AIP | Review of Scientific Instruments

An Approximate Method of Cutting Short Circular Cylindrical Arcs with Very Large Radii of Curvature in the Milling Machine

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Citation: Review of Scientific Instruments **9**, 329 (1938); doi: 10.1063/1.1752357
View online: http://dx.doi.org/10.1063/1.1752357
View Table of Contents: http://scitation.aip.org/content/aip/journal/rsi/9/10?ver=pdfcov
Published by the AIP Publishing

An Approximate Method of Cutting Short Circular Cylindrical Arcs with Very Large Radii of Curvature in the Milling Machine

In the transmission type curved crystal focusing x-ray spectrograph, a crystalline lamina is imprisoned between the convex and concave surfaces of two rigid metal plates cut in the shape of arcs of right circular cylinders of appropriate radius to impose the correct curvature on the lamina. Recently I have sought a method of cutting such circular cylindrical arcs of very large radius of curvature (say 2 meters) without the delay and expense involved in the use of a large vertical boring mill of four meters diameter. The method which I describe here might also prove of use in the production of circular cylindrical arcs of metal for lapping cylindrical lenses.

A "fly cutter" is mounted on the spindle of the milling machine so as to sweep out with the point of the cutting tool a circle of radius, a, which is but a small fraction of the radius of the cylindrical arc to be generated. (See Fig. 1.) The piece on which the approximate cylindrical arc is to be cut is mounted on the table of the milling machine which is set so that its direction of travel is not quite normal but slightly oblique to the axis of rotation of the cutter spindle as shown in Fig. 1. As the oblique circle swept out by the cutter advances through the reference system of the piece being cut, it generates in that system an elliptical cylinder which is very much flattened if the inclination between the direction of feed and the plane of the circle swept out by the cutting tool is small. The work is so situated that the approximate circular cylindrical arc comes from that part of the ellipse situated symmetrically on either side of the terminus B of its minor axis. (See Fig. 2.) In Fig. 1 the dotted position for the work gives the concave cut while the full line position gives the convex cut.

Referring again to Fig. 2, by obvious methods of analytic geometry one writes the equations of the ellipse and of the circle tangent to the ellipse at B having at that point the same radius of curvature R (the osculating circle). The slopes dy/dx of the circle and of the ellipse are then obtained and, since in the interesting region these slopes are small, their difference is a measure in radians of the small angular disagreement, δ, between the direction of the circle and the direction of the elliptical arc which approximates it at any point, x, δ being obviously a maximum at the extreme value of x on the block being cut (at its edges).

The following relations are then easy to derive:

The radius of curvature, R, of the generated cylindrical

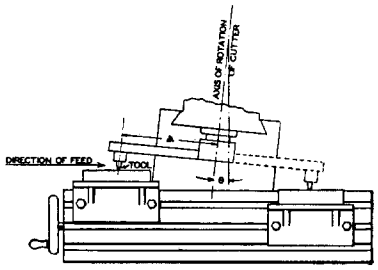

FIG. 1

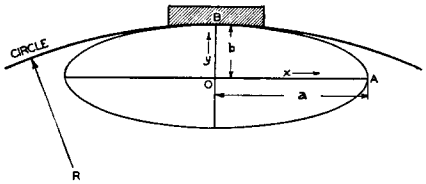

FIG. 2

surface at B is

$$R = a^2/b \tag{1}$$

a and b being, respectively, the semi-major and semi-minor axes of the ellipse. Thus with an ellipse of axial ratio $a/b = 10$ and a semi-major axis (effective radial sweep of cutter) of 20 cm, the generated radius $R = 2$ meters. The angle of obliquity θ of the cutter axis to the normal to the direction of feed is given by

$$\sin\theta = b/a. \tag{2}$$

The angular deviation, δ, which measures the maximum departure of the inclination of the elliptical surface from the inclination of the circular surface at the edges of the generated arc is

$$= (q/r)\cdot(1-q^2)^{-\frac{1}{2}}\cdot[(1-q^2)^{-\frac{1}{2}}-1]\cdot[(1-q^2)^{-\frac{1}{2}}+(r^2-1)]^{-1} \tag{3}$$

in which $r = a/b$ the axial ratio of the ellipse and $q = x/a$ is the fraction of the major axis included in the utilized arc. Evaluating this formula for δ we obtain the values given in Table I.

TABLE I. δ *in radians.*

$r/q =$	0.1	0.2	0.3	0.4	0.5
5	4×10^{-7}	16×10^{-6}	12×10^{-5}	8×10^{-4}	16×10^{-4}
10	2×10^{-7}	8×10^{-6}	6×10^{-5}	4×10^{-4}	8×10^{-4}
20	1×10^{-7}	4×10^{-6}	3×10^{-5}	2×10^{-4}	4×10^{-4}

We note from Table I that the angular error, δ, decreases much more rapidly with decrease of q than with increase of r so that if a very accurate approximation to the circle is desired, the radius of sweep of the cutter (major axis of ellipse) should be five to ten times the chord of the cylindrical arc to be generated. The inclination of the cut at its edges will then differ from a circular cylinder by only a few seconds of arc.

Thus for example we may cut a surface with 2 meter radius of curvature using a cutter with an effective sweep radius of 20 cm and an angle of inclination θ of slightly less than 6°. Under these circumstances $r = 10$ and referring to our table of δ if the chord of our utilized arc does not exceed 8 cm we shall have $q = 0.2$ and the maximum error of inclination of the surface (at its edges) will not exceed 8×10^{-6} radian or 1.6 seconds of arc.

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Aug. 22, 1938.