement of 3310 ± 12 km, a model cannot be constructed for Mars which possesses the identical heavy element composition as the earth. Mars must be a undifferentiated and nearly homogeneous body, but it is not necessary to violate compositional identity with the earth. If a somewhat larger mean radius, of the order of 3325 km, is adopted, a density model could be constructed for Mars which has core material constituting 30% of the total mass. The gross composition of Mars would then differ only slightly from that of the earth, which has a core constituting 32.5% of the total mass.

If a core is formed by downward drainage of heavy elements, we can say that Mars is in an earlier stage of evolution than the earth and has probably not gone through a completely molten stage. If a planet can segregate its iron without melting, the internal temperatures in Mars are probably lower than in the earth.

Conclusions. Astronomical data on the ter-

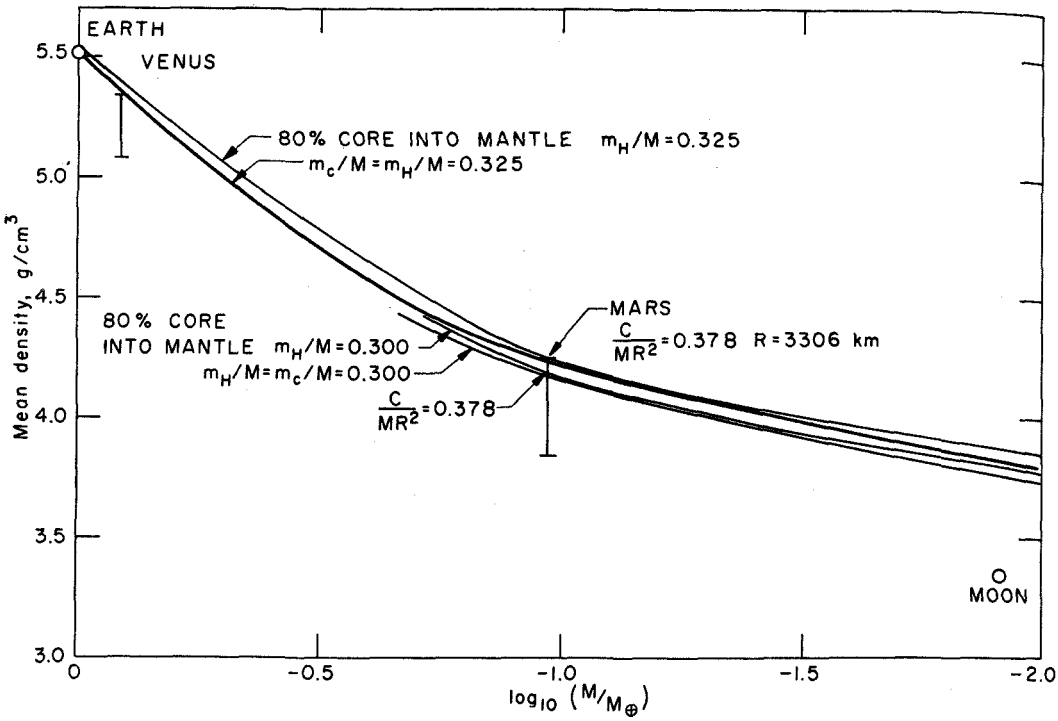

Fig. 4. Mass–mean density relation for different-sized bodies, showing effects of mixing 80% of the mass of the core into the mantle. The effect of mixing is to keep gross composition the same but to make the planet more homogeneous.

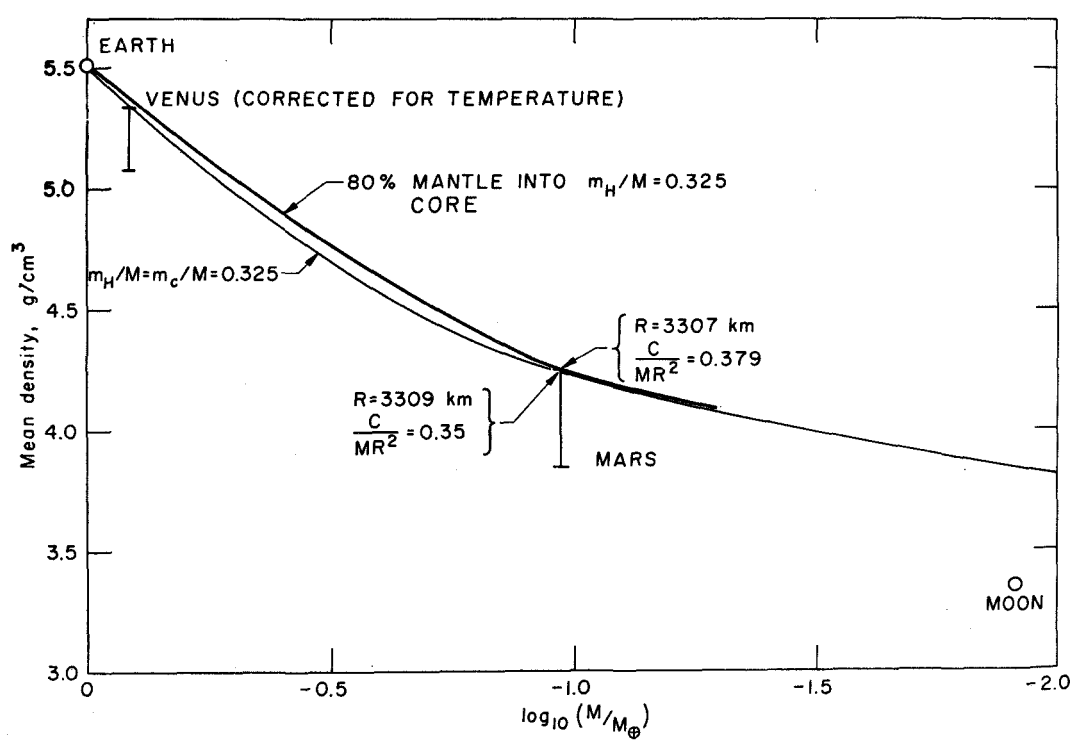

Fig. 5. Mass–mean density relation, showing effect of mixing 80% of the mass of the mantle into the core.

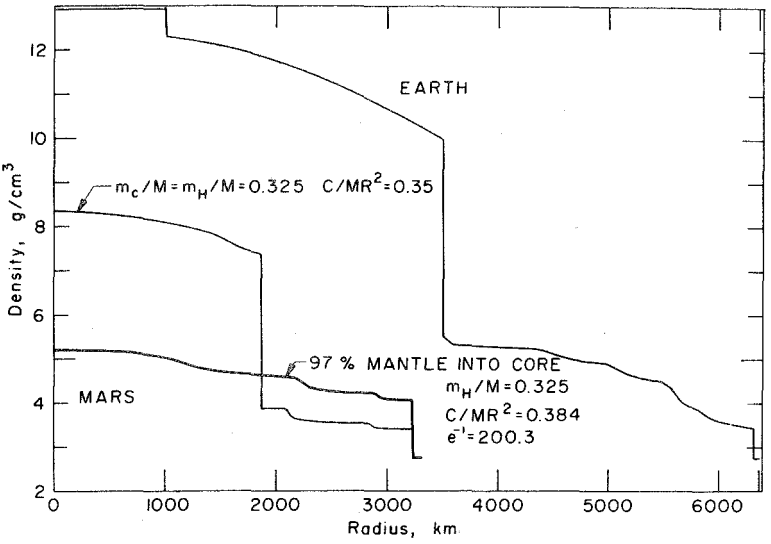

Fig. 6. The final density distribution for Mars which has the same gross composition as the earth and satisfies moment of inertia data has 97% of mass of mantle mixed into core. Differentiated model for Mars has too low a moment of inertia.

restrial planets, Earth, Venus, and Mars, can be satisfied with the hypothesis of chemical uniformity and a chemically distinct, i.e. iron-rich, core. The moon must differ in composition from the earth but can have a metallic core constituting as much as 8.5% of its total mass.

The earth has a core which is approximately 32.5% of the total mass. The mean density of Venus can be satisfied by an identical core-to-mass ratio if the lower estimates of the radius are correct. Presumed high temperatures on Venus improve the agreement.

Mars has been a crucial test in discussions of the terrestrial planets because its radius and mean density are between those of the earth and moon. It has previously proved impossible to satisfy both the hypotheses of chemical identity with the earth and a chemically distinct core for the earth. Mars has been used to discard one or the other of the above hypotheses.

We have demonstrated that Mars can be similar to an undifferentiated earth, provided that the lower value of its radius is correct. The earth's core is not required to be a pressure-induced phase change in order to keep the composition of Mars and the earth identical.

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