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Weighing the Universe with the Cosmic Microwave Background

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Variations in Ω , the total density of the Universe, leave an imprint on the power spectrum of temperature fluctuations in the cosmic microwave background (CMB). We evaluate the precision with which Ω can be determined by a CMB map as a function of sky coverage, pixel noise, and beam size. Assuming only that the primordial density perturbations were adiabatic and with no prior information on the values of any other cosmological parameters, a full-sky CMB map at 0.5° angular resolution and a noise level of $15 \mu\text{K}$ per pixel can determine Ω with a standard error of 5%. If all other cosmological parameters are fixed, Ω can be measured to better than 1%.

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Determination of the geometry of the Universe remains perhaps the most compelling problem in cosmology. Alternatively stated, what is the mean total energy density of the Universe? The answer to this question will reveal the ultimate fate of the Universe. If the density Ω (in units of the critical density $\rho_c = 3H_0^2/8\pi G$, where H_0 is the Hubble constant) is greater than unity, the Universe is closed and will eventually recollapse; if it is less than unity, the Universe will expand forever; and if $\Omega = 1$, the expansion will asymptotically decelerate to zero.

Theoretical considerations favor a critical ($\Omega = 1$) Universe, and inflation provides a generic mechanism for obtaining $\Omega = 1$. However, luminous matter provides less than 1% of this mass. Various inferences of Ω by dynamical means have hinted at substantial amounts of unseen mass, but most traditional methods of determining Ω are plagued by systematic uncertainties. Furthermore, virtually all dynamical methods of obtaining Ω give the

mean density in only nonrelativistic matter, and thus cannot discriminate between an open universe and a flat universe that is dominated by vacuum energy (i.e., a cosmological constant).

Recently, it was proposed that temperature anisotropies in the cosmic microwave background (CMB) might be used to determine the geometry of the Universe [1]. Features (known as “Doppler peaks,” or more accurately as acoustic peaks) in the CMB angular power spectrum result from acoustic oscillations in the photon-baryon fluid before the photons decouple. The characteristic wavelength of these fluctuations is the sound horizon at decoupling (the distance an acoustic disturbance propagates from $t = 0$ until decoupling), which subtends an angular scale on the sky today of $\theta \approx 1^\circ \Omega^{1/2}$. The dependence on Ω arises directly from the geometry of the Universe, and this angular scale is largely independent of other cosmological parameters. Thus, the location of the first

Doppler peak provides a robust determination of Ω . A CMB map with fine angular resolution also constrains the other cosmological parameters by measuring the angular locations and amplitudes of the higher Doppler peaks.

In this paper, we evaluate the precision with which Ω can be determined with high-resolution CMB maps [2]. We work within the context of models with adiabatic primordial density perturbations, although similar arguments apply to isocurvature models as well [3], and we expect the power spectrum to distinguish clearly the two classes of models. We also briefly consider what information on other cosmological parameters the CMB can provide.

A given cosmological theory makes a statistical prediction about the distribution of CMB temperature fluctuations, expressed by the angular power spectrum

$$C(\theta) \equiv \left\langle \frac{\Delta T(\hat{\mathbf{m}})}{T_0} \frac{\Delta T(\hat{\mathbf{n}})}{T_0} \right\rangle_{\hat{\mathbf{m}} \cdot \hat{\mathbf{n}} = \cos\theta} \\ \equiv \sum_{\ell} \frac{2\ell + 1}{4\pi} C_{\ell} P_{\ell}(\cos\theta), \quad (1)$$

where $\Delta T(\hat{\mathbf{n}})/T_0$ is the fractional temperature fluctuation in the direction $\hat{\mathbf{n}}$, P_{ℓ} are the Legendre polynomials, and the brackets represent an ensemble average over all observers and directions. The mean CMB temperature is $T_0 = 2.726 \pm 0.010$ K [4]. Since we can only observe a single microwave sky, the observed multipole moments C_{ℓ}^{obs} will be distributed about the mean value C_{ℓ} with a ‘‘cosmic variance’’ $\sigma_{\ell} \approx \sqrt{2/(2\ell + 1)} C_{\ell}$; no measurement can determine the C_{ℓ} to better accuracy than this variance.

We consider an experiment that maps a fraction f_{sky} of the sky with a Gaussian beam with full width at half maximum θ_{FWHM} and a pixel noise $\sigma_{\text{pix}} = s/\sqrt{t_{\text{pix}}}$, where s is the detector sensitivity and t_{pix} is the time spent observing each $\theta_{\text{FWHM}} \times \theta_{\text{FWHM}}$ pixel. We adopt the inverse weight per solid angle, $w^{-1} \equiv (\sigma_{\text{pix}} \theta_{\text{FWHM}}/T_0)^2$, as a measure of noise that is pixel-size independent [5]. Current state-of-the-art detectors achieve sensitivities of $s = 200 \mu\text{K}\sqrt{\text{sec}}$, corresponding to an inverse weight of $w^{-1} \approx 2 \times 10^{-15}$ for a 1-yr experiment. Realistically, however, foregrounds and other systematic effects may increase the noise level; conservatively, w^{-1} will likely fall in the range $(0.9\text{--}4) \times 10^{-14}$. Treating the pixel noise as Gaussian and ignoring any correlations between pixels, estimates of C_{ℓ} can be approximated as normal distributions with a variance (modified from Ref. [5])

$$\sigma_{\ell} = \left[\frac{2}{(2\ell + 1)f_{\text{sky}}} \right]^{1/2} [C_{\ell} + (wf_{\text{sky}})^{-1} e^{\ell^2 \sigma_b^2}]. \quad (2)$$

Given a spectrum of primordial density perturbations, the C_{ℓ} are obtained by solving the coupled equations for the evolution of perturbations to the spacetime metric and perturbations to the phase-space densities of all particle species in the Universe. We consider models with initial

adiabatic density perturbations filled with photons, neutrinos, baryons, and collisionless dark matter; this includes all inflation-based models. We begin with approximate analytic solutions for the scalar [6] and tensor [7] metric perturbations. Our calculation includes polarization [8], scale dependence of the initial perturbation spectrum [9], and the large-angle integrated Sachs-Wolfe effect from a cosmological constant [10]. To a good approximation, reionization can be parametrized by the optical depth τ to the surface of last scatter [1]; anisotropies on scales much smaller than the horizon at reionization are suppressed by $e^{-2\tau}$, while those on larger scales are unaffected. The geometry of the Universe is then accounted for by shifting the moments, $C_{\ell}(\Omega) = C_{\ell\Omega^{1/2}}(\Omega = 1)$ [1], and approximating the large-angle integrated Sachs-Wolfe effect in an open universe [11]. We do not here account for massive neutrinos (hot- or mixed-dark-matter models), but the power spectrum is altered only slightly by trading some of the nonrelativistic matter for neutrinos, and our results should be unchanged [12].

The CMB power spectrum depends upon many parameters. In the present analysis we include the following set: the total density Ω ; the Hubble constant, $H_0 = 100h \text{ km sec}^{-1} \text{ Mpc}^{-1}$; the density of baryons in units of the critical density, $\Omega_b h^2$; the cosmological constant in units of the critical density, Λ ; the power-law indices of the initial scalar- and tensor-perturbation spectra, n_S and n_T ; the amplitudes of the scalar and tensor spectra, parametrized by Q , the total CMB quadrupole moment, and $r = Q_T/Q_S$, the ratio of the tensor and scalar quadrupole moments; the optical depth to the surface of last scatter, τ ; the deviation from scale invariance of the scalar perturbations, $a_{\text{run}} \equiv dn/d \ln k$; and the effective number of light-neutrino species at decoupling, N_{ν} . Thus for any given set of cosmological parameters $\mathbf{s} = \{\Omega, \Omega_b h^2, h, n_S, \Lambda, r, n_T, a_{\text{run}}, \tau, Q, N_{\nu}\}$, we can calculate the mean multipole moments $C_{\ell}(\mathbf{s})$.

We now wish to determine the capability of CMB maps to determine these cosmological parameters. The answer to this question will depend on the measurement errors σ_l , and on the underlying cosmological theory. If the actual parameters describing the Universe are \mathbf{s}_0 , then the probability distribution for observing a CMB power spectrum that is best fit by the parameters \mathbf{s} is approximately [13]

$$P(\mathbf{s}) \propto \exp\left\{-\frac{1}{2}(\mathbf{s} - \mathbf{s}_0) \cdot [\alpha] \cdot (\mathbf{s} - \mathbf{s}_0)\right\}, \quad (3)$$

where the curvature matrix $[\alpha]$ is given by

$$\alpha_{ij} = \sum_{\ell} \frac{1}{\sigma_{\ell}^2} \left[\frac{\partial C_{\ell}(\mathbf{s}_0)}{\partial s_i} \frac{\partial C_{\ell}(\mathbf{s}_0)}{\partial s_j} \right], \quad (4)$$

with σ_{ℓ} as given in Eq. (2). The covariance matrix $[C] = [\alpha]^{-1}$ is an estimate of the standard errors that would be obtained from a maximum-likelihood fit to the data: the

standard error in measuring the parameter s_i (obtained by integrating over all the other parameters) is approximately $C_{ii}^{1/2}$. If some of the parameters are known, then the covariance matrix for the others is determined by inverting the submatrix of the undetermined parameters. For example, if all parameters are fixed except for s_i , the standard error on s_i is simply $\alpha_{ii}^{-1/2}$. In previous work, variances were estimated for small subsets of the parameters with Monte Carlo calculations [5,14]; the present approach can be used to reproduce these results.

Figure 1 displays the variance in Ω as a function of the beam width θ_{FWHM} for different noise levels and for $f_{\text{sky}} = 1$. For different values of f_{sky} , replace $w \rightarrow wf_{\text{sky}}$ and scale by $f_{\text{sky}}^{-1/2}$ [cf. Eq. (2)]. The underlying model assumed here for the purpose of illustration is “standard CDM,” given by $\mathbf{s} = \{1, 0.01, 0.5, 1, 0, 0, 0, 0, 0, Q_{\text{COBE}}, 3\}$, where $Q_{\text{COBE}} = 20 \mu\text{K}$ is the COBE normalization [15]. The solid curves show the $C_{\Omega\Omega}^{1/2}$ obtained by inversion of the full 11×11 curvature matrix $[\alpha]$ for $w^{-1} = 2 \times 10^{-15}$, 9×10^{-15} , and 4×10^{-14} . These are the sensitivities that can be attained at the given noise levels with the assumption of uniform priors (that is, including *no* information about any parameter values from other observations). The dotted curves show the $C_{\Omega\Omega}^{1/2}$ obtained by inversion of the Ω - Q submatrix of $[\alpha]$; this is the standard error on Ω that could be obtained if all other parameters except the normalization were fixed, either from other observations or by assumption. Realistically, the precision obtained will fall somewhere between these two sets of curves. Other underlying models, including

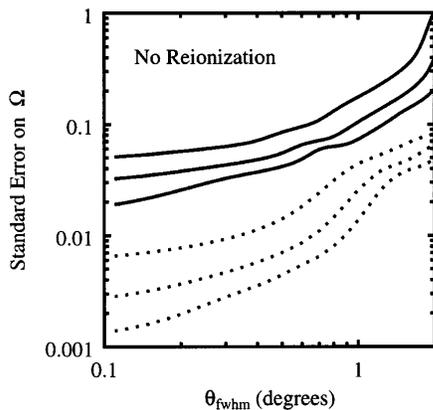


FIG. 1. The standard error on Ω that can be obtained with a full-sky mapping experiment as a function of the beam width θ_{FWHM} for noise levels $w^{-1} = 2 \times 10^{-15}$, 9×10^{-15} , and 4×10^{-14} (from lower to upper curves). The underlying model is “standard CDM.” The solid curves are the sensitivities attainable with no prior assumptions about the values of any of the other cosmological parameters. The dotted curves are the sensitivities that would be attainable assuming that all other cosmological parameters, except the normalization, were fixed. The results for a mapping experiment that covers only a fraction f_{sky} of the sky can be obtained by replacing $w \rightarrow wf_{\text{sky}}$ and scaling by $f_{\text{sky}}^{-1/2}$ [cf. Eq. (2)].

low- Ω models, give similar sensitivities. For $\Omega = 0.3$, the sensitivities are at the same overall level although the shapes of the curves are somewhat different, degrading sensitivity by a factor of 2 to 3 for beam sizes larger than a half degree. This is discussed further in Ref. [16], where we also describe the effects of varying each of the other underlying-model parameters. Although parameters other than Ω will have some weak effect on the position of the first Doppler peak, they will also alter the power spectrum at smaller angular scales. Therefore, the higher multipole moments accessible with smaller beam widths help constrain the other parameters and make the determination of Ω from the location of the first Doppler peak more precise.

Early reionization tends to wash out the structure of the power-spectrum features, decreasing the precision of the parameter estimates. To illustrate this effect, the curves in Fig. 2 show the same results as in Fig. 1, but for a reionized model in which $\tau = 0.5$. As expected, the sensitivity to Ω decreases, although it remains significant even for τ as large as one-half.

At $\ell \gtrsim 1000$, nonlinear effects become significant and linear power-spectrum calculations become unreliable. Therefore, we extend the sum in Eq. (4) only up to $\ell = 1000$. With improved calculations, the sensitivities at small beam widths could conceivably be improved.

We have also investigated the sensitivity of CMB mapping experiments to the other cosmological parameters listed above. Our results suggest that a map with 0.5° angular resolution may also provide interesting constraints to Λ with minimal assumptions, and to the other parameters with reasonable priors. In particular, the experiments should be able to distinguish between a flat matter-dominated universe and a flat cosmological-constant-dominated universe. These results will be presented in detail elsewhere [16].

Figures 1 and 2 estimate the probability of observing a set of parameters given an underlying model. Actual data will require solution of the inverse problem, estimating the probability of an underlying model given the data.

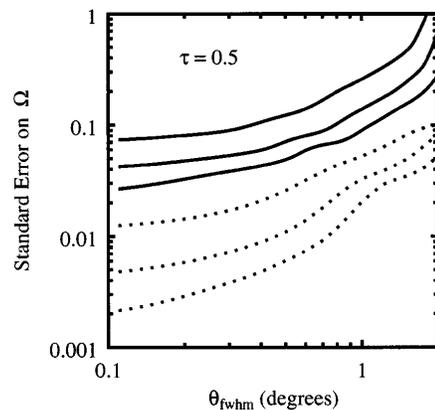


FIG. 2. Same as Fig. 1, except for a reionized model with $\tau = 0.5$.

We are currently exploring the accuracy with which all of the above parameters can be determined given a simulated data set [16,17] and to what extent parameter degeneracy in current experiments can be resolved [18]. Preliminary results show that the true maximum of the likelihood function can be recovered with good accuracy from a parameter search routine, and that the errors in Ω approach the precision obtained here.

The numerical results presented here demonstrate that Ω can be determined by realistic next-generation satellite experiments with a precision on the order of a few percent. Such a measurement will greatly solidify our knowledge of the gross properties of the Universe, have crucial bearing on the dark-matter and age problems, and will provide a stringent test of the inflationary hypothesis.

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