NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1127

A DISCUSSION OF THE APPLICATION OF THE PRANDTL-GLAUERT METHOD TO SUBSONIC COMPRESSIBLE FLOW OVER A SLENDER BODY OF REVOLUTION

By Lester Lees

Langley Memorial Aeronautical Laboratory
Langley Field, Virginia

Washington
September 1946

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE No. 1127

A DISCUSSION OF THE APPLICATION OF THE PRANDTL-GLAUERT METHOD TO SUBSONIC COMPRESSIBLE FLOW OVER A SLENDER BODY OF REVOLUTION

By Lester Lees

SUMMARY

The Prandtl-Glauert method for subsonic potential flow of a compressible fluid has generally been believed to lead to an increase in the pressures over a slender body of revolution by a factor $1/\sqrt{1 - M_1^2}$ (where M_1 is Mach number in undisturbed flow) as compared with the pressures in incompressible flow. Recent German work on this problem has indicated, however, that the factor $1/\sqrt{1 - M_1^2}$ is not applicable in this case. In the present discussion a more careful application of the Prandtl-Glauert method to three-dimensional flow gives the following results:

The Prandtl-Glauert method does not lead to a universal velocity or pressure correction formula that is independent of the shape of the body. The factor $1/\sqrt{1 - M_1^2}$ is applicable only to the case of two-dimensional flow.

The increase with Mach number of the pressures over a slender body of revolution is much less rapid than for a two-dimensional airfoil. An approximate formula from which the increase can be estimated is derived theoretically.

The increase with Mach number of the maximum axial interference velocity on a slender body of revolution in a closed wind tunnel is given approximately by the factor $1/(1 - M_1^2)^{3/2}$, rather than by the factor $1/(1 - M_1^2)^2$ previously obtained by Goldstein and Young and by Tsien and Lees.

INTRODUCTION

Because of its simplicity and applicability to both two- and three-dimensional flow, the Prandtl-Glauert method has been utilized to obtain approximate solutions for a wide variety of problems in subsonic compressible flow. The effects of wind-tunnel interference on thin airfoils and on slender bodies of revolution in subsonic flow have been thoroughly discussed in references 1 and 2. In common with many earlier writers on the subject, however, including Prandtl and von Kármán (see references 3 to 6), the authors of references 1 and 2 state without clear proof that the Prandtl-Glauert method leads to an increase by a factor of $1/\sqrt{1 - M_1^2}$ in the pressures acting on the surface of a slender body of revolution in an unbounded, uniform parallel flow. (The symbol M_1 denotes Mach number in undisturbed flow.)

The present paper was prompted by recent German work on the problem of the subsonic flow over a body of revolution, which indicates that application of the factor $1/\sqrt{1 - M_1^2}$ is entirely incorrect in this case (references 7 and 8). Some German writers (references 9 and 10) have even stated that there is no effect of compressibility on the pressures over a slender body of revolution in an unbounded stream - at least in first approximation. In the present paper, the general problem of the nearly parallel subsonic compressible flow over a closed body is treated as a straight-forward boundary-value problem. It is pointed out that previous writers (references 1 to 6) were led to erroneous conclusions for the case of three-dimensional flow either because they disregarded the boundary conditions or because they did not examine the boundary conditions with sufficient care. In order to illustrate the general method of the present paper, the problem of the subsonic flow over a slender body of revolution is discussed and the particular case of the ellipsoid of revolution is studied in detail.

The question of the application of the Prandtl-Glauert method to three-dimensional subsonic flow was treated correctly for the first time by Göthert. In reference 7, Göthert shows that the pressure coefficients on the surface of a body in a nearly parallel subsonic flow are obtained by calculating the incompressible flow over a body, the lateral coordinates of which are contracted in the ratio $\sqrt{1 - M_1^2} : 1$ as compared with the lateral coordinates of the original body, and then multiplying the pressure coefficients on the surface of this distorted body by the factor $\frac{1}{1 - M_1^2}$. The results obtained by Göthert's procedure and by the method of the present paper will be equivalent in general. It is hoped that the reasons for the essential difference between the two- and three-dimensional flow will be brought out more clearly by the present treatment than by Göthert's procedure.

The author is indebted to Dr. S. Katzoff for valuable discussions of this problem and criticism of the present paper.

ANALYSIS

Prandtl-Glauert Method for Subsonic Compressible Flow

The principal assumptions of the Prandtl-Glauert method for subsonic compressible flow are as follows:

(1) The influence of the viscosity and conductivity of the gas is neglected.

(2) The pressure is a continuous function of the density.

(3) Changes in pressure and density in the flow are small compared with the mean pressure and mean density: that is, the deviations from a uniform parallel flow are small.

The Prandtl-Glauert method thus deals with subsonic flows that are shock free, or isentropic, and therefore irrotational by Helmholtz' laws of vortex motion for a gas. In a Cartesian coordinate (x, y, z) system, the velocity potential Φ is expressed by the relation

$$\Phi(x, y, z) = U_1 x + \varphi(x, y, z) \tag{1}$$

where φ is the disturbance potential and U_1 is the velocity of the undisturbed flow. The corresponding velocity components are

$$v_x = U_1 + u, \quad v_y = v, \quad v_z = w \tag{2}$$

where

$$u = \frac{\partial\varphi}{\partial x}, \quad v = \frac{\partial\varphi}{\partial y}, \quad w = \frac{\partial\varphi}{\partial z} \tag{3}$$

and

$$\frac{u}{U_1}, \frac{v}{U_1}, \frac{w}{U_1} << 1.0$$

With the assumption that the squares and products of the disturbance velocities u, v, w are negligible compared with the velocities themselves, the exact equation of continuity is approximated by the partial differential equation for the disturbance potential

$$\left(1 - M_1^2\right)\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0 \tag{4}$$

where

M_1 Mach number in undisturbed flow $\left(\frac{U_1}{a_1}\right)$

and

a_1 speed of sound in undisturbed flow

When the Mach number is vanishingly small, equation (4) reduces to Laplace's equation for incompressible potential flow.

A solution of equation (4) must satisfy the following boundary conditions:

(1) The disturbance velocities u, v, and w must vanish in the undisturbed flow.

(2) At the surface of a solid boundary, the component of the gas velocity normal to the surface must vanish. Since $U_1 + u \approx U_1$, this condition is

$$U_1 \cos \alpha + v \cos \beta + w \cos \gamma = 0 \tag{5}$$

where α, β, and γ are the angles that the normal to the body surface makes with the x-, y-, and z-axes, respectively.

In order to solve this boundary-value problem, Prandtl and Glauert introduce new Cartesian coordinates ξ, η, ζ which are related to the coordinates x, y, and z by the affine transformation

$$\left.\begin{aligned} \xi &= x \\ \eta &= y\sqrt{1 - M_1^2} \\ \zeta &= z\sqrt{1 - M_1^2} \end{aligned}\right\} \tag{6}$$

$\phi \approx \sqrt{1-M^2}\,\varphi$

In the new ξ, η, ζ system, the disturbance potential satisfies Laplace's equation

$$\frac{\partial^2 \varphi}{\partial \xi^2} + \frac{\partial^2 \varphi}{\partial \eta^2} + \frac{\partial^2 \varphi}{\partial \zeta^2} = 0 \tag{7}$$

If the velocity potential is regarded as unchanged in the transformation, the value of φ at the point (x, y, z) in physical space is identical with the value of φ at the point (ξ, η, ζ) in the new space given by equations (6). The relations between the disturbance velocities in the ξ, η, ζ system (u', v', w') and the physical disturbance velocities are as follows:

$$\left.\begin{aligned}
u(x, y, z) &= \frac{\partial \varphi}{\partial x} = \frac{\partial \varphi}{\partial \xi}\frac{d\xi}{dx} = u'(\xi, \eta, \zeta) \\
v(x, y, z) &= \frac{\partial \varphi}{\partial y} = \frac{\partial \varphi}{\partial \eta}\frac{d\eta}{dy} = \sqrt{1 - M_1{}^2}\ v'(\xi, \eta, \zeta) \\
w(x, y, z) &= \frac{\partial \varphi}{\partial z} = \frac{\partial \varphi}{\partial \zeta}\frac{d\zeta}{dz} = \sqrt{1 - M_1{}^2}\ w'(\xi, \eta, \zeta)
\end{aligned}\right\} \quad (8)$$

The lateral coordinates in the ξ, η, ζ system are contracted, but the lateral velocity components in the ξ, η, ζ space are magnified as compared with the lateral velocity components at the corresponding point in physical space.

The boundary conditions that must be satisfied by the solution of Laplace's equation (equation (7)) can now be formulated as follows:

(1) In the undisturbed stream, u', v', and w' must vanish.

(2) At a point on the surface of the solid boundary (x_s, y_s, z_s), the lateral velocity components v_s and w_s must satisfy equation (5). These velocities are given by

$$\left.\begin{aligned}
v_s(x_s, y_s, z_s) &= \sqrt{1 - M_1{}^2}\left(\frac{\partial \varphi}{\partial \eta}\right)_{\xi_s,\eta_s,\zeta_s} \\
w_s(x_s, y_s, z_s) &= \sqrt{1 - M_1{}^2}\left(\frac{\partial \varphi}{\partial \zeta}\right)_{\xi_s,\eta_s,\zeta_s}
\end{aligned}\right\} \quad (9)$$

where

$$\xi_s = x_s, \quad \eta_s = \sqrt{1 - M_1{}^2}\ y_s, \quad \zeta_s = \sqrt{1 - M_1{}^2}\ z_s$$

The potential $\varphi(\xi, \eta, \zeta)$ is set up as a general solution of Laplace's equation that satisfies condition (1) and is appropriate to the problem. The arbitrary constants in the general solution $\varphi(\xi, \eta, \zeta)$ are determined by substituting equations (9) in equation (5). Thus, by use of the Prandtl-Glauert method the problem of the nearly parallel subsonic compressible flow over a closed body is reduced to a problem in incompressible potential flow.

The procedure outlined in the present paper was strictly carried out for the first time by Bilharz and Hölder in 1940 for the particular case of a body of revolution of 10-percent thickness ratio, with its maximum thickness at 40 percent chord (reference 11). Bilharz and Hölder, however, did not discuss the general implications of their results. Previous writers on the subject attempted to draw conclusions directly from the differential equation (4) and overlooked the necessity of satisfying the boundary conditions.

Prandtl-Glauert Method for Two-Dimensional Subsonic Flow

For two-dimensional subsonic flow, the Prandtl-Glauert method leads directly to the result that the pressures and the axial disturbance velocities on a thin cylindrical body at a given small angle of attack are increased by a factor of $1/\sqrt{1 - M_1^2}$ over the corresponding values for incompressible flow. The analysis in this case is simple largely because the contraction of the coordinates of the body surface, which is required by equations (9), never enters the problem. For two-dimensional flow, the lateral velocity component on the surface of a thin cylindrical body $v_s(x_s, y_s)$ differs from the velocity on the chord $v(x_s, 0)$ by a quantity of higher order. This fact can be seen from physical considerations or from the development of the velocity $v(x_s, y)$ in a Taylor's series

$$v(x_s, y) = v(x_s, 0) + \left(\frac{\partial v}{\partial y}\right)_{x_s,0} y_s + \ldots$$

From the approximate continuity equation

$$\frac{\partial v}{\partial y} = \left(M_1^2 - 1\right) \frac{\partial u}{\partial x}$$

and the fact that $\left(\frac{\partial u}{\partial x}\right)_{x_s,0}$ is of the order of y_s, it can be seen that $v_s(x_s, y_s)$ differs from $v(x_s, 0)$ by a quantity of the order of v_s^2.

The boundary condition that the resultant velocity on the surface must be tangential to the surface can then be satisfied by the velocity on the chord $v(x_s, 0)$. This condition is simply

$$v(x_s, 0) = U_1 \tan \beta (x_s) \tag{10}$$

where

$$-\frac{c}{2} \leqq x_s \leqq \frac{c}{2}$$

In equation (10) $\tan \beta (x_s)$ is the slope of the surface at the point (x_s, y_s) and c is the chord of the body. The boundary condition for a solution of Laplace's equation in the ξ, η plane is

$$v'(\xi_s, 0) = \frac{U_1 \tan \beta (\xi_s)}{\sqrt{1 - M_1^2}} \tag{11}$$

where

$$-\frac{c}{2} \leqq \xi_s \leqq \frac{c}{2}$$

The appropriate solution of Laplace's equation is given by a continuous distribution of "singularities" (vortices and/or sources and sinks) along the chord. The strength of these singularities per unit length is directly proportional to the factor $U_1/\sqrt{1 - M_1^2}$ in the expression for the required normal velocity $v'(\xi_s, 0)$ given by equation (11). In the limiting case of an incompressible fluid, M_1 approaches 0 and the strength of the singularities is proportional to U_1. In the present case, in which $M_1 \neq 0$, the vortex or the source-sink strength along the chord must therefore be greater than the strength for $M_1 = 0$ by a factor of $1/\sqrt{1 - M_1^2}$. Since the axial disturbance velocity $u'(\xi_s, 0)$ is also proportional to the strength of the singularities, this velocity is also increased by the factor $1/\sqrt{1 - M_1^2}$. By equation (8), therefore,

$$\frac{\left[u(x_s, 0)\right]_{M_1}}{\left[u(x_s, 0)\right]_0} = \frac{\left[u'(\xi_s, 0)\right]_{M_1}}{\left[u'(\xi_s, 0)\right]_0} = \frac{1}{\sqrt{1 - M_1^2}} \tag{12}$$

<u>regardless of the shape of the body or the angle of attack</u>.

Integration of Bernoulli's equation gives

$$\frac{C_{p_{M_1}}}{C_{p_0}} = \frac{1}{\sqrt{1 - M_1^2}} \tag{13}$$

where

$$C_p = \frac{p - p_1}{\frac{1}{2}\rho_1 U_1^2}$$

and

p local static pressure

p_1 static pressure in undisturbed flow

ρ_1 density in undisturbed flow

Prandtl-Glauert Method for Three-Dimensional Subsonic Flow

For three-dimensional subsonic flow, the application of the Prandtl-Glauert method is not quite so simple as for two-dimensional flow. The lateral velocity components on the surface of the body v_s and w_s do not differ from the lateral velocities on the axis of the body by quantities of higher order. In the case of a body of revolution, for example, the radial velocity component v_r near the axis is very nearly equal to $\frac{f(x)}{4\pi r}$, where $f(x)$ represents the strength of the source-sink distribution along the axis and r is the distance from the axis. In fact, it can be shown from the equation of continuity (equation (4)) and the condition of irrotationality that the radial velocity component v_r near the axis is of the form

$$v_r = \frac{f(x)}{r} + g(x)r \log r + \dots$$

Thus, the error made by neglecting the additional terms in the expression for v_r is in the ratio $r^2 \log r$ to $\frac{f(x)}{r}$, and this error is beyond the limits of a first approximation.

The radial velocity component thus increases indefinitely as the axis is approached - in contrast to the situation in the case of two-dimensional flow. Since the lateral velocity components on the surface of the body of revolution v_s and w_s obviously cannot be replaced by the lateral velocities on the axis, the contraction of the coordinates of the body in the ξ, η, ζ system enters the problem; consequently the shape of the body becomes important. This fact has apparently been overlooked in the discussions of the problem given in references 1 to 6.

The boundary condition at a point on the surface of the body (x_s, y_s, z_s) is

$$v_{r_s}(x_s,\ y_s,\ z_s) = \sqrt{v_s{}^2 + w_s{}^2} = U_1 \tan \beta\ (x_s) \qquad (14)$$

where

$$\frac{c}{2} \leqq x_s \leqq \frac{c}{2}$$

In equation (14) $\tan \beta\ (x_s)$ is the slope of the surface in a meridan plane and c is the length of the body. The boundary condition to be satisfied at the point $(\xi_s,\ \eta_s,\ \zeta_s)$ by a solution of Laplace's equation is

$$v'_{r_s}(\xi_s,\ \eta_s,\ \zeta_s) = \frac{U_1 \tan \beta\ (\xi_s)}{\sqrt{1 - M_1{}^2}} \qquad (15)$$

where

$$-\frac{c}{2} \leqq \xi_s \leqq \frac{c}{2}$$

The appropriate solution of Laplace's equation is given by a continuous distribution of sources and sinks along the axis between $\xi = -\frac{c}{2}$ and $\xi = \frac{c}{2}$, of strength $f(\xi)$ per unit length. The strength $f(\xi)$ is directly related to the required lateral velocity v'_{r_s} by

$$v'_{r_s}(\xi_s,\ \eta_s,\ \zeta_s) = \frac{1}{4\pi} \frac{f(\xi_s)}{\sqrt{\eta_s{}^2 + \zeta_s{}^2}}$$

$$= \frac{1}{4\pi \sqrt{1 - M_1{}^2}} \frac{f(\xi_s)}{\sqrt{y_s{}^2 + z_s{}^2}} \qquad (16)$$

Thus, from equation (15) the strength $f(\xi)$ of the source-sink distribution on the axis is proportional to $U_1 \tan \beta(\xi_s)$ for the given body, independently of the Mach number of the undisturbed flow.

(Since the equivalent source-sink distribution along the axis of a slender body of revolution is independent of Mach number in first approximation, an analysis similar to that given in reference 2 shows that the increase with Mach number of the maximum axial interference velocity on a slender body of revolution in a closed wind tunnel is given approximately by the factor $1/(1 - M_1^2)^{3/2}$ rather than by the factor $1/(1 - M_1^2)^2$ previously obtained in references 1 and 2. This result is in agreement with the conclusions reached by von Baranoff in reference 12.)

The fact that $f(\xi)$ is independent of Mach number does not mean that the axial disturbance velocity $u(x_s, y_s, z_s) = u'(\xi_s, \eta_s, \zeta_s)$ is also independent of Mach number. The point (ξ_s, η_s, ζ_s) lies closer to the axis for a Mach number $M_1 \neq 0$ than for incompressible flow. For the given source-sink distribution, the value of $u'(\xi_s, \eta_s, \zeta_s)$ near the maximum thickness of the body will therefore be larger than the corresponding value for incompressible flow. An estimate of this increase can be obtained by an examination of the incompressible field of flow around a thin ellipsoid of revolution (prolate spheroid). (See reference 13.) The disturbance velocity potential is

$$\varphi(\lambda, \mu) = A\mu\left(\frac{\lambda}{2} \log \frac{\lambda + 1}{\lambda - 1} - 1\right) \tag{17}$$

where λ and μ are elliptic coordinates and A is a constant. (The symbol λ is used instead of Lamb's ζ in order to avoid confusion.) In the vicinity of the maximum thickness of the body,

$$\left.\begin{aligned} &\lambda \rightarrow \sqrt{1 + \frac{\eta^2 + \zeta^2}{(c/2)^2}} \\ &\mu \rightarrow 0 \end{aligned}\right\} \tag{18}$$

In equation (18) c is the length of ellipsoid of revolution. The maximum axial disturbance velocity $u'_{max}(\xi, \eta, \zeta)$ is approximated by

$$u'_{max}(\xi, \eta, \zeta) \approx \frac{\sqrt{1 - \mu^2}}{\frac{c}{2}\sqrt{\lambda^2 - \mu^2}} \left(\frac{\partial\varphi}{\partial\mu}\right)_{\mu\to 0}$$

$$\approx 1 - \log 2 + \log \frac{\sqrt{\eta^2 + \zeta^2}}{c/2} \qquad (19)$$

Therefore

$$u_{max}(x_s, y_s, z_s) \approx 1 - \log 2 + \log\left(\sqrt{1 - M_1{}^2}\,\frac{t}{c}\right) \qquad (20)$$

where t/c is the thickness ratio of the body. It follows that

$$\frac{\left(u_{max}\right)_{M_1}}{\left(u_{max}\right)_0} = \frac{c_{p_{M_1}}}{c_{p_0}} = 1 + \frac{\log\sqrt{1 - M_1{}^2}}{0.31 + \log\frac{t}{c}} \qquad (21)$$

Schmieden and Kawalki in reference 8 obtained the same result by a careful application of the procedure outlined by Göthert (reference 7). The original expression given by Göthert is incorrect.

From equation (21) the effect of compressibility can be seen to vanish as the thickness ratio approaches zero - in contrast to the case of two-dimensional flow - where the first-order effect of compressibility is given by $1/\sqrt{1 - M_1{}^2}$, independently of the shape of the body or the angle of attack of the body.

For slender bodies of revolution that are symmetrical or nearly symmetrical longitudinally, equation (21) should give some indication of the first-order effect of compressibility. This equation is plotted in figure 1 for various values of the thickness ratio t/c. The effect of compressibility is evidently considerably smaller than the factor $1/\sqrt{1 - M_1{}^2}$ would predict. For an ellipsoid of revolution of 10-percent thickness ratio the estimated increase in maximum negative pressure coefficient is only 25 percent at a Mach number of 0.80. The increase in maximum pressure coefficient obtained in reference 11 by an approximate numerical procedure for a body of revolution of the same thickness ratio with its maximum thickness at 40 percent chord is 13 percent. By comparison, the factor $1/\sqrt{1 - M_1^2}$ predicts an increase of 67 percent in the axial disturbance velocity and pressure coefficient.

In passing, it should be noted that the inapplicability of the factor $1/\sqrt{1 - M_1^2}$ for three-dimensional subsonic flow is indicated by the results of the calculation of the subsonic flow over a sphere by the Janzen-Rayleigh method of iteration (reference 14). The first-order effect of compressibility on the pressures must be less for a slender body of revolution than for a thick body, such as the sphere. Since the maximum pressure coefficient on the sphere at the critical Mach number $M_1 = 0.573$ is less than the factor $1/\sqrt{1 - M_1^2}$ would predict (fig. 1), this factor cannot possibly give the correct result for a slender body of revolution.

Because the axial disturbance velocity near the maximum thickness of a slender body of revolution is small ($\approx 0.030U_1$ in the example of reference 11) and the increase of this velocity with Mach number is also small, the assumptions of the Prandtl-Glauert method are more nearly satisfied at considerably higher subsonic Mach numbers in three-dimensional flow than in two-dimensional flow. For the same reasons, the so-called critical Mach number for a slender body of revolution is quite close to unity. This conclusion is generally applicable to any three-dimensional body, the lateral dimensions of which are small compared with the dimensions in the direction of motion, for example, the highly swept-back thin wing.

CONCLUDING REMARKS

An analysis of the Prandtl-Glauert method for subsonic compressible flow around a slender body of revolution led to the following conclusions:

1. The method does not give a universal velocity or pressure correction formula that is independent of the shape of the body. The factor $1/\sqrt{1 - M_1^2}$ (where M_1

is Mach number in undisturbed flow) is applicable only to problems of two-dimensional flow.

2. The increase with Mach number of the pressure coefficients and axial velocities on the surface of a slender body of revolution is much less rapid than for a two-dimensional airfoil. An approximate estimate of this increase for slender bodies of revolution that are symmetrical or nearly symmetrical longitudinally can be obtained from the expression

$$\frac{\left(u_{max}\right)_{M_1}}{\left(u_{max}\right)_0} = 1 + \frac{\log \sqrt{1 - M_1{}^2}}{0.31 + \log \frac{t}{c}}$$

where u_{max} is maximum axial disturbance velocity and t/c is the thickness ratio of the body. (This expression is derived from a consideration of a thin ellipsoid of revolution.)

3. The so-called critical Mach number for a slender body of revolution is quite close to unity. This conclusion is generally applicable to any three-dimensional body, the lateral dimensions of which are small compared with the dimension in the direction of motion, for example, the highly swept-back wing.

4. The equivalent source-sink distribution along the axis of a slender body of revolution is independent of Mach number in first approximation. Therefore, the increase with Mach number of the maximum axial interference velocity on a slender body of revolution in a closed wind tunnel is given approximately by the factor $1/\left(1 - M_1{}^2\right)^{3/2}$ rather than by the factor $1/\left(1 - M^2\right)^2$ previously obtained by Goldstein and Young and by Tsien and Lees.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., February 13, 1946

REFERENCES

1. Goldstein, S., and Young, A. D.: The Linear Perturbation Theory of Compressible Flow, with Applications to Wind-Tunnel Interference. R. & M. No. 1909, British A.R.C., 1943.

2. Tsien, Hsue-Shen, and Lees, Lester: The Glauert-Prandtl Approximation for Subsonic Flows of a Compressible Fluid. Jour. Aero. Sci., vol. 12, no. 2, April 1945, pp. 173-187, 202.

3. Prandtl, L.: Über Strömungen deren Geschwindigkeiten mit der Schallgeschwindigkeit vergleichbar sind. Jour. Aero. Res. Inst., Tokyo Imperial Univ., No. 65, 1930, p. 14.

4. Prandtl, L.: Theorie des Flugzeugtragflügels im zusammendrückbaren Medium. Luftfahrtforschung, Bd. 13, Nr. 10, Oct. 12, 1936, p. 317.

5. von Kármán, Th.: The Problem of Resistance in Compressible Fluids. R. Accad. d'Italia, cl. sci. fis., mat. e nat., vol. XIV, 1936, pp. 17-24. (GALCIT Pub. No. 75, 1936.)

6. von Kármán, Th.: Compressibility Effects in Aerodynamics. Jour. Aero. Sci., vol. 8, no. 9, July 1941, pp. 337-356.

7. Göthert, B.: Ebene und räumliche Strömung bei hohen Unterschallgeschwindigkeiten. (Erweiterung der Prandtlschen Regel). Bericht 127 der Lilienthal-Gesellschaft für Luftfahrtforschung, 1940, pp. 97-101.

8. Schmieden, C., and Kawalki, K. H.: Beiträge zum Umströmungsproblem bei hohen Geschwindigkeiten. Bericht S 13ll. Teil der Lilienthal-Gesellschaft für Luftfahrtforschung, 1942, pp. 40-66.

9. Sauer, Robert: Theoretische Einführung in die Gasdynamik. Julius Springer (Berlin), 1943, pp. 30-34.

10. Wieselsberger, C.: Windkanalkorrekturen bei kompressibler Strömung. Bericht 127 der Lilienthal-Gesellschaft, 1940, pp. 3-7.

11. Bilharz, H., and Hölder, E.: Zur Berechnung der Druckverteilung an Rotationskörpern in der Unterschallströmung eines Gases. Teil I: Achsensymmetrische Strömung. Forschungsbericht Nr. 1169/1, Deutsche Luftfahrtforschung (Braunschweig), 1940.

12. von Baranoff, A.: Zur Frage der Kanalkorrektur bei kompressibler Unterschallströmung. Forschungsbericht Nr. 1272, Deutsche Luftfahrtforschung (Braunschweig), 1940.

13. Lamb, Horace: Hydrodynamics. Reprint of sixth ed. (first American ed.), Dover Publications (New York), 1945, pp. 139 and 141.

14. Kaplan, Carl: The Flow of a Compressible Fluid past a Sphere. NACA TN No. 762, 1940.

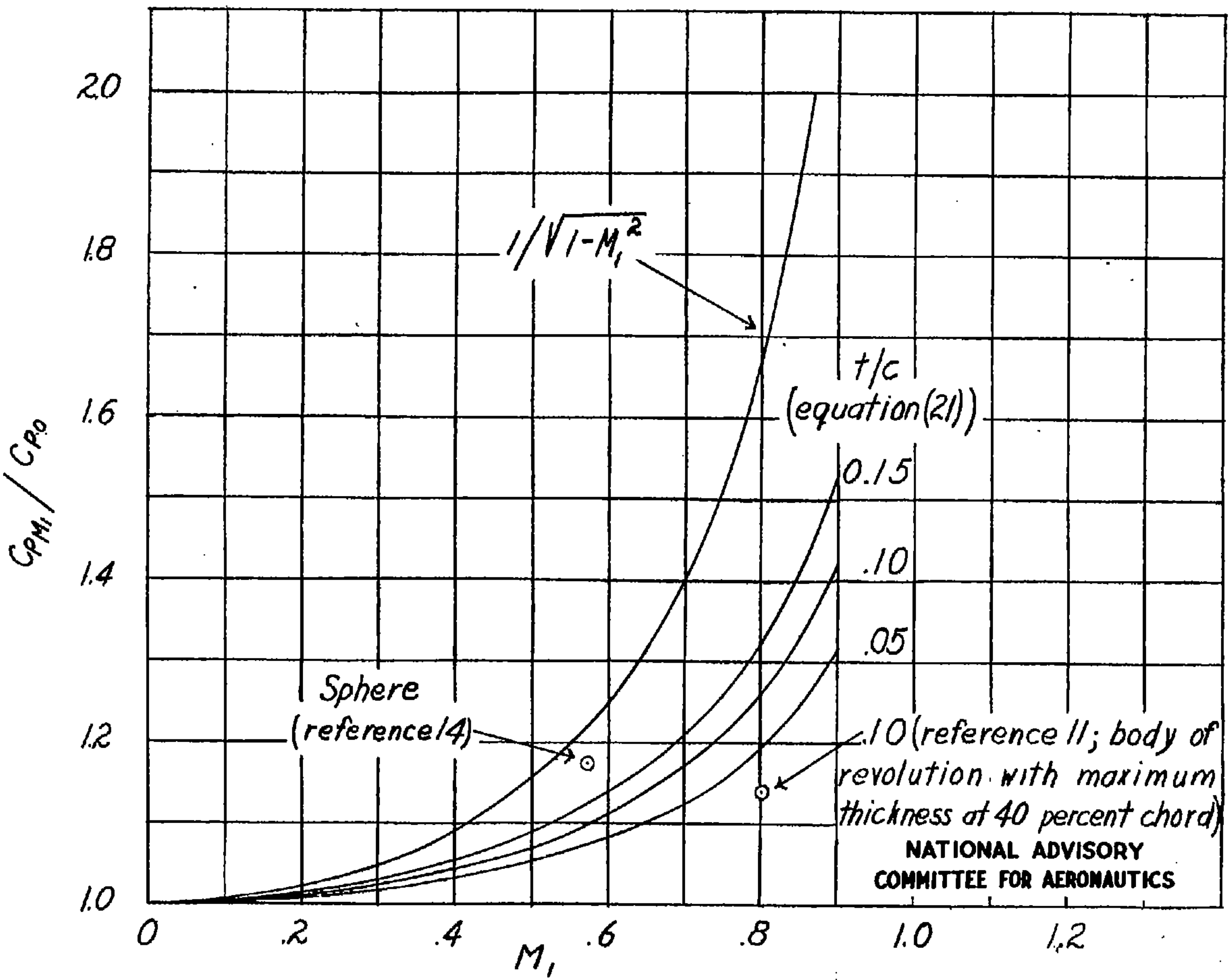

Figure 1.- Increase of maximum negative pressure coefficient with Mach number for a thin ellipsoid of revolution.