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ELEMENTARY PARTICLES OF CONVENTIONAL FIELD THEORY AS REGGE POLES

M. Gell-Mann
California Institute of Technology, Pasadena, California

and

M. L. Goldberger
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

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Composite states in nonrelativistic scattering theory lie on Regge trajectories[1] corresponding to poles in the angular momentum plane that move with varying energy. Simple approximations[2-5] indicate that composite particles in relativistic field theory have the same behavior. According to the Regge pole hypothesis,[6-9] particles like the nucleon, that have customarily been treated as elementary in field theory, also lie on Regge trajectories. Is that in accord with describing such particles by ordinary perturbation theory?

It has been thought that elementary particles behave in perturbation theory as objects of fixed angular momentum.[7-9] In reference 9 the PS-PS theory of pions and nucleons is taken as an example. (For simplicity, let us ignore here the isotopic spin of the pion.) Writing the Feynman scattering amplitude as usual in the form $A - i\gamma\cdot qB$, we have in second order

$$A = 0,$$
$$B = -g^2/(m^2-u) + g^2/(m^2-s), \tag{1}$$

where s and u are the Mandelstam variables. The first term in the expression for B corresponds to the nucleon, with $\alpha = J - \frac{1}{2} = 0$. (It is proportional to s^0.) The second term, although it represents the nucleon singularity in the s channel, corresponds in the u channel to an infinite sequence of angular momentum poles with $\alpha = -1, -2, -3$, etc., just like the Born approximation in nonrelativistic scattering by a Yukawa potential. Now in fourth order B acquires terms that vary as $s^{-1}\ln s$ (for large s and fixed u) and others that vary as s^0, but none that varies as $s^0 \ln s$. Thus the subsidiary angular momentum poles at $\alpha = -1, -2, \cdots$ may be beginning to move, as in potential scattering. For example, if $\alpha = -1$ becomes $\alpha = -1 + g^2F(\sqrt{u})\cdots$, then $s^{-1+g^2F(\sqrt{u})+\cdots}$ appears in perturbation theory as $s^{-1} + g^2F(\sqrt{u})s^{-1} \times \ln s + \cdots$; but in this order the elementary nucleon pole continues to have $\alpha = 0$.

The situation is different, however, if we replace the virtual pion in the radiative correction by a virtual neutral vector meson with mass λ and coupling parameter γ. The amplitude B then acquires terms that go, for large s and fixed u, like $s^0 \ln s$. We suggest that the nucleon pole now moves too.

The variation of α for the nucleon in perturbation theory can then be studied as follows: The contribution of a Regge pole with the parity of the nucleon is given in Eq. (4.21) of reference 9. However, in order to satisfy the symmetry condition of MacDowell[10] and Frautschi and Walecka,[11] there must be a related Regge pole with the opposite parity. The two α's become coincident at $u = 0$ and complex conjugates of each other for u negative.[12,13]

Using the two equations, we have for the complete contribution[14] at large s

$$A \cong \frac{b(\sqrt{u})(\sqrt{u}-m)}{2\sin\pi\alpha(\sqrt{u})}\left(\frac{s}{s_0}\right)^{\alpha(\sqrt{u})}\{1+\exp[-i\pi\alpha(\sqrt{u})]\}$$
$$+\frac{b(-\sqrt{u})(-\sqrt{u}-m)}{2\sin\pi\alpha'-\sqrt{u})}\left(\frac{s}{s_0}\right)^{\alpha(-\sqrt{u})}\{1+\exp[-i\pi\alpha(-\sqrt{u})]\};$$

$$B \cong \frac{-b(\sqrt{u})}{2\sin\pi\alpha(\sqrt{u})}\left(\frac{s}{s_0}\right)^{\alpha(\sqrt{u})}\{1+\exp[-i\pi\alpha(\sqrt{u})]\}$$
$$-\frac{b(-\sqrt{u})}{2\sin\pi\alpha(-\sqrt{u})}\left(\frac{s}{s_0}\right)^{\alpha(-\sqrt{u})}\{1+\exp[-i\pi\alpha(-\sqrt{u})]\}. \tag{2}$$

Now with α equal to zero plus a correction of order γ^2 and b of order $\gamma^2 g^2$, we can write

$$A \cong [b(\sqrt{u})/\pi\alpha(\sqrt{u})](\sqrt{u}-m)[1+\alpha(\sqrt{u})\ln(s/s_0)+\cdots]$$
$$+[b(-\sqrt{u})/\pi\alpha(-\sqrt{u})](-\sqrt{u}-m)$$
$$\times[1+\alpha(-\sqrt{u})\ln(s/s_0)+\cdots],$$
$$B \cong [-b(\sqrt{u})/\pi\alpha(\sqrt{u})][1+\alpha(\sqrt{u})\ln(s/s_0)+\cdots]$$
$$-[b(-\sqrt{u})/\pi\alpha(-\sqrt{u})][1+\alpha(-\sqrt{u})\ln(s/s_0)+\cdots]. \tag{3}$$

We compare these expressions with the results of perturbation theory at large s for fixed u:

$$A \cong 0+(\gamma^2 g^2/8\pi^2)mI_0(u)\ln(s/s_0),$$
$$B \cong -[g^2/(m^2-u)]+(\gamma^2 g^2/8\pi^2)[I_0(u)-I_1(u)]\ln(s/s_0), \tag{4}$$

where

$$I_n(u) \equiv \int_0^1 \frac{x^n dx}{\lambda^2(1-x)+(m^2-u)x+ux^2-i\epsilon}. \tag{5}$$

We obtain

$$\alpha(\sqrt{u}) = 0+(\gamma^2/8\pi^2)(\sqrt{u}-m)[\sqrt{u}I_0(u)-(\sqrt{u}+m)I_1(u)]+\cdots,$$
$$b(\sqrt{u}) = -(\gamma^2 g^2/16\pi)\{I_0(u)-[(\sqrt{u}+m)/\sqrt{u}]I_1(u)\}+\cdots. \tag{6}$$

It is easy to verify that these quantities obey a number of rules characteristic of the Regge pole formalism. Both α and b are real between $u=0$ and the first physical threshold at $u=(m+\lambda)^2$, where they become complex; they both obey dispersion relations in $\sqrt{u}$. The imaginary part of α is positive. The real part of α increases through $\sqrt{u}=m$ up to threshold.

Some properties are unphysical, but these seem to be attributable to perturbation theory. The real part of α increases to $+\infty$ at threshold. Above threshold the imaginary part decreases from $+\infty$ to a constant as $u\to\infty$, while the real part decreases from a finite value and goes logarithmically to $-\infty$ as $u\to\pm\infty$.[15] The infinities at threshold are characteristic of this order of perturbation theory in the case of the Schrödinger equation with a Yukawa potential [where $\alpha=-1-i\gamma^2/(\text{energy})^{1/2}$, see reference 6]; they should not be taken seriously.

If the nucleon has really turned into a Regge pole as a result of vector meson radiative corrections, then a study of higher order corrections can confirm the fact. The coefficients of powers of $\ln(s/s_0)$ to each order in γ^2 must be such as to agree with the expansion of $\exp[\alpha(\sqrt{u})\ln(s/s_0)]$, where $\alpha(\sqrt{u})$ is a power series in γ^2. We shall assume that the exponential character of the higher corrections will be confirmed, for example by the method of Sudakov.[16]

In ordinary quantum electrodynamics of electrons and photons (but with massive photons), the electron must then be a Regge pole, with corrections to $\alpha=0$ of order e^2, much as in Eq. (6). The leading terms in the amplitude at large s and fixed u are[17]

$$-e^2\gamma_\nu[(-i\gamma\cdot P+m)/(m^2-u)]\gamma_\mu+(e^4/8\pi^2)\ln(s/s_0)\gamma_\nu\{-i\gamma\cdot P[I_0(u)-I_1(u)]+mI_0(u)\}\gamma_\mu, \tag{7}$$

where P is the initial electron four-momentum minus the final photon four-momentum, while ν and μ are the initial and final photon polarizations respectively.

In the simpler case of the electrodynamics of spin-zero particles (still with massive photons), the lowest order term $4e^2P_\mu P_\nu(m^2-u)^{-1}$ acquires the radiative correction $-(e^4/2\pi^2)P_\mu P_\nu[I_0(u)-2I_1(u)+I_2(u)]\ln(s/s_0)$, so that we have (with $\alpha=J$) the result

$$\alpha(u) = 0+(e^2/8\pi^2)(u-m^2)[I_0(u)-2I_1(u)+I_2(u)]+\cdots,$$
$$b = -(e^4/2\pi)[I_0(u)-2I_1(u)+I_2(u)]+\cdots. \tag{8}$$

Again these are reasonable forms for α and b except for the wild misbehavior at threshold (and possibly for the logarithmic dependence of $\mathrm{Re}\,\alpha$ at infinity). At threshold $\mathrm{Im}\,\alpha$ should go like $p^{3+2\,\mathrm{Re}\alpha}$ (where p is the barycentric momentum in the u channel), b should be essentially constant, and the threshold unitarity relation,

$$\mathrm{Im}\alpha \cong \frac{-p^{3+2\alpha}}{8\pi^2} b \frac{(m+\lambda)}{\lambda^2} \frac{\sqrt{\pi}\,\Gamma(\alpha+1)}{(2\alpha+1)\Gamma(\alpha+\frac{1}{2})}, \tag{9}$$

should hold. Presumably, the higher corrections fix up the situation at threshold, as in potential theory.

As the photon mass, λ, tends to zero all of our expressions become infrared divergent. This comes as no surprise, of course, since the divergence of the Compton amplitude in fourth order must be present in order to cancel the corresponding soft photon emission in the double Compton cross section. We have not fully analyzed the requisite modifications of the Regge formalism for this limiting situation and hope to return to the question elsewhere.

For finite photon mass, the photon is presumably on a Regge trajectory if the electron is. If we consider the correction to the amplitude for the exchange of a photon between two electrons (order e^2) arising from the exchange of three photons (order e^6), the motion of the photon pole should be apparent. Again infrared divergences may arise as $\lambda \to 0$.

The S-matrix or dispersion relation approach to relativistic quantum theory employs dispersion and unitarity formulas abstracted from conventional field theory. If it is really true that the Regge pole boundary conditions are also contained in conventional field theory, as the present work suggests, then there is no evidence for any conflict between the two points of view.

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COMMENTS ON CHEW'S BOOTSTRAP RELATIONSHIP*

F. E. Low

CERN, Geneva, Switzerland and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts

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In a recent Letter,[1] Chew has shown the existence of a certain reciprocity between the nucleon and the (3, 3) resonance. Essentially, he calculates the reduced width γ_{33} in terms of the "reduced width" of the nucleon, $\gamma_{11}=3f^2$, and finds[2]

$$\gamma_{33} \cong A_{33,11}\gamma_{11}/(1-A_{33,33}) = \tfrac{1}{2}\gamma_{11}, \tag{1}$$

where $A_{\alpha\beta}$ is the P-wave pion-nucleon crossing matrix. Reciprocally, he may calculate γ_{11} in terms of γ_{33}; he finds

$$\gamma_{11} \cong A_{11,33}\gamma_{33}/(1-A_{11,11}) = 2\gamma_{33}. \tag{2}$$

Since $\frac{1}{2}\times 2 = 1$, Eq. (1) and Eq. (2) are consistent with each other (and with experiment), from which