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2014 New J. Phys. 16 113020

(http://iopscience.iop.org/1367-2630/16/11/113020)

New Journal of Physics
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Deutsche Physikalische Gesellschaft DPG | IOP Institute of Physics

Superfluid optomechanics: coupling of a superfluid to a superconducting condensate

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Received 30 June 2014, revised 12 September 2014
Accepted for publication 6 October 2014
Published 11 November 2014
New Journal of Physics **16** (2014) 113020
doi:10.1088/1367-2630/16/11/113020

Abstract
We investigate the low loss acoustic motion of superfluid ^{4}He parametrically coupled to a very low loss, superconducting Nb TE_{011} microwave resonator, forming a gram-scale, sideband resolved, optomechanical system. We demonstrate the detection of a series of acoustic modes with quality factors as high as 1.4×10^7. At higher temperatures, the lowest dissipation modes are limited by an intrinsic three phonon process. Acoustic quality factors approaching 10^{11} may be possible in isotopically purified samples at temperatures below 10 mK. A system of this type may be utilized to study macroscopic quantized motion and as a freqency tunable, ultra-sensitive sensor of extremely weak displacements and forces, such as continuous gravity wave sources.

Keywords: superfluid, quantum measurement, gravity wave experiments

1. Introduction

The study of the detection of motion at quantum mechanical limits has received great attention over the past 35 years [1–3] with much of the early thought and effort focused on the engineering of very large-scale, ultra-sensitive gravitational wave antennas. Given the recent success in preparing and measuring nano- and micron-scale mechanical structures at quantum limits [4–7], it is intriguing to consider what is required to accomplish quantum behavior with larger, gram-scale objects.

1367-2630/14/113020+09$33.00

When considering such experiments, a key parameter is the coupling rate to dissipative thermal environments which sets the timescale for energy decay, decoherence times [8], lifetimes of number states, limits of cooling to the ground state [9], position sensitivity [10], and force sensitivity. Furthermore, fascinating experiments have recently been accomplished with small quantum condensates of atomic vapors coupled to optical resonators [11, 12]. In light of this, superfluid ^{4}He, a condensate which can easily be prepared in macroscopic quantities and demonstrates frictionless motion at zero-frequency [13], is an intriguing material to consider for the study of quantized macroscopic motion [3], quantized mechanical fields [14], and even Planck-scale physics [15, 16].

Acoustic dissipation of first sound of superfluid ^{4}He is a very well studied and understood process [17–19]. For temperatures below 0.6 K, the attenuation is dominated by the non-linearities of the compressibility of liquid helium, which couples the low frequency acoustic mode with thermal phonons, and leads to an acoustic attenuation coefficient of [19]

$$\alpha = \frac{\pi^2}{30}\frac{(G+1)^2}{\rho_4 \hbar^3 c_4^6}\left(k_\mathrm{B}T\right)^4 \omega\left(\arctan(2\omega\tau) - \arctan(\Delta E\tau)\right), \tag{1}$$

where $G = (\rho/c)\partial c/\partial\rho = 2.84$ is the Grüneisen's parameter [20], k_B is the Boltzmann constant, $\rho = 145\ \mathrm{kg\,m^{-3}}$ is the density, $\hbar$ is the Planck constant, $c_4 = 238\ \mathrm{m\,s^{-1}}$ is the speed of sound [20], ω is the frequency of the acoustic wave, T is the temperature, $\tau = 1/(0.9 \times 10^7\ T^5)$ is the thermal phonon lifetime [21], $\Delta E = 3\gamma\bar{p}^2\omega$ is the energy discrepancy between the initial and final states in the 3PP, $\bar{p} = 3k_\mathrm{B}T/c$ is the average thermal momentum, and $\gamma \approx -10^{48}$ $(\mathrm{s\,kg^{-1}\,m^{-1}})^2$ is a constant which characterizes the weak non-linearity of the dispersion relation for low momentum phonons [22, 23][1]. In the low temperature limit, $\omega\tau > 1$, and the absorption is analogous to the Landau–Rumer regime in solids [24, 25]. Here the arctan functions simplify to a factor of $\pi/2$ and the quality factor $Q = \omega/c\alpha$ is frequency independent. For a 6 kHz mode, we will realize this limit for temperatures below 350 mK. At very low temperatures, $T < 40$ mK, we may reach the limit where $|\Delta E\tau| \gg 1$. In this case the second term in equation (1) contributes to the attenuation, at most increasing α by a factor of 2 [26]. This limit could prove difficult to reach, however, as the thermal phonon lifetimes may be limited by boundary scattering. In the high temperature limit of equation (1), $\omega\tau < 1$, and absorption is in the Akheiser regime [25, 27]. However, this limit is complicated by the effects of proton scattering, which are not included in the above equation and which become important at temperatures above ≈0.6 K.

Assuming a sample temperature of 10 mK and that the thermal phonon lifetimes are limited such that $|\Delta E\tau| \ll 1$, equation (1) leads to an acoustic quality factor of $Q = 5 \times 10^{10}$, which would exceed the highest recorded mechanical quality factors [28–30] of 10^9. A 5 kHz acoustic mode in this condition would have an extraordinary number state lifetime of $\tau_N = \hbar Q/(k_\mathrm{B}T) \approx 36$ s.

A parametric system is formed when low frequency mechanical motion modulates a higher frequency electromagnetic resonance: an optical cavity with a mechanical resonator as a mirror, or a microwave cavity coupled to a mechanical element are prototypical examples of parametric opto-mechanical systems [3]. In this work, we couple the acoustic motion of superfluid ^{4}He to a microwave resonance through the modulation of the density and resulting modulation of the

[1] $\epsilon(p) = cp\left(1 - \gamma p^2 \ldots\right)$. With this form, $\gamma < 0$ is known as anomalous dispersion.

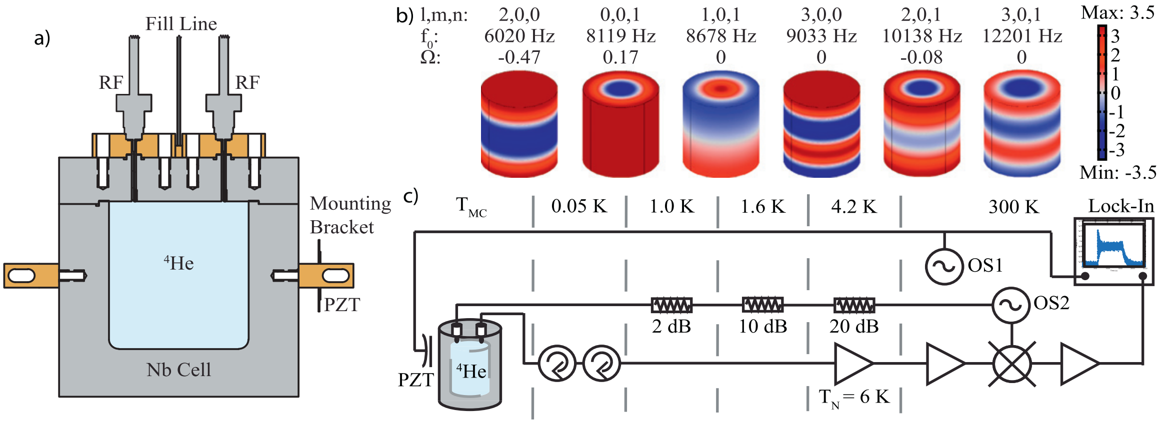

Figure 1. (a) A cross-section of the parametric system: a niobium cavity filled with superfluid ^{4}He. The resonator is formed from two pieces (body and lid) with the microwave ports and helium fill line located on the lid and sealed to the lid with indium. The lid is sealed to the body with indium which has a T_c of 3.4 K. The volume of the cavity is 39.3 cm^3 and is filled with 5.70 g of ^{4}He. (b) Shows the pressure profile of the lowest frequency superfluid acoustic resonances computed with finite element analysis; pressure nodes are shown in white. Shown above the mode profiles are the mode numbers (l,m,n) which represent the number of nodes in the longitudinal, azimuthal, and radial directions, respectively, the frequency (f_0), and the coupling constant (Ω) to the TE_{011} mode. (c) Shows the low temperature microwave measurement circuit, where OS1 is at audio frequency and used to drive the piezoelectric actuator (PZT) which drives the acoustic mode, and OS2 is a microwave oscillator used to pump the SCR.

permittivity. We find [31] that the change in the resonant microwave frequency due to an acoustic wave with pressure amplitude P is

$$\frac{\partial \omega_C}{\partial P} = -\frac{1}{6}\omega_C \kappa_{He}\left(\epsilon_R + 2\right)\left(\epsilon_R - 1\right)\Omega, \tag{2}$$

where ω_C is the microwave cavity frequency and $\kappa_{He} = 1.2 \times 10^{-7}$ Pa^{-1} and $\epsilon_R = 1.057$ are the compressibility and relative permittivity of liquid helium. Ω is the geometric coupling between the acoustic wave and the energy stored in the microwave field

$$\Omega = \frac{\int f(r, \theta, z)\,|E(r, \theta, z)|^2\ \mathrm{d}V}{\int \epsilon_R\,|E(r, \theta, z)|^2\ \mathrm{d}V}, \tag{3}$$

where $P \cdot f(r, \theta, z)$ and $E(r, \theta, z)$ are the spatial functions of the pressure field of the acoustic mode and electric field of the microwave mode. For the TE_{011} mode coupled to the acoustic mode at 8119 Hz (see figure 1), we compute $\Omega = 0.17$.

The coupling between the motion of the helium and the microwave field is relatively weak in comparison to nano- and micro-scale optomechanical realizations: the single quanta frequency shift is given by $\frac{\partial\omega_C}{\partial P}\Delta P_{SQL} = -4.2 \times 10^{-8}$ where $\Delta P_{SQL} = \sqrt{\hbar\omega/(\kappa_{He} V_{eff})} = 1 \times 10^{-9}$ Pa is the amplitude of the zero point fluctuations of the acoustic field, $V_{eff} = 39.3$ cm^3 is the effective volume of the acoustic mode and $\omega = 2\pi \times 8$ kHz [31]. However, as a result of the extremely low levels of dissipation of the microwave resonator and the very low dielectric loss of the helium ($\epsilon'' < 10^{-10}$) [32], we do expect to be able to pump this system sufficiently hard to detect the acoustic field at the standard quantum limit (SQL) [31]. Furthermore, mechanical coupling between the superfluid and the Nb cell is also expected to be weak given the large difference in speeds of sound ($c_{\mathrm{Nb}} = 3480\ \mathrm{m\,s^{-1}}$): the lowest acoustic resonance of the helium is lower in frequency than the lowest vibrational mode of the Nb cell. In addition, Nb has been shown to be an excellent mechanical material at low temperatures (Q of 4×10^7 at 50 mK at kHz [33]). Finite element analysis shows that only $\approx 4 \times 10^{-4}$ of the acoustic energy resides in the Nb cell with the superfluid excited at the 6020 Hz acoustic resonance (figure 1(b)), limiting [31] the quality factor of the acoustic helium mode to 10^{11}. In future experiments, it may be possible to pre-fill the cell and thus eliminate acoustic losses from the helium fill line. For the cell to be filled at 4.2 K, the pressure required at 77 K is 2.3×10^7 Pa (230 bar) and at 300 K is 9×10^7 Pa (900 bar). This would be achievable if the lid were welded in place. Given that the acoustic wavelength in silver at these frequencies is ~ 26 cm, acoustic radiation through the mechanical support which provides thermal contact to the cell could be reduced through the engineering of acoustic band-gap structures. Other designs utilizing very low loss dielectric materials such as sapphire are also possible and under development [31].

Our superconducting cavity resonator (SCR) is formed from a billet of superconductivity-grade Nb [2] which is machined into a cylindrical cavity with internal dimensions: 1.78 cm radius and 3.95 cm length (figure 1(a)). After machining, the Nb pieces were polished and etched in acid ($HF : HNO_3 : HPO_3 :: 1 : 1 : 2$) for 40 min which removed approximately 100 μm of material [34]. Microwaves are coupled in and out of the SCR through loops recessed into the lid. We choose to work with the TE_{011} mode; it is typically the highest Q mode in a cylindrical cavity resonator as there are no currents between the lid and the body of the cylinder, minimizing the effect of attaching the lid on the quality factor. Realizing very low loss in the microwave resonator is important for both producing single side-band resolution and for reducing the power dissipated into the low temperature cirucuit, which will limit the pump strength one can apply before heating becomes an issue. Measurements at 1.7 K show a TE_{011} resonance of 10.89 GHz (10.60 GHz when filled with ^{4}He) with an internal loss rate of $\kappa_{\mathrm{int}} = (2\pi) \times 31$ Hz. The SCR is coupled to the mixing chamber of the dilution refrigerator (base temperature of 5.5 mK) with copper brackets.

For our first experiments with this system, we increased the microwave coupling to $\kappa_{\mathrm{in}} = \kappa_{\mathrm{out}} = (2\pi) \times 680$ Hz, enabling sideband resolution for kilo-hertz acoustic modes. We mounted the SCR to the fridge with two copper brackets, attached to opposite sides of the SCR. The cell was filled at 4.0 K through a 0.5 mm ID capillary with normal isotopic purity ^{4}He (^{3}He/^{4}He $\approx 10^{-6}$). As the cell is cooled below 1 K, the helium undergoes a substantial thermal contraction ($\Delta\rho/\rho = 0.13$), which lowers the free surface of the liquid helium into a 7.4 cm^3 volume placed at the mixing chamber, before the SCR. This avoids the heat load into both the helium and the refrigerator from a filled capillary. The remaining heat load from the helium film in the capillary limits the base-temperature of the refrigerator to 33 mK. The superfluid cools through the walls of the SCR with a thermal time constant expected to be ~ 10 s, which is anticipated to be largely temperature independent [35]. Future work will include the addition of a low temperature valve [36] to the fill line which will avoid both the heat flow and acoustic coupling into the superfluid cell.

[2] ATI Wah Chang, 1600 NE Old Salem Road, P.O. Box 460, Albany, Oregon 97321.

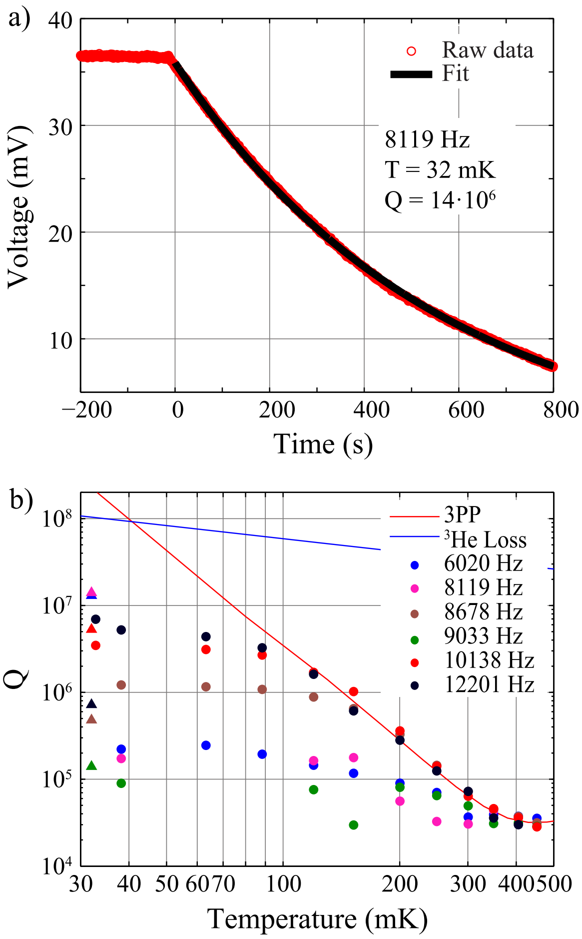

Figure 2. (a) The free decay of the helium acoustic resonance at 8119 Hz, demonstrating a quality factor of 1.4×10^7. (b) Shows the measured quality factors for a number of the lowest frequency acoustic modes, versus refrigerator temperature. The data represented with triangles shows the low temperature quality factors realized in our second run (same colors used for data from first run). The red line shows the dissipation expected from the three-phonon process which scales as T^4 (equation (1)), and the blue line shows the dissipation expected from ^{3}He impurities at a concentration of 10^{-6}, assuming in both cases a mode frequency of 8119 Hz.

We pump the parametric system with a microwave source, red-detuned from the SCR resonance ($\omega_p = \omega_C - \omega$) and measure the up-converted sideband appearing at the cavity frequency [9] due to the acoustic motion which is stimulated with a piezoelectric transducer attached near the SRC. We find a series of acoustic resonances within 1% of the expected frequencies for a right cylindrical resonator. We measure the quality factor by recording the undriven, free decay. Figure 2(a) shows such a trace for a 8119 Hz mode demonstrating a Q of 1.4×10^7. Figure 2(b) shows the measured quality factors for a number of the acoustic modes versus temperature.

For the highest Q acoustic modes measured in this run, the Q approximately follows the dissipation rate versus temperature due to the expected three-phonon process (equation (1)), but eventually saturates below 100 mK. At this point we do not have a clear understanding of this saturation or the difference in Q between the modes: from equation (1) we would expect the Q

Table 1. A comparison of the highest quality factors for various acoustic modes measured in the first and second run of this experiment, Q(first) and Q(second) respectively.

Frequency (Hz)	Q(first) $/10^6$	Q(second) $/10^6$
6020	0.25	13
8119	0.18	14
8678	1.2	0.48
9033	0.09	0.14
10138	3.5	5.3
12201	7.0	0.72

to be frequency independent when $\omega\tau > 1$. As the frequency approaches the first acoustic mode of the niobium cell (15 kHz), the helium acoustic resonances become more difficult to isolate, and the Q's start to degrade. Above 15 kHz, some modes are extremely low Q ($Q < 10^3$). We have also estimated the dissipation due to ^{3}He impurities [31] and show this on figure 2(a). We believe the highest Q acoustic modes are not yet limited by this effect, which will scale as $1/\omega$ and is inconsistent with our data. We have obtained a sample of isotopically purified helium [37] (concentration of ^{3}He is 10^{-10} that of ^{4}He), to mitigate this effect in future experiments.

For the second run of this experiment, we decreased the size of the helium fill line at the SCR to reduce losses from acoustic transmission. From the SCR, the initial section of fill line is 5 cm long and only 125 μm in diameter, a factor of 4 smaller than that used in the initial run. Two additional capillaries, the first 16.5 cm long and 900 μm diameter and the second 13 cm long and 250 μm diameter, connect it to the small ballast volume used to fill the cell. To reduce heating effects from higher temperature stages, the fill line from the ballast volume to 1.6 K was increased to a 1 m length and decreased to a 250 μm diameter. We also attached the SCR to the mixing chamber with a single copper bracket mounted to the midpoint of the cell, and used smaller, flexible semi-rigid coaxial lines (reduced from 2.2 mm to 0.9 mm diameter). After these changes, we found the following low temperature quality factors, listed in table 1.

As is clear from this table, we realize a dramatic improvement in the lower frequency resonances compared to our first run, by a factor of 50 for the 6 kHz mode. At this point, the origin of the dominant source of dissipation is not known and will be the focus of our next experiments. More careful design considerations must be placed on mechanical contacts to the cell (fill line, microwave contact, thermal attachment of the SCR to the refrigerator). We do know from thermometry at each stage that the superfluid film flow in the fill line is heating the lower stages of the refrigerator and preventing both operation at the base temperature of 5.5 mK and thermalization of the superfluid in the cell to the mixing chamber. We are in the process of adding sintered silver heat exchangers which can properly thermalize helium to lower temperatures [35].

The measurement shown in figure 2(a) is realized with a microwave pump strength producing $n_\mathrm{p} \approx 10^7$ microwave photons inside the SCR, leading to a dissipation in the SCR of 10 fW, and in the helium itself of <0.5 fW. At 100 mK (10 mK), this would lead to an increase of the temperature of the helium of 20 pK (20 nK). This low level of heating allows for the application of large amplitude microwave signals until the sensitivity is limited by the phase noise of the microwave sources. With our lowest phase noise source (Agilent E8257D-$\mathcal{L}$(8 kHz) = −110 db$_c$/Hz) and an 8 kHz, $Q = 1.4 \times 10^7$ acoustic resonance, the pressure resolution is 2×10^{-4} Pa, which corresponds to a noise temperature of 2×10^{4} K. Utilizing a sapphire cryogenic reference (77 K, unloaded $Q = 6 \times 10^{7}$) a much lower phase noise source is possible: $\mathcal{L}$(8 kHz) = −156 db$_c$/Hz [38]. Improvement to $\mathcal{L}$(8 kHz) = −220 db$_c$/Hz could be achieved by filtering this sapphire source with a $Q = 1 \times 10^{9}$ SRC, similar to that used here to contain and couple to the helium. This would yield a detection noise temperature of 10 mK with $n_{\mathrm{p}} = 4 \times 10^{12}$.

Optomechanical back-action effects such as linewidth broadening and cooling [9] are possible and potentially very useful in this system. Assuming an improved microwave resonator with $\kappa_{\mathrm{in}} = \kappa_{\mathrm{out}} = \kappa_{\mathrm{int}} = 2\pi \times 3$ Hz, we expect an optical damping rate of $\Gamma_{\mathrm{opt}}/(2\pi) = 8 \times 10^{-3}$ Hz for a pump strength of $n_p = 4 \times 10^{14}$, which would broaden the acoustic resonance by a factor of 5×10^{4} (assuming an 8 kHz acoustic resonance at T = 10 mK, $Q = 5 \times 10^{10}$.) Together with an ultralow phase noise source ($\mathcal{L}$(8 kHz) = −220 db$_c$/Hz) this damping would be extremely useful to make sensitive measurements with much shorter measurement times: 3 s versus 1.7 days in the example here. Taking into account heating of the helium and the dilution refrigerator (cooling power of 400 μW at 100 mK), we would expect to side-band resolved cool the 8 kHz acoustic mode to an occupation factor of $n_{\mathrm{m}} \sim 2$ assuming a starting temperature of 10 mK ($n_{\mathrm{m}} = 3 \times 10^{4}$) and $n_{\mathrm{p}} = 4 \times 10^{14}$. Starting from 20 mK or 30 mK, one expects cooling to occupation factors of $n_{\mathrm{m}} \sim 8$ and $n_{\mathrm{m}} \sim 19$, respectively, limited by the phase noise of the pump. Note that actively cooling the 8 kHz mode to $n_{\mathrm{m}} = 2$ will reduce the Q to 6×10^{6} and will not affect the sensitivity of this system to forces. Further details of these parameters and effects will be provided in a longer manuscript.

The possibility to produce an extremely sensitive inertial sensor can be understood through the following order of magnitude estimate. Suppose the helium acoustic resonance is modeled as a simple mass–spring system contained inside the Nb cavity, where $m = 5.7$ g is the mass of the superfluid and $k = m\omega^2 = 10^{7}\,\mathrm{N\,m^{-1}}$. The thermal motion of this system at 80 mK is $x_{\mathrm{th}} = \left(k_b T/(m\omega^2)\right)^{1/2} = 3 \times 10^{-16}$ m. If one drove this system by shaking the Nb cavity at the acoustic resonance, with an amplitude x_0, the motion of the helium would be $x = Q \times x_0$. Turning this expression around, and assuming both our current device parameters (8 kHz, $Q = 1.4 \times 10^{7}$, 80 mK) and that the detection is limited by the thermal noise of the acoustic field, we would expect the displacement sensitivity of the Nb container to be 2×10^{-23} m (in the current setup, we are limited by our detection noise temperature of 2×10^{4} K to a sensitivity of 10^{-13} m); assuming 10 mK and $Q = 5 \times 10^{10}$, we expect sensitivity to motion of the outside container of $x_0 = x_{\mathrm{th}}/Q = 2 \times 10^{-27}$ m.

This remarkable level of sensitivity would correspond to a strain sensitivity of $h \approx 5 \times 10^{-26}$ assuming our current device geometry; this is comparable to the sensitivities achieved by LIGO [39] and resonant bar detectors such as NAUTILUS [40]. This level of strain is below the level predicted to exist at Earth due to the gravitational waves from rapidly spinning, nearby pulsars [41–43]. Although this extreme sensitivity would only be obtained around the narrow acoustic resonance (broadened significantly through backaction effects), it is possible to tune the acoustic resonance by as much as 50% by pressurizing and modifying the speed of sound of the helium [20, 44]. This is a significant advantage in comparison to past implementations of resonant detectors [43, 45]. This frequency control could also enable parametric amplification techniques which could both preamplify weak signals and provide sensitive detection at frequencies other than the resonant acoustic frequency.

Acknowledgments

We would like to acknowledge helpful conversations with Rana Adhikari. We acknowledge funding provided by the Institute for Quantum Information and Matter, an NSF Physics Frontiers Center (NSF IQIM-1125565) with support of the Gordon and Betty Moore Foundation (GBMF-1250) NSF DMR-1052647, and DARPA-QUANTUM HR0011-10-1-0066. LD acknowledges support from the NSF GRFP under grant no. DGE-1144469.

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