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Estimate of the Distance to the Solar Wind Termination Shock From Gradients of Anomalous Cosmic Ray Oxygen

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The radial gradient of anomalous cosmic ray oxygen measured instantaneously between Voyager 2 and Pioneer 10 during 1985–1988 is correlated with the inferred tilt of the heliospheric neutral sheet. This is consistent with a simple model in which the radial gradient is related to the length of the neutral sheet between the two spacecraft. With this model we show that the radial gradient and the tilt of the neutral sheet near the solar wind termination shock can be inferred from the Voyager and Pioneer observations. By comparing the time history of the inferred tilt with that derived from solar observations, we estimate that the termination shock was at 62^{+5}_{-7} AU at solar minimum in 1987. At solar maximum the shock should be located at ~90 AU due to the increased pressure of the solar wind.

INTRODUCTION

There is increasing observational and theoretical evidence that during periods when the north polar magnetic field of the Sun is directed inwardly cosmic ray nuclei drift inward along the heliospheric neutral sheet when the tilt of the sheet is $\lesssim 30°$ [*Jokipii et al.*, 1977; *Jokipii and Thomas*, 1981; *Smith and Thomas*, 1986; *Smith*, 1990; *Cummings et al.*, 1990; *Lopate and Simpson*, 1991; *Potgieter*, 1992]. Reducing the tilt of the neutral sheet decreases the distance the particles have to drift inward along the sheet, thereby increasing the observed intensity.

The location of the termination shock is determined by the interaction of the solar wind with the local interstellar medium. Because the dynamic pressure of the solar wind varies by approximately a factor of 2 over the solar cycle [*Lazarus and McNutt*, 1989], the distance of the shock from the Sun should change by $(2)^{1/2}$, with the minimum occurring at solar minimum. It is believed that the anomalous cosmic rays (ACR) are accelerated at the termination shock [*Pesses et al.*, 1981]. This suggests that the radial and temporal variation of ACR fluxes and radial gradients may provide an estimate of the location of the shock.

Average radial gradients of cosmic rays in the outer heliosphere are generally computed from the logarithmic ratio of particle fluxes at two spacecraft divided by their separation distance. The "instantaneous" local gradient is determined by comparing fluxes observed simultaneously by the two spacecraft. A "shifted" gradient can also be determined by comparing fluxes observed at times differing by the convection time between the spacecraft. Both kinds of gradients have been used in the literature in various studies [See *Lockwood and Webber*, 1984].

The shifted gradient is usually regarded as an estimate of the local gradient that has been "corrected" for the propagation of transient solar modulation effects outward from the Sun at the solar wind speed. However, in the context of a simple model in which the radial gradient is a function of the tilt of the current sheet (for $qA < 0$, appropriate for the 1987 solar minimum time period), we will show that the shifted gradient is an estimate of the radial gradient near the termination shock. We have used this model previously to explain the observed time variation of the radial gradient of ACR oxygen in the outer heliosphere during 1984–1989 [*Cummings et al.*, 1990]. A preliminary account of this work can be found in the work by *Stone et al.* [1991].

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OBSERVATIONS

The fluxes of ACR oxygen at Voyager 2 (V2) and Pioneer 10 (P10) in the energy range 7.1–17.1 MeV/nucleon are shown in Figure 1. In Figure 2 we compare the time variation of the instantaneous radial gradient (Figure 2*c*) and the separation distance between the spacecraft with the variation in the average tilt angle of the neutral sheet between V2 and P10. The tilt angle midway between V2 and P10 is shown in Figure 2*a* and was estimated from the near-sun neutral sheet tilt value (J. T. Hoeksema, private communication, 1990), taking into account the delay time corresponding to an outward propagation at 400 km/s. The separation distance, both radially and along the neutral sheet, is shown in Figure 2*b*.

In Figure 3 we show the radial gradients as a function of the estimated average tilt of the neutral sheet between V2 and P10. The radial gradient is well-correlated with the neutral sheet for tilts $\lesssim 30°$ [see *Cummings et al.*, 1990]. This strong correlation led *Cummings et al.* [1990] to adopt a model in which the differential gradient is specified along the neutral sheet. In this model, positively charged particles drift and diffuse inwards along the neutral sheet during the $qA < 0$ portion of the solar cycle [*Jokipii et al.*, 1977]. The time variation of the tilt angle causes a corresponding variation in the distance between the two spacecraft along the neutral sheet, leading to a variation in the P10/V2 intensity ratio, and hence to a variation in the measured radial gradient given by $G_r = \ln(f_{P10}/f_{V2})/(r_{P10} - r_{V2})$, where f is flux and r is heliocentric radial position. In this model the differential

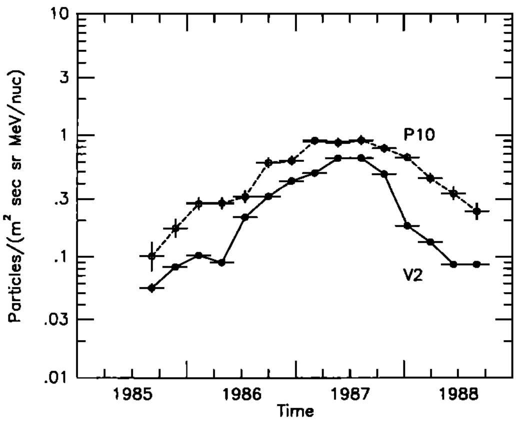

Fig. 1. The 78-day average fluxes of ACR oxygen at P10 and V2 versus time. The energy interval is 7.1–17.1 MeV/nucleon.

gradient along the neutral sheet is assumed to be given by $1/f(\partial f/\partial s) = C/r$, where C is a constant and s is distance along the neutral sheet. This model for a wavy neutral sheet is a consequence of the relationship derived by *Levy* [1978] for a flat neutral sheet, assuming $\kappa_\perp \propto r$ and the latitudinal gradient is independent of r [see *Cummings et al.*, 1990]. The solid line in Figure 2*c* shows the radial gradient computed from

$$G_r = \frac{\int_{r_{V2}}^{r_{P10}} \frac{1}{f}\frac{\partial f}{\partial r}\,dr}{(r_{P10}-r_{V2})} = \frac{\int_{r_{V2}}^{r_{P10}} \frac{1}{f}\frac{\partial f}{\partial s}\,ds}{(r_{P10}-r_{V2})} = \frac{\int_{r_{V2}}^{r_{P10}} \frac{C}{r}\,ds}{(r_{P10}-r_{V2})} \qquad (1)$$

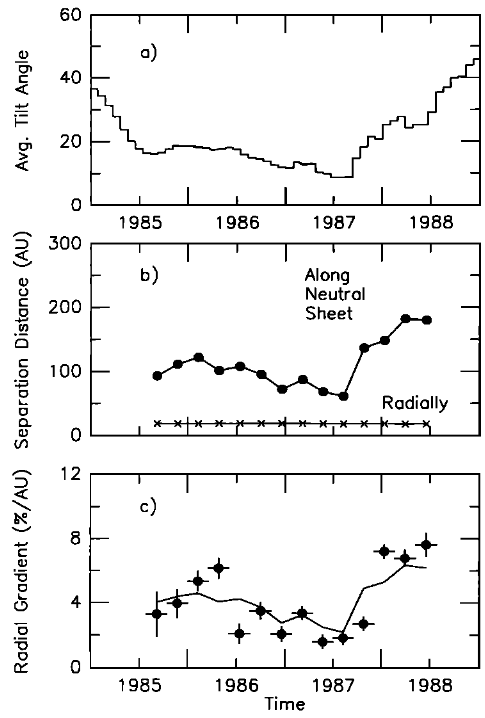

Fig. 2. (*a*) Estimated tilt of neutral sheet between V2 and P10 versus time. (*b*) V2 and P10 separation distance both radially (crosses) and along the neutral sheet (solid circles). (*c*) Instantaneous radial gradient between V2 and P10 of ACR oxygen in the energy interval 7.1–17.1 MeV/nucleon (solid circles). The solid line is calculated from a model as described in the text.

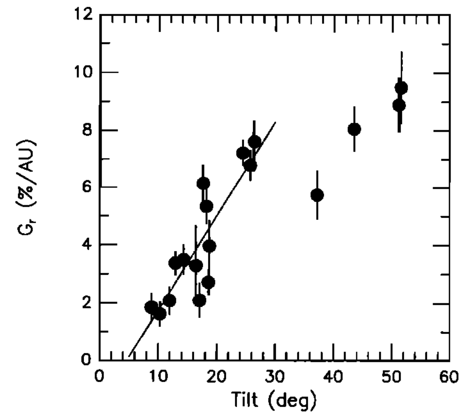

Fig. 3. Radial gradient of ACR oxygen between the radial positions of V2 and P10 versus neutral sheet tilt angle for 18 time periods between 1985 and 1989. The radial separation of the V2 and P10 spacecraft was ~18 AU for all periods, centered on an average radial position of 32 AU. The dotted line is a least squares fit to the data for tilts ≲30°.

(See *Cummings et al.* [1990] for a method of evaluating this integral.) The parameter C was determined by least squares fit to be 0.21, corresponding to a gradient of 21%/AU at 1 AU. The parameters of the wavy neutral sheet were assumed to be given by the tilt angle and a wavelength of 6.3 AU, corresponding to the 27-day solar rotation period and a solar wind velocity of 400 km/s. Note that from Figure 2*b* the radial separation distance $r_{P10} - r_{V2}$ (hereinafter ΔR) is ~18 AU throughout the period.

We used 78-day intervals in Figure 2*c* to facilitate the calculation of shifted radial gradients, since 78 days is approximately the propagation time of the solar wind between V2 and P10 during this time period. In Figure 4 we show the measured shifted radial gradients for 14 78-day epochs during 1985–1988.

DISCUSSION

Since in our model the particles are propagating inward along the neutral sheet from the solar wind termination shock, the shifted radial gradient (G_r^*) may be expressed in the following way:

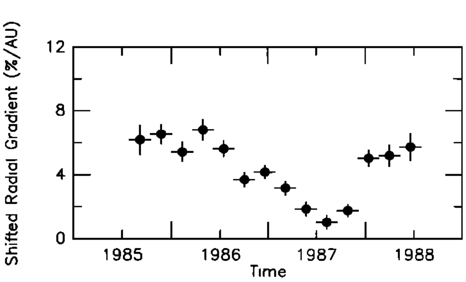

Fig. 4. Shifted radial gradients of ACR oxygen between V2 and P10 versus time.

$$G_r^*\Delta R = \int_{r_{V2}}^{R_S-\Delta R} \frac{C\,ds}{r}\bigg|_{t_0} + \int_{R_S-\Delta R}^{R_S} \frac{C\,ds}{r}\bigg|_{t_0} - \int_{r_{P10}}^{R_S} \frac{C\,ds}{r}\bigg|_{t_0+78\text{ days}} \qquad (2)$$

where the integral is along the neutral sheet and R_S is the heliocentric radial position of the solar wind termination shock (the assumed source of the ACR nuclei). The principal quantity that varies with time is the path length along the neutral sheet between the two spacecraft due to the time dependence of the tilt (see Figure 2*a* and 2*b*).

In the outer heliosphere, where $r \gg 1$ AU, the distance along the neutral sheet between two points at radial distances R and $R + \Delta R$ scales linearly with R. Thus for a given tilt and neutral sheet wavelength,

$$\frac{\int_R^{R+\Delta R} \frac{ds}{r}}{\Delta R}$$

is independent of R. The effect of this is that the first and third integrals in (2) cancel because they are over the same part of the neutral sheet.

This is illustrated in Figure 5 where we show two hypothetical "snapshots" of the neutral sheet. In this example we assume that R_S is at 100 AU and is fixed in time. The lower curve shows the neutral sheet 78 days after the upper curve. The heavy solid curves represent the same part of the neutral sheet, taking into account its propagation at the solar wind speed. The first integral of (2) is over the heavy solid upper curve, which is equal to the third integral which is over the heavy solid lower curve. Thus the result of (2) is the second integral, which is over the dashed portion of the upper curve in Figure 5:

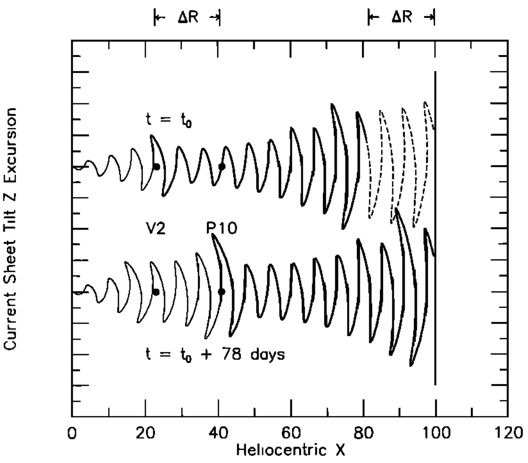

Fig. 5. Schematic cross section of wavy neutral sheet at two different times. The heavy solid curves represent the same portion of the sheet, taking into account propagation. The vertical line is the assumed shock position. The integral $\int ds/r$ over a portion of the upper curve is equal to the integral over the corresponding portion of the lower curve, for example, over the heavy portions of the two curves. See text for discussion.

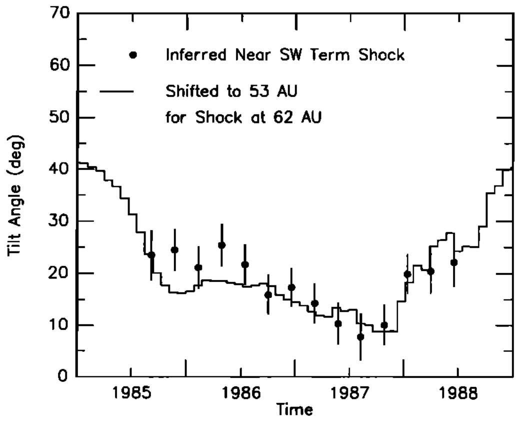

Fig. 6. Tilt angles inferred near the solar wind termination shock (solid circles) and compared with the Hoeksema data shifted to 53 AU at 400 km/s (histogram). The inferred tilt angles correspond to a position ~9 AU inside the shock; thus the inferred shock position is ~62 AU.

$$G_r^* = \frac{\int_{R_S-\Delta R}^{R_S} \frac{C\,ds}{r}\Big|_{t_0}}{\Delta R}. \qquad (3)$$

Thus within the context of our simple model the shifted gradient G_r^* is the average gradient within $\Delta R = 18$ AU of the termination shock at $t = t_0$.

With the assumptions that both the shape of the neutral sheet and $\kappa_\perp$ scale as r, the dependence of G_r on Θ displayed in Figure 3 should be independent of radius, and the best fit straight line given by $G_r(\mathrm{AU}^{-1}) = 0.0033\Theta - 0.015$ can be applied to infer a neutral sheet tilt angle Θ^* near the termination shock that would give rise to the measured G_r^*:

$$\Theta^* = \frac{(G_r^* + 0.015)}{0.0033} \qquad (4)$$

Note that the use of this empirical relationship does not require that the neutral sheet have the simple sinusoidal shape shown in Figure 5, only that the amplitude of the shape scales with radius.

Using (4) and the observed shifted gradients from Figure 4, we have inferred the tilt angle near the termination shock, Θ^*, for each epoch of 78 days. In deriving the appropriate errors we added an estimated 3.2° systematic uncertainty in quadrature to the statistical error. This value was estimated by using (4) (with G_r for G_r^* and Θ for Θ^*) to estimate the tilt angle Θ between V2 and P10 from the instantaneous gradients for each epoch and comparing it to the "observed" values (Figure 2*a*).

The inferred tilt angles near the termination shock for 14 epochs are shown in Figure 6, together with a curve which represents the Hoeksema tilt angle data shifted to 53 AU, the best fit value ($\chi_\nu^2 = 1.0$). Because $\Delta R = 18$ AU, the data points represent the tilt angle ~9 AU inside the shock and the inferred shock position is ~62 AU. The range of shock positions corresponding to the formal uncertainty (1σ) in the least squares fit is 55–67 AU. Figure 7 shows χ^2 as a function of shock distance.

The termination shock is expected to move outward

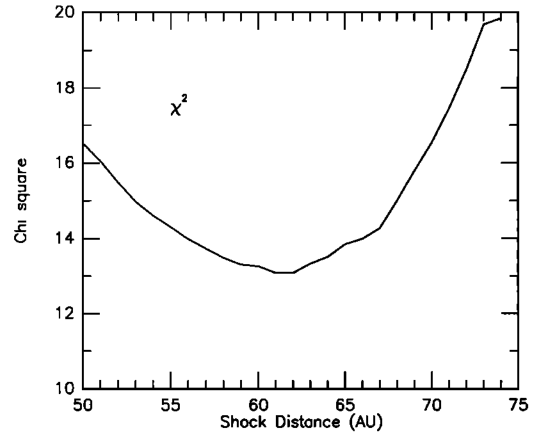

Fig. 7. The χ^2 value computed from the data in Figure 6.

during solar maximum conditions when the solar wind pressure increases. *Lazarus and McNutt* [1989] find a variation of approximately a factor of 2 in the solar wind pressure as measured from the plasma detector on V2 from solar minimum to solar maximum. In that case, considerations of pressure balance with the local interstellar medium implies that the termination shock should move by a factor of $(2)^{1/2}$ [*Holzer*, 1989]. Thus we expect that at solar maximum the termination shock may be as distant as ~90 AU.

On the basis of the time history of the solar wind pressure presented by *Lazarus and McNutt* [1989] we estimate that the next period of minimum solar wind pressure, and when the shock can be expected to be in the vicinity of ~62 AU, is ~1998. The P10 and V1 spacecraft will both reach ~72 AU by the end of 1998. Hence it appears reasonable that either or both spacecraft have a good chance of encountering the shock by the turn of the century.

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