$$\left| \frac{\partial \Theta_h(0)}{\partial u_s} - i\pi q_s^h \Theta_h(0) \right| < n \sum m_s \exp - nw \sum_j (m_j + c_j^h)^2$$

which I say $\longrightarrow 0$. Much as above it comes down to showing that

$$n \sum_{m=1}^{+\infty} me^{-nwm} \longrightarrow 0.$$

As $m < e^m$, this sum may be replaced by

$$n\sum e^{-(nw-1)m} < \frac{n}{e^{nw} - e} \longrightarrow 0.$$

Hence the limit of our expression is $i\pi q_s^h$.

3. Referring then to the matrix (1), among its determinants of order $p + 1$ there is one whose limit is

$$(i\pi)^p \begin{vmatrix} l \\ q_s^h \end{vmatrix}, \ (h = 1, 2, \ldots p + 1; s = 1, 2, \ldots p).$$

Since after all the q's are arbitrary integers, we may so choose them that this determinant $\neq 0$. This proves our theorem.

[1] See the Encyklopädie article II B 7, Abelsche Functionen, by Krazer-Wirtinger, also the forthcoming bulletin of the National Research Council; selected topics in algebraic geometry, Ch. 17, §1.

THE DIRECTION OF EJECTION OF X-RAY ELECTRONS

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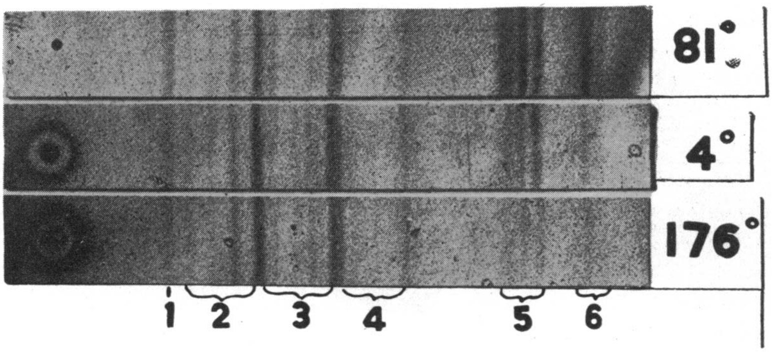
No conclusive evidence that the direction in which photo-electrons are ejected by X-rays depends in any way upon the nature of the atom from which the ejection takes place has as yet been brought forward. Auger,[1] using the C. T. R. Wilson cloud expansion-chamber method, showed that the most probable direction of ejection in a gas is a function of the frequency of the incident X-rays, but the variations which he found in this most probable direction with the nature of the gas used (oxygen or nitrogen, argon, krypton, xenon) were probably less than the experimental error, particularly as heterogeneous X-rays were used and the frequency of the X-rays which were most effective in ejecting electrons may have varied from gas to gas. Loughridge[2] concluded that the most probable direction of ejection was the same for water-vapor, air and argon, but the absorption energies of even the K-shells of all these atoms is so small that at best only a small effect would be expected in these cases. Bothe[3] using the point-discharge ion-counter made observations on air and on gases the molecules of which contained I, Br and Cl atoms. His results show small variations with the nature of the gas, but again as heterogeneous X-rays were used and his maxima were not sharp the variations were probably less than the experimental accuracy.

If the electron orbits inside the atom, postulated by Bohr with such amazing success, have physical reality, then it is to be expected that the momentum of the electron in these orbits will have some effect upon the direction it will be moving when removed from the atom. In fact, all the attempts which have been made to take the momentum of the electron in its atomic orbit into account in a theory of the direction of ejection of photo-electrons have led to the conclusion that (for circular orbits at least) the most probable direction of ejection of an electron whose absorption energy (i.e., the energy necessary to remove it to infinity) is only a little less than the energy of the incident quantum will be greatly different (as much as 90°) from the most probable direction of ejection of an electron whose absorption energy is negligible compared to the energy of the incident quantum (see theories of Auger and Perrin[4] and of Bothe[5]). That this is not the case for electrons ejected from K-orbits has been proved by Auger[1] who found that with 45 k.v. X-rays the most probable direction of ejection of the K-electrons of xenon (whose absorption energy is 35 k.v.) was not appreciably different from that of the K-electrons of oxygen or nitrogen (whose absorption energy is negligible). In their recent theory of the space-distribution of X-ray electrons Auger and Perrin[4] point out that this constitutes a very serious difficulty for the old conception of electronic orbits and attempt to evade it by making use of the new wave-mechanics which permits the moment of momentum of the K-electron about the nucleus to be put equal to zero. The difficulty cannot be evaded in this way, however, as the writers have recently obtained evidence which proves that the most probable direction of ejection of the L- as well as the K-electrons does not change as the absorption energy increases. The purpose of this paper is to present this evidence.

The method which was used so successfully by de Broglie, Whiddington and Robinson, to obtain magnetic spectra of the electrons ejected by X-rays, has recently been modified by one of us[6] in such a way as to make it possible to study the velocity and number of the electrons as a function of the angle of ejection. Magnetic spectra taken in this manner of electrons ejected from exceedingly thin metallic films show that the most probable direction of ejection is a little forward of perpendicular to the direction of the X-ray beam and that this most probable direction is approximately the same for all electrons whether from the K-, L-, M- or N-levels and whether the absorption energy of the level is large or small. This is well shown in figure 1, in which magnetic spectra of the electrons emerging at three different angles from a thin film of gold traversed by primary X-rays from molybdenum are reproduced. The lines are all relatively strong in the spectrum taken at 81°, i.e., a little forward of perpendicular to the X-ray beam. At 4° (measured from the forward direction of the X-ray beam) all the lines, except those due to the fluorescent radiation of the gold itself which should be the same in all directions, are much weaker and at 176° they have practically disappeared. It is to be noted espe-



Magnetic spectra of electrons emerging at various angles from a thin sputtered film of gold traversed by primary x-rays from molybdenum

I. L_{III} electrons ejected by MoK_{α}
2. M electrons ejected by AuL_{α_1}
3. M electrons ejected by AuL_{β_1}
4. M electrons ejected by AuL_{γ_1}
5. M electrons ejected by MoK_{α}
6. N&O electrons ejected by MoK_{α}

FIGURE 1

cially that the intensity of the line numbered 1, which is due to electrons from the circular L-orbit whose absorption frequency is 289×10^{16} as compared with a frequency of 421×10^{16} for the incident X-rays, falls off as the angle departs from 80° in practically the same way as do the intensities of the lines numbered 5 and 6 which are due to electrons from the outer orbits whose absorption frequencies are small.

The theory of Auger and Perrin demands that the most probable direction of ejection of the L_{III} electrons of gold by the K_{α} lines of molybdenum be in the neighborhood of 35°. A number of spectra of the L_{III} line have been taken at small intervals between 0° and 90° and it is certain that the intensity is greatest close to 80° and that it falls off regularly as the angle departs therefrom.

We conclude, therefore, that the momentum of the electron in its orbit within the atom does not affect the direction of ejection in the way demanded by the theory of Auger and Perrin. While this does not constitute definite evidence against the physical reality of electronic orbits, it is a serious difficulty for the conception.

A detailed discussion of the general question here raised in the light of new experimental results will be published elsewhere.

[1] Auger, P., *J. Physique Rad.*, **8,** 1927 (85–92).

[2] Loughridge, D. H., in press.

[3] Bothe, W., *Zs. Physik,* **26,** 1924 (59–73).

[4] Auger, P., and Perrin, F., *J. Physique Rad.*, **8,** 1927 (93–111).

[5] Bothe, W., *Zs. Physik,* **26,** 1924 (74–84).

[6] Watson, E. C., in press.

THE CHARACTER OF THE GENERAL, OR CONTINUOUS SPECTRUM RADIATION

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The impacts of electrons against atoms produce two different kinds of radiation—the general, or continuous spectrum and the line spectra. By far the greatest amount of energy radiated belongs to the general, or continuous spectrum and not to the line spectra. In the X-ray region, in particular, the continuous, or general radiation spectrum carries most of the radiated energy. In spite of this, and because the line spectra have an important bearing on atomic structure, the general radiation has received much less attention than the line spectra in the recent development of fundamental ideas in physical science. The experiments described in this note deal with the general radiation and not the line spectra.

Some time before the appearance of X-ray spectrometers in research work carried on in this country the writer performed a number of experiments on the general X-radiation. The object of these experiments was to determine whether the impacts of electrons, all of which had practically the same velocity when they struck the target of an X-ray tube, would produce homogeneous, monochromatic radiation. The experiments showed that the general radiation coming from a tube operated at a constant voltage was by no means homogeneous. The advent of the Bragg's X-ray spectrometer enabled us to show the now well-known fact