

by a reversible analytic transformation will carry the field and associated topological set into an analytic field and associated topological set.

The application of these results to deformations, and coincidences of transformations of the same class does not need elaboration.

¹ Veblen, "Analysis Situs," *The Cambridge Colloquium*, Part II, § 24.

² Eisenhart, "Riemannian Geometry," *Princeton University Press*.

³ Hopf, "Über die Curvatura integra geschlossener Hyperflächen," *Math. Ann.*, **95**, (1925), 340-351; Brouwer, "Über Abbildung von Mannigfaltigkeiten," *Ibid.*, **71**, (1911), 97-115.

⁴ Hopf, "Vectorfelder in n -dimensionalen Mannigfaltigkeiten," *Ibid.*, **96** (1926), 225-249.

⁵ Lefschetz, "Manifolds with a Boundary and Their Transformations," *Trans. Amer. Math. Soc.*, **29** (1927), 429-463.

*REMARK ON THE NUMBER OF CLASSES OF BINARY
QUADRATIC FORMS OF A GIVEN NEGATIVE
DETERMINANT*

BY E. T. BELL

DEPARTMENT OF MATHEMATICS, CALIFORNIA INSTITUTE OF TECHNOLOGY

Communicated March 31, 1928

On p. 254 of Mathews' "Theory of Numbers," Part I, 1892 (all that was published), we find the following clear statement of a desideratum that has often been expressed. ". . . leads to the conclusion that in the series 1, 2, 3, . . . $(p - 1)/2$ (p is an odd prime), there are more quadratic residues of p than non-residues. It does not appear that any independent proof of this proposition has ever been discovered. If any such proof could be found, it is not impossible that it might lead to a determination of h (the number of classes described in the title of this note) without the use of infinite series. Similar remarks apply to the other formulae for negative determinants."

The material for at least four such required proofs is contained in papers published by the writer during the past ten years. As the implied proofs are buried as mere details in other work, it is of interest to disinter one. On p. 113 of the *Tôhoku Mathematical Journal*, vol. **19**, Nos. 1, 2 (May, 1921), will be found the sufficient means for constructing a proof of the theorem depending only upon the rudiments of elementary algebra. In the paper cited, free use is made of elliptic theta identities. This is not necessary. In fact, as is evident, and is indeed well known, all basic identities of the kind used in the paper are obtainable from strictly elementary considerations; they have been so obtained by J. V. Ouspensky in a series of memoirs in the *Bulletin de l'Académie des Sciences de l'URRS*,

1925. Anyone who is interested may easily reconstruct the somewhat tedious entirely elementary proof for himself. I have constructed such a proof in detail. It is not worth reproduction. The thing can be done, but it does not seem worth while to do it.

The hope expressed by Mathews that h may be determined without the use of infinite series can be artificially realized on this basis. The gain in clarity by such a procedure is at best doubtful, and the like applies to the indicated proof concerning the distribution of quadratic residues. It seems rather futile to recast what is actually a matter of simple algebra in terms of so-called pure arithmetic when the latter is no more than a sophisticated disguise of the former.

CONCERNING CERTAIN TYPES OF NON-CUT POINTS, WITH
AN APPLICATION TO CONTINUOUS CURVES¹

BY HARRY MERRILL GEHMAN

DEPARTMENT OF MATHEMATICS, YALE UNIVERSITY

Communicated April 5, 1928

In our study of irreducible continua, we have been led to consider certain special types of non-cut points of a continuum. As these special types may be of use also in other connections, it seems best to set down here some of their properties, especially since an application is made in this paper to continuous curves, and in the following paper to irreducible continua.

In the following definitions, M is a continuum in space of any finite number of dimensions, P is a point of M , and D is a subset of M . If $M - P$ is connected, P is said to be a *non-cut point* of M ; if $M - P$ is not connected, P is said to be a *cut point* of M . The set $M - D$ is said to be the complement of D . If the complement of D is closed, D is said to be an *open subset* of M . If D is a connected open subset of M , then D is said to be a *domain* in M , or an *M -domain*.

We shall say that a point P of M is a *point of type 1*, if given any positive number ϵ there exists an M -domain containing P of diameter less than ϵ whose complement is connected. We shall say that P is a *point of type 2*, if given any positive number ϵ there exists an open set in M containing P of diameter less than ϵ whose complement is connected. For convenience, we shall call a non-cut point of M a *point of type 3*.

THEOREM 1. *If P is a point of type 1 of a continuum M , it is a point of type 2, but not conversely. If P is a point of type 2, it is a point of type 3, but not conversely.*