Supplementary material for "Beyond the spin model approximation for Ramsey spectroscopy"

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Change of Basis Formulas

The exact solutions rely on a change of basis between the individual-particle coordinate basis with wavefunctions $\psi_{n_1}(z_1)\psi_{n_2}(z_2)$ and the center of mass-relative coordinate basis with wavefunctions $\Psi_{n_R}(R)\Psi_{n_r}(r)$. To convert between these bases we introduce raising operators acting on the vacuum state $|0,0\rangle$, which is the same in both bases: $|n_1=0,n_2=0\rangle = |n_R=0,n_r=0\rangle$. We use the usual form of a raising operator $\hat{a}^\dagger = (\hat{x}-i\hat{p})/\sqrt{2}$ to define

$$\hat{a}_R^\dagger = \frac{1}{\sqrt{2}}(\hat{a}_{z_1}^\dagger + \hat{a}_{z_2}^\dagger), \quad \hat{a}_r^\dagger = \frac{1}{\sqrt{2}}(\hat{a}_{z_1}^\dagger - \hat{a}_{z_2}^\dagger)$$

We can create a particular state out of the vacuum to convert between the two bases:

$$|n_R,n_r\rangle = \frac{(\hat{a}_R^\dagger)^{n_R}(\hat{a}_r^\dagger)^{n_r}}{\sqrt{n_R!n_r!}}|0,0\rangle, \quad |n_1,n_2\rangle = \frac{(\hat{a}_{z_1}^\dagger)^{n_1}(\hat{a}_{z_2}^\dagger)^{n_2}}{\sqrt{n_1!n_2!}}|0,0\rangle$$

These binomials need to be expanded and re-grouped in the form

$$\langle R,r|n_R,n_r\rangle = \sum_{i=0}^{n_R+n_r} c_i^{n_R,n_r}\psi_i(x)\psi_{n_R+n_r-i}(y), \quad \langle x,y|n_1,n_2\rangle = \sum_{i=0}^{n_1+n_2} d_i^{n_1,n_2}\Psi_i(R)\Psi_{n_1+n_2-i}(r)$$

Grouping the terms, we find

$$c_i^{n_R,n_r} = \sqrt{\frac{i!(n_R+n_r-i)!}{2^{n_R}2^{n_r}n_R!n_r!}}\sum_{j=\max[0,n_r-i]}^{\min[n_r,n_R+n_r-i]}(-1)^j\binom{n_R}{n_R+n_r-i-j}\binom{n_r}{j} \tag{1}$$

$$d_i^{n_1,n_2} = \sqrt{\frac{i!(n_x+n_y-i)!}{2^{n_x}2^{n_x}n_x!n_y!}}\sum_{j=\max[0,n_y-i]}^{\min[n_y,n_x+n_y-i]}(-1)^j\binom{n_x}{n_x+n_y-i-j}\binom{n_y}{j}$$

Dependence of Ramsey Dynamics on Laser Pulses

Eq. (4) in the main text gives the generic form of the Ramsey dynamics in terms of functions $A(\tau)$, $B(\tau)$, and $C(\tau)$. These functions depend on the laser pulses through functions f_i, given by:

$$\begin{aligned}
f_1 &= \sin(\Delta\theta_1^{n_1,n_2})\sin(\Delta\theta_2^{n_1,n_2})\cos(\bar{\theta}_1^{n_1,n_2})\cos(\bar{\theta}_2^{n_1,n_2}) \\
f_2 &= \cos(\Delta\theta_1^{n_1,n_2})\cos(\Delta\theta_2^{n_1,n_2})\sin(\bar{\theta}_1^{n_1,n_2})\sin(\bar{\theta}_2^{n_1,n_2}) \\
f_3 &= \cos(\Delta\theta_1^{n_1,n_2})\cos(\bar{\theta}_2^{n_1,n_2})\sin(\Delta\theta_1^{n_1,n_2})\sin(\Delta\theta_2^{n_1,n_2}) \\
f_4 &= -\sin(\Delta\theta_1^{n_1,n_2})\sin(\Delta\theta_2^{n_1,n_2})\sin(\bar{\theta}_1^{n_1,n_2})\sin(\bar{\theta}_2^{n_1,n_2}) \\
f_5 &= -\cos(\Delta\theta_1^{n_1,n_2})\cos(\Delta\theta_2^{n_1,n_2})\cos(\bar{\theta}_1^{n_1,n_2})\cos(\bar{\theta}_2^{n_1,n_2})
\end{aligned} \tag{2}$$

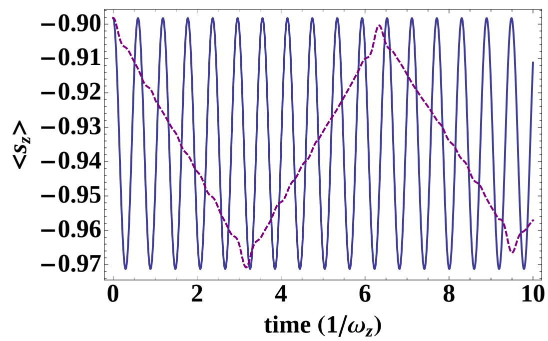

FIG. 1: Ramsey dynamics [see Eq. (4) in the main text] with $\delta = 0$ predicted by the 1D spin model (solid) and the exact solution (dashed) for an initial ($n_1 = 6$, $n_2 = 3$) mode configuration. Strong interactions ($u^{1,0}_{\uparrow\downarrow} = 100\omega_z$) are assumed during the dark time.

Calculation of Ramsey Dynamics

Here we briefly sketch the derivation of Eq. (5)$\Delta E_s(n_r) = \hbar u^{1,0}_{\uparrow\downarrow} \frac{\Gamma(n_r/2+1/2)}{\sqrt{\pi}\Gamma(n_r/2+1)} \Big(1 + \mathcal{O}(u^{1,0}_{\uparrow\downarrow}/\omega_z)\Big)$.

After the dark time, we expand the singlet back into individual-particle coordinates which is the convenient basis to calculate the action of a second laser pulse, with effective pulse area $\bar{\theta}_2^{n_1,n_2}$ and inhomogeneity $\Delta\theta_2^{n_1,n_2}$. The observable $\langle \hat{s}_z \rangle$ is calculated as the population difference between the $|t_{\uparrow\uparrow}\rangle$ and $|t_{\downarrow\downarrow}\rangle$ spin states, summed over each spatial mode. However, only the triplet contributes to the $\langle \hat{s}_z \rangle$ dynamics, and the triplet only contains the original spatial modes $|n_1, n_2\rangle$ and $|n_2, n_1\rangle$, so a major simplification can be made by only summing over these two modes. This simplification is what allows us to calculate the analytic form of Eq. (5).

We can also now understand better why the dependence of the dynamics on the second pulse area is not affected by dark-time interactions. Interactions induce mode changes and introduce new frequencies only to the singlet. The triplet is what determines the dynamics, however, where only the original modes $|n_1, n_2\rangle$ and $|n_2, n_1\rangle$ are present. The second pulse area affects these modes in exactly the same manner as in the spin model treatment of Refs. [1–4].

Strongly Interacting Dynamics

Fig. (1) in the supplementary materials shows the Ramsey dynamics predicted by both the spin model and exact calculation for the case of strong interactions. The spin model predicts oscillations at the interaction frequency, which are much faster than the true dynamics which oscillate at the trapping frequency. For fermions, when interactions become very large, the trapping frequency is the only remaining energy scale in the system.

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