COSMIC RAY PERPENDICULAR DIFFUSION COEFFICIENT AND DRIFT VELOCITY CALCULATED FROM PIONEER/VOYAGER OBSERVATIONS

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Abstract

The radial and latitudinal gradients measured by the Pioneer 10 and Voyager 1 and 2 spacecraft, reported by Cummings and Stone (1990), provide a unique opportunity to calculate the cosmic ray perpendicular diffusion coefficient and drift velocity.

1. Introduction. During the 1986/87 solar minimum Voyager 2 had reached the approximate radial position of 25 AU, while cruising near the ecliptic plane, Voyager 2 was at ≈ 30 AU and ≈ 30° North of the ecliptic plane, while Pioneer 10 was at ≈ 40 AU, near the ecliptic plane. Cummings *et al.* (1990) noted that the radial and latitudinal gradients of anomalous cosmic ray oxygen from 7.1 to 17.1 MeV/nuc, observed by these spacecraft in the four-year period around this solar minimum, are strongly correlated. Figure 1 is a repetition of their Figure 4 in Paper SH6.4-10, with a linear regression line drawn through the data points.

This Paper presents a model to calculate the cosmic ray perpendicular diffusion coefficient and drift velocity from these observations. This is essentially an extension of the suggestion of Levy (1978) that the ratio of these parameters is given by the ratio of the two gradients. The importance of the method is that the results are independent of the source of the particles, the boundary distance of the modulation region and other poorly known boundary conditions.

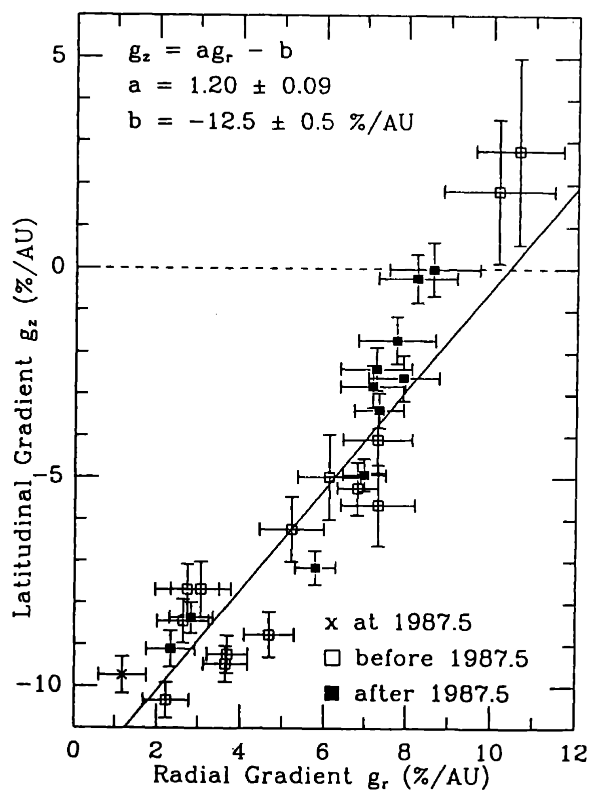

Figure 1: The relationship between the latitudinal and radial gradients observed on Pioneer 10 and Voyagers 1 and 2 from 1984 to 1989.

2. The Model. The steady-state comic-ray transport equation states that $\nabla.S=Q$, where Q represents a source/sink function due to adiabatic cooling, and where S is the cosmic ray flux density given by

$$S = -4\pi p^2 \left(K.\nabla f + V\frac{p}{3}\frac{\partial f}{\partial p}\right). \quad (1)$$

The omni-directional cosmic ray distribution function is denoted by $f(r,p)$ in terms of momentum p, V is the radial solar wind velocity, and K is the drift/diffusion tensor.

Singly charged anomalous cosmic ray oxygen ions in the kinetic energy range $7.1 < T < 17.1$ MeV/nuc have rigidities in the range $1.8 < P < 2.9$ GV, and adiabatic cooling is relatively unimportant at these high rigidities. Therefore, setting $Q = 0$, the nearly divergence-free flux density, $\nabla.S \approx 0$, leads to approximate flux conservation through a closed surface in the form

$$\int_{C.S.} \mathbf{S}.d\mathbf{a} \approx 0 \qquad (2)$$

As closed surface, consider the sides of the box in Figure 2, which is centered about the ecliptic plane, with Voyager 2 positioned at the center of its back surface at radial distance $r \approx 20$ AU. Pioneer 10 is at the center of its front surface at $r+\delta r \approx 40$ AU, while Voyager 1 is on the top surface at $r \approx 25$ AU, $\theta \approx 60°$ (latitude $\approx 30°$ North). Although Pioneer 10 is actually separated by $\approx 180°$ in azimuth (ϕ) from the Voyagers, this is of no consequence for the model: the observations of Figure 1 are based on time averages of at least a solar rotation, and the model therefore employs azimuthal symmetry.

Conservation statement (2) for this box becomes

$$(r+\delta r)^2\delta\phi\int_f S_r(r+\delta r)\sin\theta d\theta \;-\; r^2\delta\phi\int_b S_r(r)\sin\theta d\theta \;=\; 2\delta\phi\int_t S_\theta \sin\theta\, r dr, \qquad (3)$$

where the limits f, b, and t on the integrals stand for front, back, and top respectively. This states that the net outward (or inward) radial flux is supplied (or drawn off) by equal, but oppositely directed latitudinal fluxes at the top and bottom. Due to the assumed azimuthal symmetry, there is no *net* contribution through the two sides of the box.

If, in the momentum range of interest, f is a power law of the form $f \propto p^{-\gamma}$ the radial and latitudinal components of the flux density (1) may be written as

$$S_r \propto -(K_\parallel\cos^2\psi + K_\perp\sin^2\psi)g_r \;-\; K_T\sin\psi\, g_z \;+\; \gamma V/3 \qquad (4)$$

$$S_\theta \propto -K_T\sin\psi\, g_r \;+\; K_\perp\, g_z, \qquad (5)$$

the factor of proportionality being the average value of $4\pi p^2 f$ throughout the box. The radial and latitudinal gradients are defined as $g_r = \partial ln f/\partial r$ and $g_z = -g_\theta/r = -\partial ln f/r\partial\theta$, respectively. The diffusion tensor has the elements $K_\parallel$ and $K_\perp$, which describe diffusion parallel and perpendicular to the field lines, while $K_T = \beta P/(3B)$ gives the drift velocity $\mathbf{v}_d = \nabla \times (K_T\mathbf{B}/B^2)$. Notice that the factor 2 in the last term of (3) is justified by the fact that in (5) both K_T and g_z switch sign at $\theta = \pi/2$ (on a 27-day average basis). The magnetic field used in this model is the usual Parker spiral structure with magnitude $B = B_e(r_e^2\cos\psi_e)/(r^2\cos\psi)$. Henceforth the subscript "e" will refer to quantities at Earth. For a solar wind speed V = 400 km/s, the spiral angle is given by $\tan\psi = (r/r_e)\sin\theta$. Inside the box the field is very nearly azimuthal with $\tan\psi \geq 10\sqrt{2}$, $\sin\psi \geq 0.997$, and $\cos\psi \leq 0.07$. We therefore set $\sin\psi = 1$ and note that the term $K_\parallel\cos^2\psi$ in (4) is

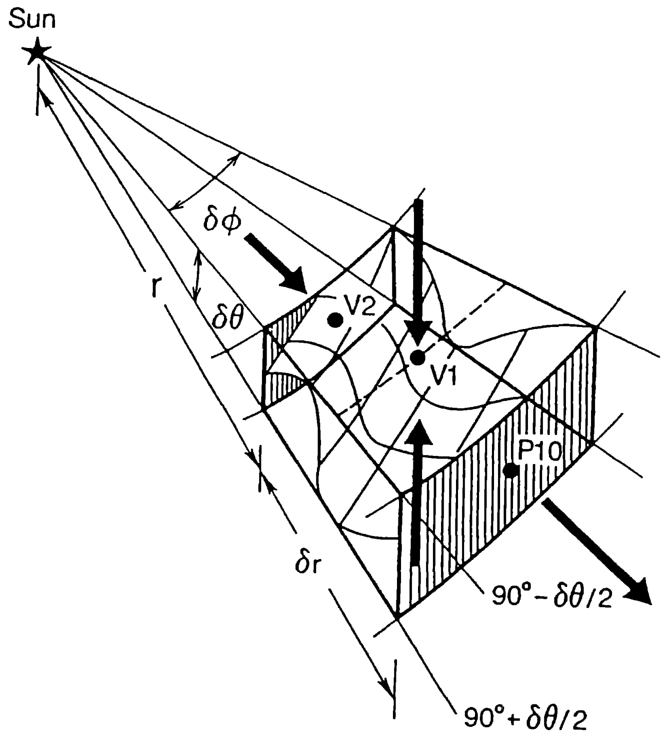

Figure 2: A Volume in the heliosphere with Voyager 2 and Pioneer 10 on the back and front surfaces respectively, and Voyager 1 on top at latitude 30° North. It contains the wavy neutral sheet entirely. Fluxes are shown with thick black arrows.

negligible, even though $K_{\parallel}/K_{\perp}$ may be $\gtrsim$ 100. The calculation therefore does not yield information on the value of $K_{\parallel}$.

Because the dimensions of the box are several tens of AU, we allow for spatial variations of $K_{\perp}$. Since

$$K_T = \frac{\beta P}{3B} = \sqrt{2}K_{Te}\left(\frac{r}{r_e}\right)^2 \cos\psi \approx \sqrt{2}\frac{K_{Te}}{\sin\theta}\left(\frac{r}{r_e}\right), \quad (6)$$

the same spatial dependence for $K_{\perp}$, i.e. $K_{\perp} = K_{\perp e}\,|B_e|/|B|$, simplifies the algebra significantly. (Notice, however, that K_T switches sign across the ecliptic while $K_{\perp}$ does not.) This choice reduces (4) and (5) to

$$S_r \propto -(\sqrt{2}/\sin\theta)(r/r_e)[K_{\perp e}g_r + K_{Te}g_z] + \gamma V/3 \quad (7)$$

$$S_\theta \propto -(\sqrt{2}/\sin\theta)(r/r_e)[K_{Te}g_r - K_{\perp e}g_z]\,. \quad (8)$$

Substitution of these flux densities into the conservation equation (3) then gives

$$[3R(K_{\perp e}g_r + K_{Te}g_z) - \gamma V r_e/\sqrt{2}]\delta\theta = 2R(K_{Te}g_r - K_{\perp e}g_z) + 6\,\overline{\cos\beta}\,K_{Te}, \quad (9)$$

where the approximation $\sin\delta\theta/2 \approx \delta\theta/2$ has been made, and where

$$R \equiv [(r+\delta r)^3 - r^3]/[(r+\delta r)^2 - r^2]\,. \quad (10)$$

The last term, $6\,\overline{\cos\beta}\,K_{Te}$, of (9) is the contribution due to neutral sheet drift. This should, of course, have been included in (4) and (5), but it is easier to add it *post hoc*. Burger and Potgieter (1989), and references therein, have shown that all particles within two gyroradii (R_L) from the sheet experience, on average, a neutral sheet drift with magnitude $v/6$, directed along the sheet, and perpendicular to B. This velocity has a radial component $(v/6)\cos\beta\sin\psi \approx (v/6)\cos\beta$, where β is the angle between the sheet and the radial direction. This velocity, spread over a surface with height $4R_L$ and width $r\sin\theta\delta\phi$, then gives a radial neutral sheet flux $(v/6)\cos\beta\times 4R_L\times r\sin\theta\delta\phi = 2K_T\cos\beta\, r\sin\theta\delta\phi$. This flux is added to the first two terms of (3), and pure algebraic manipulation leads to the last term of (9), where the bar denotes an appropriate average value over the dimensions of the box.

The solution of (9) for g_z in terms of g_r is

$$g_z = a g_r - b\,, \quad (11)$$

$$\text{where } a = \frac{2K_{Te} - 3K_{\perp e}\delta\theta}{2K_{\perp e} + 3K_{Te}\delta\theta} \quad \text{and} \quad b = -\frac{6K_{Te}\overline{\cos\beta} + \gamma V r_e\delta\theta/\sqrt{2}}{(2K_{\perp e} + 3K_{Te}\delta\theta)R}\,. \quad (12)$$

The observed values of a and b are the regression coefficients shown on Figure 1. Inverting this pair of equations gives the values of $K_{\perp e}$ and K_{Te} in terms of these coefficients as

$$K_{\perp e} = \frac{(3a\,\delta\theta - 2)\,\gamma V r_e\delta\theta/\sqrt{2}}{bR(4 + 9\delta\theta^2) + 6\,\overline{\cos\beta}(2a + 3\delta\theta)} \quad \text{and} \quad K_{Te} = K_{\perp e}\frac{2a + 3\delta\theta}{2 - 3a\,\delta\theta}\,. \quad (13)$$

3. Results. Together with the regression coefficients a and b, the following numerical values were used in (13). The solar wind speed was V = 400 km/s, the height of the box was $\delta\theta \approx 60^\circ \approx 1$ radian, while from (10) it follows that for $r = \delta r = 20$ AU, R = 140/3 AU. From Cummings and Stone (1987) we estimate that the average spectral index for the anomalous oxygen spectra during 1985 to 1989 lay in the range $\gamma = (3\pm1)$. For tilt angle values $10^\circ \lesssim \alpha \lesssim 60^\circ$, the average value of $\cos\beta$ is in the range $0.35 \gtrsim \overline{\cos\beta} \gtrsim 0.05$ inside the box. Substitution of these values (using appropriate average values $\gamma = 3$ and $\overline{\cos\beta} = 0.1$) into (13) gives

$$K_{\perp e} = 2.6\times10^{19}\ \text{cm}^2\text{s}^{-1} \qquad \text{and} \qquad K_{Te} = -\,8.7\times10^{19}\ \text{cm}^2\text{s}^{-1}. \quad (14)$$

These values are for a median $T \approx 10$ MeV/nuc, $P \approx 2.2$ GV, $\beta \approx 0.15$, and $\beta P \approx 0.33$. If we assume that $K_{\perp}$ has the standard rigidity dependence as being proportional to βP, and noting the assumed spatial dependence (just after eq. (6)), the final result for $K_{\perp}$ is

$$K_{\perp} = 7.8\times10^{19}\beta P\ |B_e|/|B|\ \ \text{cm}^2\text{s}^{-1}. \quad (15)$$

It is somewhat unfamiliar to relate to numerical values of K_{Te}. From (6) and (14) this value can, however, be converted to the strength of the Parker spiral field at Earth as

$$B_e = 9.2\ \text{nT} \quad (16)$$

It must be noted that these results are essentially order-of-magnitude estimates valid to within, perhaps, the proverbial factor of 2 for the following reasons: the observational uncertainties do not allow for a relationship between the gradients which is of higher degree than a linear one; the "box", over which both the observations and the calculations must be averaged, is uncomfortably large; the spatial dependence of $K_\perp$ need not be as chosen after eq. (6); and, variations in the spectral index γ alone can cause uncertainties of $\lesssim$ 30%. (The results are, however, insensitive to the precise geometry of the sheet. Varying $\cos\beta$ between 0.1 and 0.5 causes variations of $\lesssim 10\%$ in $K_\perp$ and B_e.) With these reservations in mind we interpret the results as follows.

4. Discussion. The results (14) to (16) confirm our present understanding of the modulation process as being due to convective, diffusive, and drift transport in a Parker spiral type field in the heliosphere which contains a field reversal at a wavy neutral sheet near the ecliptic plane.

First, the minus sign on K_{Te} in (14) is applicable to the so-called $qA < 0$ drift cycle from $\approx$ 1980 to $\approx$ 1990. This sign was in no way imposed on the calculation, but comes out naturally from the observed regression coefficients a and b. (The sense of the four flux arrows in Figure 2 are arbitrary.)

Winterhalter *et al.* (1988) reported observed values of B_e ranging from 5.5 to 8.5 nT from solar minimum to maximum. The calculated value in (16), which is most representative of solar minimum conditions, is therefore considerably higher than the observed value of 5.5 nT. This apparent discrepancy agrees with our experience from numerical modulation modeling, that drift effects must be scaled down. Burger and Moraal (1990) explain why this is so in terms of the effects of long wavelength turbulence, which tends to increase the effective scalar magnitude of the fields experienced by the particles. Jokipii and Kota (1989) have shown that this effect may reduce particle transport in the polar regions quite dramatically.

From the point of view of modeling the modulation process numerically, the ability to calculate a value for $K_\perp$ is particularly gratifying. This is one of the many free parameters available in the modeling process (see, *e.g.*, Moraal (1990) for such an attempt) and any estimate to pin it down contributes significantly to our understanding of the modulation.

When the values of $K_{\perp e}$ and K_{Te} in (14) are substituted back into (12), but with the sign of K_{Te} reversed, the model predicts that in the previous ($qA > 0$) modulation cycle

$$g_z = 0.31 g_r + 9.1 \ \%/\text{AU} \quad \text{or} \quad g_z = 0.03 g_r + 0.8 \ \%/\text{degree}, \text{ at } 5 \text{ AU}. \qquad (17)$$

This predicted latitudinal gradient is always positive, but with a four times weaker correlation with g_r than in the $qA < 0$ cycle. The second form, valid at 5 AU, also agrees quantitatively with observations made in the mid-1970's (see summaries by, *e.g.*, Fillius (1989) and McKibben (1989)), which suggest that g_z was $\approx 1\%$ per degree in the inner heliosphere.

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