azimuthal plane (x,y) normal to the axis (z) of the cylinder and containing its midpoint (0,0,0).

$$\hat{e}''\cdot\mathbf{A}(\hat{r}'',\hat{r})=f^2/(g+g'), \tag{2}$$

where[4,5]

$$g=Si(4kl)+jCi(4kl)-j\ln(\gamma 4kl)+4[\cos kl][kl\cos kl-\sin kl]\left[\times ZCi(2kl)-jSi(2kl)-\ln(\gamma 2kl)+\ln\frac{4l}{a}\right]-j2[\cos^2 kl]\times[\exp(-2jkl)-1]-\frac{4ka}{\pi}\sin^2 kl,$$

$$g'=-j\frac{4\pi}{k\eta}Z_i[kl(2+\cos 2kl)-\tfrac{3}{2}\sin 2kl],$$

$f=2(\sin kl-kl\cos kl)$,
$\gamma=1.781072$,
Z_i = skin impedance per unit length[6] of cylinder,
$l=\frac{1}{2}$ length of cylinder,
$k=2\pi/\lambda$ for incident wave,
a = radius of cylinder $\ll\lambda$,
$\eta=\mu^{\frac{1}{2}}\epsilon^{-\frac{1}{2}}$ = impedance of free space.

It should be noted that this expression is independent of the azimuthal angle $[\phi=\cos^{-1}(\hat{r}''\cdot\hat{r})]$, since the shadow remanant becomes isotropic for bodies of very small thickness.[7] The back scattering cross section is proportional to $|\mathbf{A}|^2$, and this result has been checked with experimental back scattering measurements[4] on fine wires for lengths less than λ. The experimental results agree with the theoretical to within 4% for cases where losses cause as much as a 75% reduction in scattering.[4] This indicates that the result given in Eq. (2) may be used in Eq. (1) to give the total cross section to about the accuracy quoted above.

A partial check on this may be made from the measurements quoted in reference 4. The absorption cross section σ_a of the cylinder may be computed by integration of the current squared times the skin resistance over the cylinder, where current is taken to be the same function as was used in deriving Eq. (2), with the constants determined by the variational method. This yields

$$\sigma_a/\lambda^2=(4\pi R_i g/k\eta f^2)|\mathbf{A}|^2, \tag{3}$$

where

$$R_i=\mathrm{Re}(Z_i).$$

The scattering cross section computed from Eqs. (1) and (2) is given by

$$(\sigma_s/\lambda^2)_{\mathrm{th}}=(\sigma_T-\sigma_a)/\lambda^2. \tag{4}$$

The scattering cross section may be obtained from the experimental measurements by integrating the differential cross section over the total solid angle. The ϕ dependence is constant, and the θ dependence of a half-wave dipole is known from antenna theory. Therefore,

$$(\sigma_s/\lambda^2)_{\mathrm{ex}}=(1/1.64)\frac{\sigma_B}{\lambda^2}, \tag{5}$$

where σ_B = scattering cross section in azimuthal plane and 1.64 = gain of half-wave dipole in azimuthal plane.

Measurements of back scattering from a 0.001-in. diam bismuth wire illuminated by 10 cm radiation[4] yield the following value at first resonance,

$$(\sigma_s/\lambda^2)_{\mathrm{ex}}=0.121\pm0.005.$$

The calculated value for this case is

$$(\sigma_s/\lambda^2)_{\mathrm{th}}=0.127,$$

yielding a discrepancy of 5.1%, similar to those obtained in the back scattering results.[4] The scattering cross section of a wire with the same dimensions as above but of perfect conductivity is

$$\sigma_s/\lambda^2=0.515,$$

indicating the correction offered by the variational expression for the scattered amplitude.

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Spontaneous Emission from an Inverted Spin System

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THIS paper summarizes the results of calculations made in an attempt to explain the origin of the observed modulation effects in two-level maser experiments.[1,2]

In a spontaneously oscillating spin system any transverse magnetization, which couples rf power out of the spin system, is due to the presence of a transverse rf field, which is in turn induced by the transverse magnetization. Any consideration of the dynamics of the spin system or the build-up of the radiation field must consequently treat the two systems simultaneously.

Our analysis uses a model first formulated by Bloembergen and Pound,[3] except that our problem is inherently nonlinear[4] and we cannot make the simplifying assumption Mz = const, which linearizes the equations of motion, and are forced to have recourse to an IBM 704.

The physical situation is that of an inverted, homogeneously broadened, magnetization of amplitude M_0 placed inside a microwave cavity which can support a circularly polarized transverse magnetic field. If we take the spin precession frequency γH_0 to be equal to the cavity resonant frequency $(LC)^{-\frac{1}{2}}$, assume that no external power is fed into the cavity, and that the spin lattice relaxation time T_1 is infinite, we can write a set of three nonlinear simultaneous differential equations.

$$\frac{2j\omega}{T_R|M_0\gamma|}M_1+\frac{2}{T_R|M_0\gamma|}\frac{dM_1}{dt}+Q\frac{dH_1}{dt}+\omega H_1=0, \tag{1}$$

$$\frac{dM_1}{dt}-j\gamma M_z H_1+\frac{M_1}{T_2}=0, \tag{2}$$

$$\frac{dM_z}{dt}=\mathrm{Im}[\gamma M_1{}^* H_1]. \tag{3}$$

Equations (1)–(3) appear in a slightly different form in footnote 3. M_1 and H_1 are complex and are defined by

$$M_x+jM_y=M_1e^{j\omega t} \tag{4}$$

and

$$H_x+jH_y=H_1e^{j\omega t}, \tag{5}$$

where M_x, M_y, H_x, and H_y are the transverse components of the magnetization and magnetic field, respectively. γ is the magnetogyric ratio. T_R is the radiation damping time defined by

$$T_R=(2\pi\eta|\gamma M_0|Q)^{-1}.$$

η is the sample filling factor, Q is the loaded "Q" of the cavity. T_2 is the spin-spin relaxation time which for a homogenous Lorentizian line is given by

$$T_2=2/\gamma\Delta H=2/\Delta\omega,$$

where ΔH is the separation in gauss between the half power points. M_1^* is the complex conjugate of M_1. Since M_z, H_1, and M_1 are functions of time. Equations (1)–(3) constitute a set of nonlinear differential equations. The solution was carried out with the aid of an IBM 704. For numerical purposes it was necessary to consider the following set of equations which are the real and imaginary parts of Eqs. (1)–(3)

$$aM_r+b\frac{dM_i}{dt}+Q\frac{dH_i}{dt}+\omega H_i=0, \tag{6}$$

$$-aM_i+b\frac{dM_r}{dt}+Q\frac{dH_r}{dt}+\omega H_r=0, \tag{7}$$

$$\frac{dM_i}{dt}+|\gamma|M_zH_r+CM_i=0, \tag{8}$$

$$\frac{dM_r}{dt}-|\gamma|M_zH_i+CM_r=0, \tag{9}$$

and

$$\frac{dM_z}{dt}+|\gamma|M_rH_i-|\gamma|M_iH_r=0, \tag{10}$$

where

$$M_1=M_r+jM_i,$$

$$H_1=H_r+jH_i,$$

$$a=\frac{2\omega}{T_R|M_0\gamma|}=4\pi\omega\eta Q,$$

$$b=\frac{2}{T_R|M_0\gamma|}=4\pi\eta Q,$$

and

$$c=1/T_2.$$

Since we are dealing with an inverted magnetization, our first boundary condition is $M_z(0)=-M_0$. This constitutes an unstable equilibrium and some mechanism is necessary to trigger the spontaneous emission. This could be either some residual transverse magnetization or some cavity current present at $t=0$. We used the residual current for a trigger and put $H_r(0)=0.1M_0$. The magnitude of $H_r(0)$ was found to have little effect on the subsequent behavior of the magnetization.

The values of ω, Q, and M_0 were chosen to approximate the experimental conditions prevailing at the two-level maser experiment performed by Chester, Wagner, and Castle of Westinghouse.[2] These are

$$M_0=5\times10^{-4}\text{ gauss }(10^{18}\text{ spins at }4.20\text{K}),$$
$$Q=10\,000,$$

and

$$\omega=2\pi\times10^{10}\ (f=10\text{ kMc}).$$

The resulting behavior of H_1^2, which is equal to $H_r^2+H_i^2$, is shown in the lower trace of Fig. 1. H_1^2 is proportional to the power coupled out of the cavity and is to be compared with the experimental observations in which the spontaneous emission power following inversion was monitored.[1,2] The upper trace shows the oscillatory return of M_z to its equilibrium, $M_z=+M_0$, value. The peaks of M_z coincide with these of H_1^2 as required by conservation of energy considerations. It was found that as long as $T_r<T_2$ which is the start oscillation condition for a two-level maser oscillator, T_2 had a negligible effect on the shape of the curves and could be assumed to be infinite. The behavior of H_1^2 bears a close resemblance to the experimental results—especially those of the Westinghouse Group.[2] A significant difference remains—the time interval between power peaks in Fig. 1 is $\sim$0.1 μsec while the observed results give $\sim$1 μsec. This difference may be due to the actual inhomogenous nature of the line, in contrast to the homogenous line assumed in our analysis. The envelope curve, traced through the successive oscillation peaks, decays with a characteristic time corresponding to the cavity Q, i.e.,

$$T_{\text{envelope}}\approx Q/\omega.$$

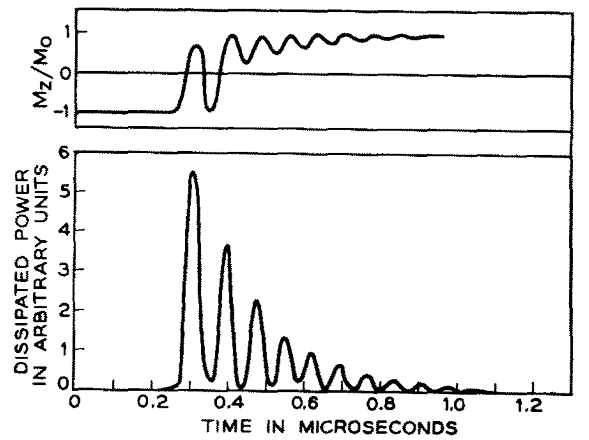

FIG. 1. Upper trace: $M_z(t)/M_0$ vs t following spin inversion at $t=0$. $M_0=5\times10^{-4}$. Lower trace: $(H_i^2+H_r^2)/M_0^2$ which is proportioned to the power coupled out of the cavity is shown as a function of the time following the spin inversion, $M_0=5\times10^{-4}$.

This feature is retained regardless of the magnitude of M_0. The interval between peaks depends on M_0. This dependence is not very strong and using a value of M_0 which is smaller by a factor of 15 than that used in Fig. 1 yielded a value of $\sim$0.3 μsec which is closer to the experimentally observed values.

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Backscattering of Carbon-14 Beta Radiation

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IN connection with studies of radiostearic acid monolayers,[1] we have been concerned about the backscattering of C^{14} β-particles by various substrates. It is commonly stated[2] that the percentage saturation backscattering of β-particles is independent of β-energy for radiation of sufficiently high energy ($E_{max}>0.6$ Mev). Zumwalt[3] has shown that the same saturation backscattering results are obtained with Geiger-Müller counters for radiation of lower energy if the observed results are corrected for air- and window-absorption of the scattered radiation. Glendenin and Solomon[4] have pointed out that the same sort of correction brings results for S^{35} β-particles (0.17 Mev) into line with those for more energetic radiation.

It is reasonable to suppose that the same results would be obtained with C^{14} (0.15 Mev), but we have found no reported experimental verification of this. Because of the considerable importance of this isotope in chemical and biological studies, it seems desirable to record the results which we have obtained. Yaffe and Justus[5] have reported some results for C^{14} β-radiation with Geiger-Müller counters. Beischer[6] has found that autoradiographic observations lead to similar backscattering results for C^{14} and more energetic radiation. In the present experiments we have compared the backscattering of β-radiation from C^{14} and Tl^{204} (0.76 Mev) with a mica end-window Geiger-Müller counter under conditions of similar geometry, and have measured the absorption (in aluminum) of the backscattered radiation.

The sources were evaporated deposits of C^{14} labelled stearic acid ($\sim$10 mC/g) and $TlNO_3$ ($\sim$50 mC/g) on vinylite films of