Optical Model Description of Parity-Nonconserving Neutron Resonances

S. E. Koonin and C. W. Johnson
W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125

P. Vogel
Norman Bridge Laboratory of Physics, California Institute of Technology, Pasadena, California 91125
(Received 26 March 1992)

We calculate the average helicity dependence of the low-energy neutron-nucleus cross section in terms of a phenomenological optical model. Comparison with recent measurements on ^{232}Th and ^{238}U by the TRIPLE Collaboration shows that the effective parity-nonconserving interaction is two orders of magnitude larger than estimates based on standard meson-exchange models. The helicity dependence is predicted to oscillate with target mass due to shape resonances.

PACS numbers: 25.40.Dn, 11.30.Er, 24.10.Ht, 24.60.Dr

Recent experiments [1,2] have demonstrated the generic nature of parity nonconservation (PNC) in resonant low-energy p-wave scattering from heavy nuclei; longitudinal analyzing powers approaching 10% have been observed. Such data might offer a measure of the PNC nucleon-nucleon interaction complementary to that obtained from nucleon-nucleon scattering [3] or from discrete states in light nuclei [4]. However, the complex and unknown nature of the compound nucleus states involved is problematic.

Previous theoretical treatments of PNC resonant neutron scattering include the mixing of isolated $p_{1/2}$ and $s_{1/2}$ compound nuclear states [5], resonances embedded in a coherent background [6,7], and random matrix models [8,9]. However, none of these can account for the fact that, at least for the heaviest targets with $A \sim 235$, the longitudinal analyzing power of the many resonances measured is predominantly positive. This indicates that a coherent mechanism is operative, and several recent papers [10–12] have made specific proposals for such a mechanism.

In this Letter, we consider an optical model approach to PNC neutron scattering. The optical potential is a venerable description of the average properties of compound nuclear states [13]. Further, it has a direct connection to the underlying nucleon-nucleon (NN) interaction [14]. Our approach is similar in spirit to Bowman *et al.* [10], but is more direct, realistic, and quantitative.

We describe the interaction of a neutron with a spin-zero target of mass A as the sum of a complex strong (parity conserving) and weak (PNC) potential,

$$V = V_S + V_{\rm PNC} , \qquad (1)$$

where for the strong interaction we take the surface-absorption Buck-Perey form [15]:

$$V_S = V_0 f_a(r) + V_{ls} r_0^2 \frac{1}{r}\frac{df_a}{dr} \mathbf{l}\cdot\mathbf{s} + iW_0 f_b(r)[1 - f_b(r)] . \qquad (2)$$

Here, $f_c(r) \equiv (1 + e^{(r-R)/c})^{-1}$ is the Woods-Saxon function with diffuseness c and radius $R = 1.27A^{1/3}$ fm, $a = 0.65$ fm, $b = 0.47$ fm, $V_0 = -48.0$ MeV, $V_{ls} = 14.8$ MeV, and $W_0 = -44$ MeV; $\mathbf{l}$ and $\mathbf{s}$ are the neutron angular momentum and spin operators, respectively. For the PNC interaction, which we will treat in first-order perturbation theory, we take

$$V_{\rm PNC} = \tfrac{1}{2}\epsilon_7 \times 10^{-7} \{ f_a(r), \boldsymbol{\sigma}\cdot\mathbf{p}c \} , \qquad (3)$$

where $\boldsymbol{\sigma} = 2\mathbf{s}$ and $\mathbf{p} = -i\hbar\nabla$ is the momentum operator. Estimates of $V_{\rm PNC}$ (discussed in detail below) indicate

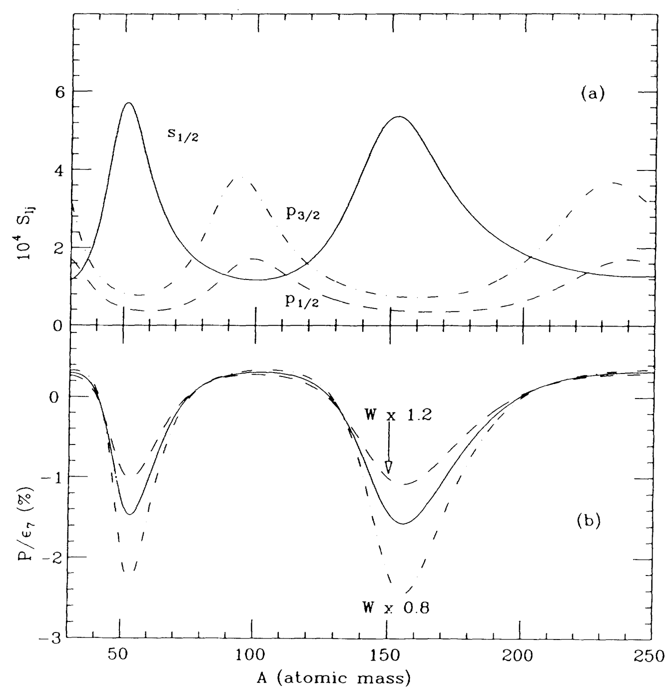

FIG. 1. (a) s- and p-wave neutron strength functions as calculated from the Perey-Buck potential [15]. (b) Predicted helicity dependence of the $p_{1/2}$ cross section at a neutron energy of 1 eV (solid curve). The dashed and dot-dashed curves show the effect of arbitrarily increasing or decreasing the strength of the absorptive potential by 20%.

that the dimensionless coupling constant ϵ_7 is at most of order unity.

An important constraint on our model is that it correctly describes the average low-energy neutron-nucleus cross section. For a neutron with wave number k and energy $E=\hbar^2k^2/2m$, the energy-averaged cross section in a particular partial wave with neutron orbital angular momentum l and total angular momentum j can be obtained from the radial wave functions ψ_{lj} describing scattering by V_S and normalized to

$$\psi_{lj}(r) \underset{r\to\infty}{\longrightarrow} \frac{1}{r}(A_{lj}e^{i(kr-l\pi/2)}-e^{-i(kr-l\pi/2)})\,. \tag{4}$$

In particular, $\sigma_{lj}=(\pi/k^2)(2j+1)(1-\mathrm{Re}A_{lj})$. Alternatively, the average cross section can be expressed in terms of the average properties of the compound nuclear states:

$$\sigma_{lj}=\frac{\pi}{k^2}\frac{(2j+1)}{2}2\pi\left\langle\frac{\Gamma_{lj}}{D_{lj}}\right\rangle, \tag{5}$$

where Γ_{lj} is the resonance width, D_{lj} is the resonance spacing, and $\langle\cdots\rangle$ denotes an average over resonances. Here, we have neglected the shape-elastic cross section, which is a small fraction of the compound-elastic cross section for the targets and energies of interest here.

The strength function in a particular partial wave, S_{lj}, is defined in terms of the average resonance properties, normalized to a neutron energy of 1 eV and corrected for penetration [16]. For the s and p waves important here,

$$S_{s1/2}=\frac{\langle\Gamma_{s1/2}\rangle}{\langle D_{s1/2}\rangle},\quad S_{p1/2}=\frac{1}{3}\frac{\langle\Gamma_{p1/2}\rangle}{\langle D_{p1/2}\rangle}\frac{1}{\Pi},$$
$$S_{p3/2}=\frac{2}{3}\frac{\langle\Gamma_{p3/2}\rangle}{\langle D_{p3/2}\rangle}\frac{1}{\Pi}. \tag{6}$$

Here, $\Pi\equiv k^2R^2/(1+k^2R^2)$ is the p-wave penetrability, with $R=1.35A^{1/3}$ fm. In Fig. 1(a) we show the $l=0,1$ strength functions calculated from V_S for nuclei with $30\le A\le 250$. The classic shape resonances are evident, with the $l=0,1$ partial waves out of phase with each other. A small spin-orbit splitting can be discerned by comparing the $p_{1/2}$ and $p_{3/2}$ curves. The overall agreement with the experimental data [16] is generally good, although the calculation overestimates the s-wave strength function at the $A=100$ minimum by about a factor of 2. Inclusion of nuclear deformation [15] improves the agreement with data near the maxima of the strength functions; for simplicity, we do not consider these effects here.

By treating the PNC interaction in first-order perturbation theory, the average longitudinal asymmetry, normalized to the average $p_{1/2}$ cross section, is found to be

$$P\equiv\frac{\sigma_+-\sigma_-}{\sigma_{p1/2+}+\sigma_{p1/2-}}=\frac{2\pi}{\sigma_{p1/2}k^2}\frac{m}{\hbar^2k}\,\mathrm{Im}\int_0^\infty dr\,r^2\left[\psi_{s1/2}V_p\psi'_{p1/2}-\psi'_{s1/2}V_p\psi_{p1/2}+2\psi_{s1/2}\frac{V_p}{r}\psi_{p1/2}\right]. \tag{7}$$

Here, $V_p(r)\equiv\epsilon_7\times10^{-7}f_a(r)\hbar c$ is the radial dependence of V_{PNC}, and $\psi'\equiv d\psi/dr$. We note that other authors have derived similar formulas for P or other PNC observables [5-7]. However, these have consistently (and incorrectly) been interpreted as describing the properties of a single compound nucleus state, rather than as an average over many resonances.

The average asymmetry given by Eq. (7), $P=\langle\Delta\sigma\rangle/2\langle\sigma_{p1/2}\rangle$, depends smoothly on energy. On the other hand, the experimentally determined quantity, $P_{\mathrm{expt}}=\Delta\sigma/2\sigma_{p1/2}$, is known only at, or very near, $p_{1/2}$ resonances. Nevertheless, there are several arguments to support direct comparison of these two quantities. First, P_{expt} is essentially energy independent over the width of a p-wave resonance (see, for example, Fig. 1 of Ref. [2]). Moreover, if the usual resonance expansion of the scattering amplitude is perturbed in first order by a parity nonconserving interaction, H_{PNC}, one finds

$$\langle P_{\mathrm{expt}}\rangle=\left\langle\sum_s\frac{\langle s|H_{\mathrm{PNC}}|p\rangle}{E_s-E_p}\frac{\gamma_s}{\gamma_p}\right\rangle, \tag{8}$$

where the average is over $p_{1/2}$ resonances, the sum is over $s_{1/2}$ resonances, $\gamma_{s,p}$ are the reduced widths, and $E_{s,p}$ are the resonance energies. On the other hand, P defined by Eq. (7) is found to be

$$P=\left\langle\frac{\gamma_s}{\gamma_p}D_p\left[\frac{1}{D_p}\sum_s\frac{\langle s|H_{\mathrm{PNC}}|p\rangle}{E_s-E_p}+\frac{1}{D_s}\sum_p\frac{\langle p|H_{\mathrm{PNC}}|s\rangle}{E_p-E_s}\right]\right\rangle. \tag{9}$$

Since $D_p\approx D_s$ and H_{PNC} is Hermitian but purely imaginary, so that $\langle s|H_{\mathrm{PNC}}|p\rangle=-\langle p|H_{\mathrm{PNC}}|s\rangle$, the two terms in Eq. (9) add, and $P\approx P_{\mathrm{expt}}$. (Note that γ_s/γ_p is also purely imaginary [10].) Similar terms arising from the mixing of the $p_{1/2}$ channel into the $s_{1/2}$ channel and the $s_{1/2}$ channel into the $p_{1/2}$ channel arise in the derivation of Eq. (7), and also contribute equally to P. It is important to note, however, that the actual measured quantity is $\Delta\sigma$, which is greatly enhanced at $p_{1/2}$ resonances. Hence, it is very difficult to determine P_{expt} at off-resonance energies.

Simple considerations show [10] (and our numerical calculations confirm) that $P\sim E^{-1/2}$. Thus, in analogy with the treatment of strength functions, we henceforth normalize P to a neutron energy of 1 eV. We also note that P is directly proportional to ϵ_7, the dimensionless weak coupling constant.

In Fig. 1(b) we show P/ϵ_7 for nuclear masses $30\le A\le 250$. The oscillations evident are due to the interplay of the s- and p-wave shape resonances; the change of sign of P is related to the changes in the number of nodes in $\psi_{s1/2}$ and $\psi_{p1/2}$ as the mass number A increases. Since the PNC amplitude is proportional to the square root of the ratio of the s- and p-wave strengths, the helicity dependence is expected to be largest for those targets where the p wave is antiresonant while the s wave is resonant. Note also that the imaginary potential has greatest influence at the s-wave resonances (where the wave function is largest inside the nucleus), and acts in the expected way of decreasing P as the absorption is increased. These results are essentially unchanged if the surface absorption potential [2] is replaced by one with volume absorption [13].

The TRIPLE Collaboration's data were taken with ^{238}U [1] and ^{232}Th [2] targets. For these nuclei, the Perey-Buck $s_{1/2}$ strength functions are (in units of 10^{-4}) 1.27 and 1.30, respectively, while the experimental values [17] are 1.2 ± 0.1 and 0.84 ± 0.07. Similarly for the p-wave strength function, $S_p=(S_{p1/2}+2S_{p3/2})/3$, the optical model gives $S_p=5.29$ for both nuclei, while the experimental values are 1.7 ± 0.3 and 1.48 ± 0.07. (This overestimate of the p-wave strength by a factor of 3 is reduced when deformation is accounted for [15].) The measured value of $P=(8\pm6)\%$ implies [in view of Fig. 1(b)] that $\epsilon_7=27\pm21$.

While we make quantitative predictions for the mass dependence of the size and magnitude of the average longitudinal analyzing power, only individual resonances for any one target have been examined in other mass regions. For $\epsilon_7>0$, our predictions for the sign agree with the data for ^{81}Br (0.88 eV) and ^{117}Sn (1.33 eV), but disagree for ^{111}Cd (4.53 eV) and ^{139}La (0.73 eV) [16]. We emphasize that systematic measurements of a number of resonances in one mass region are needed to compare quantitatively with our predictions.

The experimental value for the strength of the PNC interaction can be compared to the predictions of meson-exchange models. In the limit of CP conservation and low energy, one can write the most general one-meson-exchange potential in terms of six parity-violating meson-nucleon coupling constants $F_{\pi,0,1,2}$ and $G_{0,1}$; the resulting PNC NN potential can be approximated by an effective one-body potential of the form of Eq. (3) [4] [note that the sign of G_1 in Eq. (20) of that reference is wrong]. We assume the isovector density is proportional to the isoscalar density.

The meson-theoretical value of ϵ_7 is modulated by two sets of quantities: W^x, the reduction in the exchange terms due to finite meson range ($x=\pi,\rho,\omega$), and Γ^x ($\Gamma^x=G^x$ of Ref. [18]), the reduction of both direct and exchange terms due to short-range NN correlations. Approximate values for W^x and Γ^x can be obtained in analytic models and are confirmed numerically [4,18].

Taking both the ρ and ω masses to be 770 MeV/c^2, one has $W^\pi=0.13$ and $W^\rho=W^\omega=0.79$, while $\Gamma^\pi=0.7$ and $\Gamma^\rho=\Gamma^\omega=0.3$. Because we are considering heavy nuclei, there is also the possibility of in-medium effects [19]. We have therefore recalculated W^x and Γ^x with the nucleon and vector-meson masses reduced by 20% (the pion mass does not change). Denoting these results by an asterisk, we find $W^{\rho^*}=W^{\omega^*}=0.71$ and $\Gamma^{\rho^*}=\Gamma^{\omega^*}=0.4$.

Desplanques, Donohue, and Holstein (DDH) [20] quote a "reasonable range" of values for the PNC coupling constants and include a benchmark "best value." All experiments performed to date involving light nuclei or nucleons are consistent with the DDH range [4]. In Table I we summarize experimental and theoretical values of ϵ_7. The large variation of the theoretical values with effective meson mass is because of large cancellations between F_π and F_0.

Bowman *et al.* [10] sought to explain the average magnitude and sign of P by considering mixing of each $p_{1/2}$ resonance with the single-particle $s_{1/2}$ states in the neighboring oscillator shells. Their calculated P/ϵ_7 for $A\sim235$ is 10 times larger than ours, presumably due to their use of harmonic oscillator wave functions and a reduced energy denominator attributed to deformation, neglect of the absorptive potential, and use of a different radial form of $V_{\rm PNC}$. [They replace $f_a(r)$ by unity.]

The unexpectedly large value of ϵ_7 implied by our optical model analysis can be attributed to one of two causes. The first is an enhancement of the two-body PNC interaction in the nuclear medium. Indeed, recent speculations [21] have questioned the applicability of the DDH framework in nuclei when strange quark matrix elements and multipion exchange terms are included. (However, the situation is complicated by a random-matrix analysis that claims that the extracted root-mean-square PNC matrix element between compound nuclear states in ^{239}U is consistent with DDH [9].) As such an enhancement would have profound implications for our understanding of PNC interactions, it is important to consider the second possible explanation: a more complicated reaction mechanism than we have assumed.

Flambaum has recently proposed [11] that inelastic excitations of the target enhance $V_{\rm PNC}$ beyond that expected from the NN PNC interaction. We find this implausible, as there is no such renormalization of the strong interaction (i.e., the magnitude of V_S is that given by the

TABLE I. Values of the PNC coupling constant ϵ_7. "Best values" and "reasonable range" are quoted for DDH; values calculated with meson masses reduced as described in the text are indicated by an asterisk.

PNC neutron scattering	
This work	27 ± 21
Bowman *et al.* [10]	2.8 ± 2.1
Theory	
DDH [20]	0.02 ($-0.47\to0.42$)
DDH*	0.10 ($-0.99\to0.97$)
AH [4]	0.05
AH*	0.16

product of the nuclear density and the *NN* scattering lengths, not 10^2 times larger). Further, Flambaum's mechanism depends upon V_{PNC} being most effective at the surface; indeed, he argues that P depends only upon $d\rho/dr$, rather than ρ. However, our numerical calculations show that P calculated from Eq. (7) varies very little as the surface diffuseness of V_p is changed.

Auerbach has recently suggested [12] that collective isoscalar and isovector spin-dipole excitations of the target result in a coherence of the PNC resonance widths analogous to that associated with isobaric analog states. However, he makes no prediction as to the sign or magnitude of P, and it is difficult to see how the requisite large enhancements would arise.

In summary, we have considered a simple yet realistic and quantitative description of PNC neutron-nucleus scattering in terms of the optical model. A one-body PNC potential some two orders of magnitude larger than is expected from meson-exchange models is required to explain the experimental observations. Hence, either an as yet unidentified reaction mechanism is operating or the two-body PNC interaction is modified in heavy nuclei. Experiments aimed at observing the target mass and energy dependence of the PNC predicted by the model would be a first step toward resolving this discrepancy.

We are grateful for discussions with G. E. Brown and H. A. Bethe and with other members of the Caltech nuclear theory effort. This work was supported in part by National Science Foundation Grants No. PHY90-13248 and No. PHY91-15574 and by Contract No. DE-F603-88ER-40397 with the U.S. Department of Energy.

[1] J. D. Bowman *et al.*, Phys. Rev. Lett. **65**, 1192 (1990).
[2] C. M. Frankle *et al.*, Phys. Rev. Lett. **67**, 564 (1991).
[3] D. E. Nagle *et al.*, in *Proceedings of the Third International Symposium on High Energy Physics with Polarized Beams and Polarized Targets, Argonne, 1978*, AIP Conf. Proc. Vol. 51 (AIP, New York, 1979), p. 24; R. Balzer *et al.*, Phys. Rev. C **30**, 1409 (1984).
[4] E. G. Adelberger and W. C. Haxton, Annu. Rev. Nucl. Part. Sci. **35**, 50 (1985).
[5] O. P. Sushkoo and V. V. Flambaum, Pis'ma Zh. Eksp. Teor. Fiz. **32**, 402 (1980) [JETP Lett. **32**, 377 (1980)]; V. E. Bunakov and V. P. Gudkov, Nucl. Phys. **A401**, 93 (1983).
[6] G. Karl and D. Tadić, Phys. Rev. C **16**, 1726 (1977).
[7] S. Noguera and B. Desplanques, Nucl. Phys. **A457**, 189 (1986).
[8] A. Müller, E. D. Davis, and H. L. Harvey, Phys. Rev. Lett. **65**, 1329 (1990).
[9] M. B. Johnson, J. D. Bowman, and S. H. Yoo, Phys. Rev. Lett. **67**, 310 (1991).
[10] J. D. Bowman *et al.*, Phys. Rev. Lett. **68**, 780 (1992).
[11] V. V. Flambaum, Phys. Rev. C **45**, 437 (1992).
[12] N. Auerbach, Phys. Rev. C **45**, R514 (1992).
[13] H. Feshbach, C. Porter, and V. F. Weisskopf, Phys. Rev. **96**, 448 (1954).
[14] *Microscopic Optical Potentials*, Lecture Notes in Physics Vol. 89, edited by H. V. Geramb (Springer, Berlin, 1979).
[15] B. Buck and F. Perey, Phys. Rev. Lett. **8**, 444 (1962).
[16] S. F. Mughabghab, *Neutron Cross Sections* (Academic, Orlando, 1984), Vol. 1.
[17] V. P. Alfimenkov *et al.*, Nucl. Phys. **A398**, 93 (1983); V. W. Yuan *et al.*, Phys. Rev. C **44**, 2187 (1991).
[18] A. Griffiths and P. Vogel, Phys. Rev. C **43**, 2844 (1991).
[19] G. E. Brown and M. Rho, Phys. Rev. Lett. **66**, 2720 (1991).
[20] B. Desplanques, J. F. Donoghue, and B. R. Holstein, Ann. Phys. (N.Y.) **124**, 449 (1980).
[21] J. Dai, M. Savage, J. Liu, and R. Springer, Phys. Lett. B **271**, 403 (1991); D. Kaplan and M. Savage, UCSD Report No. PTH92-04, 1992 (to be published).