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### Measurements of conductivity near the metal-insulator critical point

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We present measurements of the electrical conductivity at low temperatures of bulk samples of Si:P under uniaxial stress controlled to bring the samples within 0.1% of the metal-insulator transition. As the metal approaches the critical point, we find that the power law of the temperature correction to the conductivity predicted for weak Coulomb interactions continues to fit, but that its sign, size, and range of validity change. Its size defines a diffusion temperature which tends towards zero at the critical density, at which point the power law itself appears to change.

We present here some new aspects of the behavior<sup>1</sup> of Si:P at milli-Kelvin temperatures in the metallic state near the metal-insulator transition. We find that the results can be described by extending the description<sup>2,3</sup> of interacting electrons in a weakly disordered metal into the region of strong scattering near the critical point.<sup>4-7</sup> Previous experiments<sup>3</sup> well above  $n_c$  and first-order perturbation theory<sup>2</sup> gave the temperature dependence of the conductivity as

$$\sigma(n, T) = \sigma(n, 0) + \sigma_1(T/T_D)^{1/2}, \quad (1)$$

where  $\sigma_1$  varies slowly with the electron density  $n$ , and the diffusion temperature

$$T_D \equiv T_F^2 k_B \tau / \hbar, \quad (2)$$

where  $\tau$  is an effective, short-range, elastic scattering time. This description may be extended<sup>6,7</sup> into the region of strong scattering provided that  $T$  is below a crossover temperature  $\Delta/k_B$ , where the two terms in Eq. (1) are comparable,

$$\Delta/k_B = T_D [\sigma(n, 0) / \sigma_1]^2. \quad (3)$$

Generally, the characteristic conductivity is the Ioffe-Regel value<sup>4</sup>

$$\sigma_{\text{IR}} = ne^2 / \hbar k_F^2, \quad (4)$$

with the Fermi wave vector  $k_F = (e\pi^2 n / \nu')^{1/3}$ . [For Si:P, with the number of valleys as  $\nu' = 6$  and  $n$  as the critical density,  $n_c = 3.7 \times 10^{18} \text{ cm}^{-3}$ ,  $\sigma_{\text{IR}} = 230 (\Omega \text{ cm})^{-1}$ .]

Our results provide the first indication that  $T_D$

tends toward 0 as  $n$  approaches  $n_c$ . We also confirm that the  $T^{1/2}$  term changes sign and that the crossover temperature tends toward 0. The analysis requires very small rounding of the transition and no minimum metallic conductivity for  $n/n_c - 1 > 10^{-3}$ .

Previous studies have described the critical point of the metal-insulator transition in disordered systems by presenting results along two paths in a density-temperature ( $n - T$ ) phase diagram. Both paths approach  $n_c$  by varying  $n$  at  $T = T_c = 0 \text{ K}$ : one from the metallic side<sup>1,8</sup> using the conductivity,  $\sigma(n, 0)$ , and the other from the insulating side<sup>9</sup> using the dielectric susceptibility,  $4\pi\chi(n, 0)$ . We also present here an estimate along an orthogonal direction: measuring  $\sigma(n_c, T)$ , i.e., lowering  $T$  toward 0 K at  $n = n_c$ . We also confirm results<sup>1</sup> on  $\sigma(n, 0)$  to within  $\sim 0.1\%$  of  $n_c$ .

Figure 1 shows  $\sigma(n, T)$  of a sample of Si:P as a function of  $T$  at a series of values of uniaxial stress  $S$ . The data are plotted as a function of  $T^{1/2}$  following Eq. (1). Various other experimental results provide evidence for a correction to the conductivity<sup>10,11</sup> or tunneling conductance<sup>12,13</sup> varying as the square root of  $T$  or magnetic field or voltage. All of the curves shown in Fig. 1 are for  $n$  or  $S$  within 1% of the metal-insulator transition.

The values of  $S$  are obtained from measurements of a calibrated capacitor<sup>1</sup> with one flexible plate attached to one end of the sample. The relative uncertainty in measuring this  $S$  is  $\leq 10^{-5}$  and does not limit the experimental resolution. The values of  $T$  are measured by a He<sup>3</sup> melting-curve thermometer linked

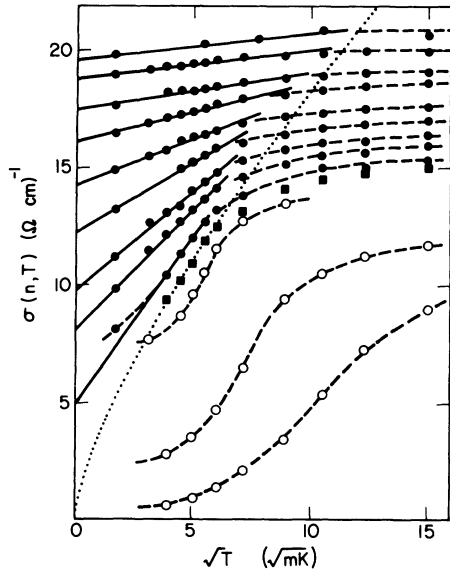


FIG. 1. Electrical conductivity as a function of temperature  $T$  for  $3 \leq T \leq 225$  mK, and for a series of values of uniaxial stress  $S$  applied to a crystal of Si:P with P concentration below  $n_c \sim 3.75 \times 10^{18} \text{ cm}^{-3}$ . The data are plotted vs  $\sqrt{T}$  to make extrapolations to  $T=0$  K using the solid circles. The open circles are not included in these extrapolations because they are affected by rounding. The dotted line is  $1.03 \sigma_{\text{IR}}(T/T_F)^{0.37}$ , near which we find deviations from the  $\sqrt{T}$  behavior. The values of  $S$  for the circles shown, from the top, are approximately 8.03, 7.82, 7.56, 7.36, 7.15, 7.00, 6.83, 6.71, 6.59, 6.33, 5.73, 5.12 kbar. The solid squares are at an interpolated  $S$ , where  $n = n_c$ .

thermally to the sample by the walls of the stress chamber. The  $\sigma(n, T)$  values were obtained with a current of  $\leq 2.5 \times 10^{-7}$  A applied to two spot contacts, with the resulting voltage measured from two separate contacts using a preamplifier with input resistance 100 M $\Omega$  and capacitance 15 pF. Since the sample dimensions between voltage leads were  $(1.5 \times 1 \times 0.4) \text{ mm}^3$ , the applied electric fields were  $\sim 10^{-7}$  V/cm. A lock-in amplifier was used in phase with the applied current at a frequency of 11 Hz. Under these conditions, we found no rounding (arising from, e.g., measuring frequency and current) within our experimental accuracy in the region above either  $\sigma(n, T) \sim 10 (\Omega \text{ cm})^{-1}$  or  $T \sim 10$  mK.

In Fig. 1, the solid lines are linear fits to the data in the low- $T$  limit. The dashed lines are guides to the eye in the region where we do not find a fit to the  $\sqrt{T}$  form. The dotted line is given by

$$\sigma(n_c, T) = 1.03 \sigma_{\text{IR}}(T/T_F)^{0.37} \quad (5)$$

and is obtained from a fit to the data at  $S = S_c$  and  $T < 36$  mK. The Fermi temperature  $T_F = 100$  K according to the free-electron formula. Data at  $n = n_c$  were obtained by interpolation between stresses

within 0.1% of  $S_c$  using  $n_c$  from the analysis of Fig. 2. This temperature dependence at  $n = n_c$  is a crude estimate because the fitting range is only  $10 \leq T \leq 36$  mK, limited at low  $T$  by rounding effects and at high  $T$  by a change to a slower variation. We estimate the uncertainty from these factors in the exponent in Eq. (5) to be  $\sim 30\%$ , although additional systematic uncertainty arising from the determination of  $n_c$  is significant. We note that the extension of the dotted line to  $T > 36$  mK roughly coincides with the region of the crossover from  $T^{1/2}$  to a slower  $T$  dependence and serves to emphasize the tendency of  $\Delta$  toward 0 as  $n$  approaches  $n_c$ .

The results for  $\sigma(n, T)$  as a function of  $n/n_c - 1$  are shown in Fig. 2 at  $T = 30$  and 3 mK and in the limit as  $\sqrt{T} \rightarrow 0$ , with  $S/S_c$  converted to  $n/n_c$  linearly as described previously.<sup>1</sup> The solid circles were taken at constant  $T$  as  $S$  was varied, in contrast to most of the data in Fig. 1, where  $S$  was constant and  $T$  was varied; the two data sets are consistent. The open circles are the zero- $T$  intercepts of the solid lines shown in Fig. 1. The solid curve is a fit to these points given by

$$\sigma(n, 0) = 0.83 \sigma_{\text{IR}}(n/n_c - 1)^{0.51}, \quad (6)$$

consistent with previously published results<sup>1,8</sup> in uncompensated Si:P for the wide range  $10^{-3} \leq n/n_c - 1 \leq 1$ . The uncertainty in the exponent in Eq. (6) is  $\sim 10\%$ , considering only the statistical scatter of the experimental points; additional systematic uncertainty is present related to the

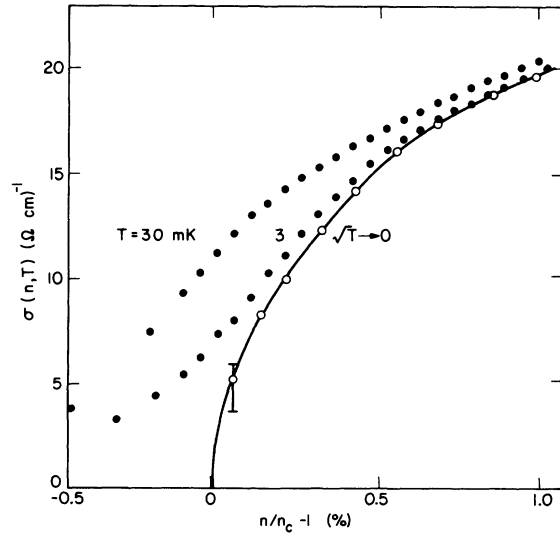


FIG. 2. Conductivity of Si:P as a function of relative concentration within 1% of the critical point. The open circles are the results of the extrapolations vs  $\sqrt{T}$  shown in Fig. 1, and the solid circles are data at  $T = 30$  and 3 mK. The solid line is a fit to the form  $(n/n_c - 1)^{1/2}$ . The values of  $n/n_c$  are obtained from the applied stress.

method of extrapolation, restriction of the region for fitting (dotted line in Fig. 1), and rounding effects. The uncertainty in the absolute magnitude of  $\sigma$  is  $\sim 20\%$  and arises primarily from difficulty in determining the effective sample size. (The voltage contacts were roughly hemispheres of radius  $\sim 0.2$  mm, or  $\sim 15\%$  of the distance between them.) In Fig. 2, we have not shown our estimates of  $\sigma(n, 0)$  for  $n < n_c$ , since here a different method is needed to extrapolate  $T$  to 0 K. Our best estimate is that  $\sigma(n, 0) = 0$  for  $n$  below our fitted  $n_c$ . The transition appears continuous and sharply defined at  $n_c$  within our experimental resolution of  $10^{-3}$ .

To summarize the three paths of approach to the critical point in uncompensated Si:P we define the following exponents:

$$\alpha = 0.37 \pm 0.1 ,$$

from  $\sigma(n_c, T)$  [Eq. (5)];

$$\nu = 0.51 \pm 0.05 ,$$

from  $\sigma(n, 0)$  [Eq. (6)]; and

$$\zeta = 1.15 \pm 0.15 ,$$

from  $4\pi\chi(n, 0)$  (Ref. 9).

As noted previously,<sup>9</sup>  $\zeta/\nu = 2.25 \pm 0.5$ . The exponents  $\nu$  and  $\zeta$  describe the data for a range of  $n/n_c$  of about a factor of 2 (and, for  $\nu$ , as close as  $n/n_c - 1 = 10^{-3}$ ). Results<sup>14</sup> in compensated Ge:Sb suggest  $\nu$  may shift from  $\frac{1}{2}$  toward 1, but measurements have been made only for the range  $0.2 \leq n/n_c - 1 \leq 4$ . Better measurements<sup>13</sup> on  $a - \text{Si}_{1-x}\text{Nb}_x$  alloys give  $\nu \sim 1$  for  $0.01 \leq n/n_c - 1 \leq 0.5$ , and studies<sup>15</sup> of  $a - \text{Si}_{1-x}\text{Au}_x$  give  $\nu = 0.84$  for  $0.02 \leq n/n_c - 1 \leq 3$ , consistent with other results.<sup>16</sup>

Figure 3 shows the slopes of the temperature corrections to  $\sigma(n, 0)$ ,  $m \equiv d\sigma/dT^{1/2}$ , at low  $T$  (solid lines in Fig. 1), as a function of relative density above  $n_c$ . To represent the relative density, we have plotted  $\sigma(n, 0)/\sigma_{\text{IR}}$  on the  $x$  axis. The solid circles are from Fig. 1 and the open circles are from the series of samples reported previously.<sup>3</sup> The solid line through the open circles in Eq. (1), where the prediction<sup>2</sup> for Coulomb interactions in the weak scattering limit gives

$$m = A_T \sigma_{\text{IR}} (4/3 - \lambda F) \sqrt{1/T_D} , \quad (7)$$

with  $F \equiv x^{-1} \ln(1+x)$ ,  $x \equiv (2k_F/K)^2$ ,  $A_T$  is a constant,  $\lambda = 2$ , and the Thomas-Fermi screening wave vector  $K$  is given by the free-electron formula for a material with a single-valley, isotropic band structure. Bhatt and Lee<sup>17</sup> show that Si:P can be described by moderate anisotropy and small intervalley scattering so that  $\lambda = 4$  and  $x = 0.5 (n/10^{18})^{1/3}$ . Far from  $n_c$ ,  $T_D$  varies as  $T_F$  times the diffusion coefficient. In the limit as  $n \rightarrow n_c$  the formulas are expected to

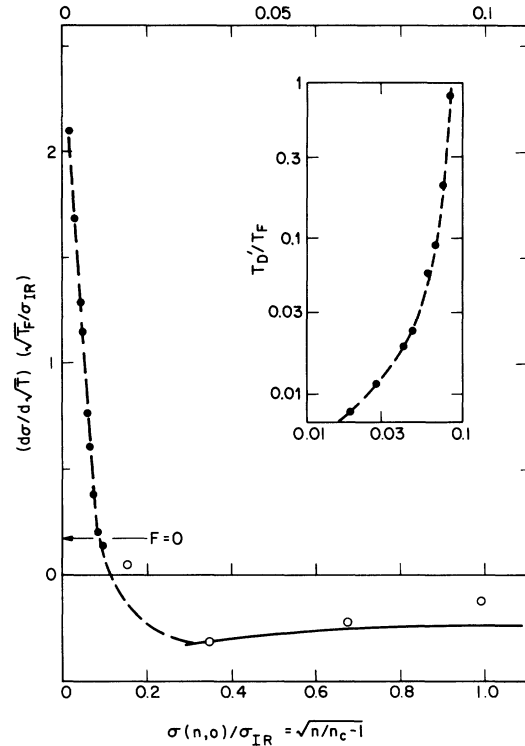


FIG. 3. Magnitude of the temperature dependence of  $\sigma(n, T)$  (defined by  $d\sigma/d\sqrt{T}$ ) as a function of  $\sigma(n, 0)$ . These slopes  $m$  are normalized to the Ioffe-Regel conductivity  $\sigma_{\text{IR}} = 230 (\Omega \text{ cm})^{-1}$  and the Fermi temperature at  $n_c$ ,  $T_F = 100$  K. The solid circles are the slopes of the solid lines in Fig. 1 and the open circles are similar slopes from more metallic samples at  $S = 0$  (Ref. 3). The insert shows the same solid circles replotted as an effective diffusion temperature  $T'_D \equiv (0.18\sigma_{\text{IR}}/m)^2$  as a function of  $\sigma(n, 0)$  on a log-log scale.

break down;  $F$  will go to zero, and  $\sigma_1$  to a constant as the interactions become effectively long range.<sup>7</sup> In this case,  $\sigma_1$  will change sign<sup>3</sup> as observed in Fig. 3.

Near  $n_c$ ,  $T_D$  is difficult to separate from the behavior of  $F$ , but if we set  $F = 0$  (its minimum value theoretically) and  $T_D = T_F$ , we find  $m(\sqrt{T_F}/\sigma_{\text{IR}}) = 0.18$ . From this point in Fig. 3, the observed slopes are seen to increase by a factor  $\sim 10$  which then may arise from  $T_D$  dropping by a factor of  $\sim 10^2$ . This drop in  $T_D$  is illustrated in the insert to Fig. 3, where we plot  $T'_D/T_F$ , defined by Eq. (7) with  $F = 0$ , i.e.,

$$T'_D \equiv (0.18\sigma_{\text{IR}}/m)^2 . \quad (8)$$

The two temperatures,  $T'_D$  and  $T_D$  are not equal, but become so near  $n_c$  as the variation in  $F$  becomes unimportant. Thus, in the inset to Fig. 3 our best estimate of the critical variation of  $T_D$  comes from the smallest values of  $\sigma(n, 0)$ , where the results show, very roughly,  $T_D \propto \sigma(n, 0)$ . Our previous results<sup>3</sup> in

the weak scattering regime were fitted assuming  $T_D \propto \sigma(n, 0)$ . In contrast, tunneling results have been interpreted<sup>12,13</sup> as showing no variation in  $T_D$ .

A phenomenological analysis<sup>6</sup> suggests that  $T_D$  has a critical variation, but that its form is highly uncertain, varying as something between  $1/\sigma(n, 0)$  and  $\sigma(n, 0)$ . Scaling theories of localization<sup>5</sup> can account for some of the observed behavior by assuming a suitable inelastic scattering time, but we know of no such evaluation that can account for the sign change observed in the  $T^{1/2}$  term. An extension<sup>7</sup> of second-

order perturbation theory including localization and Coulomb interactions gives  $T_D \propto \sigma(n, 0)^{4/3}$ . This theory predicts behavior qualitatively in agreement with all the results presented here, including the exponents  $\alpha \sim 0.3$ ,  $\nu \sim 0.6$ ,  $\zeta \sim 1.4$ , and  $\zeta/\nu \sim 2.3$ . In summary, our results seem to confirm the importance of both the random potential and Coulomb interactions in the metal-insulator transition.

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